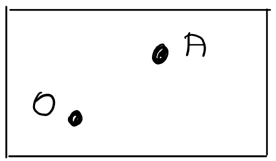
CINEMATICA 2D

Goal: moto dei corpi su un piano → punti

1. Sistema di coordinate



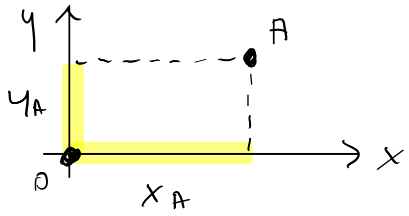
a. punto di riferimento ≡ origine

b. coppia di assi (o direzioni orientate)

c. procedura per far corrispondere A a a. e b.

Coordinate cartesiane:

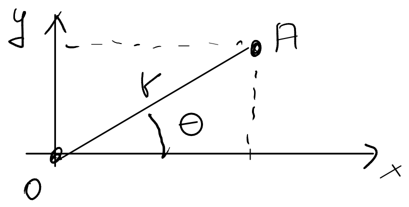
assi x e y ⊥ e fissi



$\begin{cases} x_A \equiv \text{distanza proiezione di A su asse x rispetto O} \\ y_A \equiv \qquad -\text{''}- \qquad y \qquad -\text{''}- \end{cases}$

\+ segno + se dal lato della freccia e − viceversa

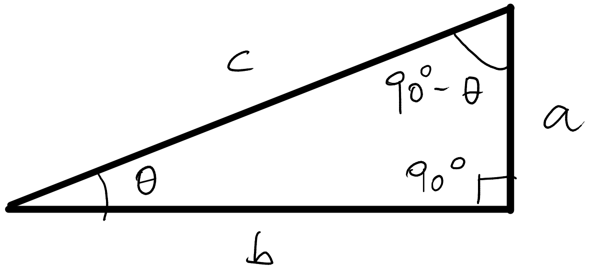
Coordinate polari:



$\begin{cases} r \equiv \text{distanza tra O e A} > 0 \\ \theta \equiv \text{angolo compreso tra } \overline{OA} \text{ e asse x} \end{cases}$

$$\begin{cases} x_A = r\cos\theta \\ y_A = r\sin\theta \end{cases} \leftrightarrow \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \arctan(y/x) \end{cases}$$

Funzioni trigonometriche:

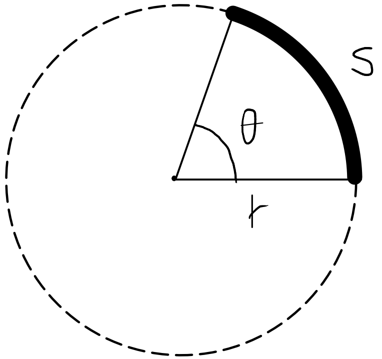


$$\begin{cases} \text{sen}\,\theta \equiv \dfrac{a}{c} \\ \cos\theta \equiv \dfrac{b}{c} \\ \tan\theta \equiv \dfrac{a}{b} \end{cases} \Rightarrow \begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \tan\theta = \dfrac{\sin\theta}{\cos\theta} \end{cases}$$

Radiante:

unità di misura dell'angolo

$$\theta \equiv \frac{s}{r} \qquad [\theta] = 1$$



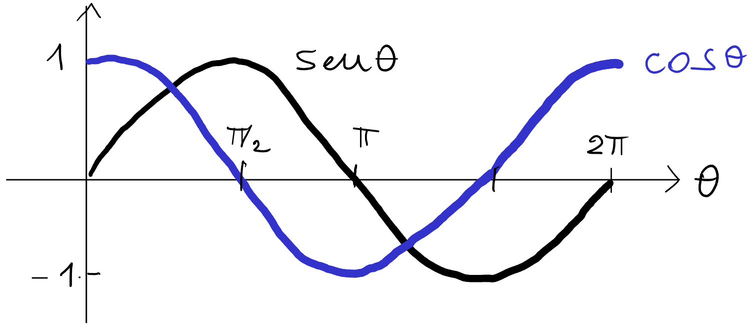
$\theta = 360°$ $\quad s = 2\pi r$ $\quad \theta = 2\pi$

$\theta = 180°$ $\quad s = \pi r$ $\quad \theta = \pi$

$\theta = 90°$ $\quad s = \frac{\pi}{2} r$ $\quad \theta = \frac{\pi}{2}$

...

$\theta = 1°$ $\quad s = \dfrac{2\pi r}{360}$ $\quad \theta \approx 0.0174$



Angoli notevoli

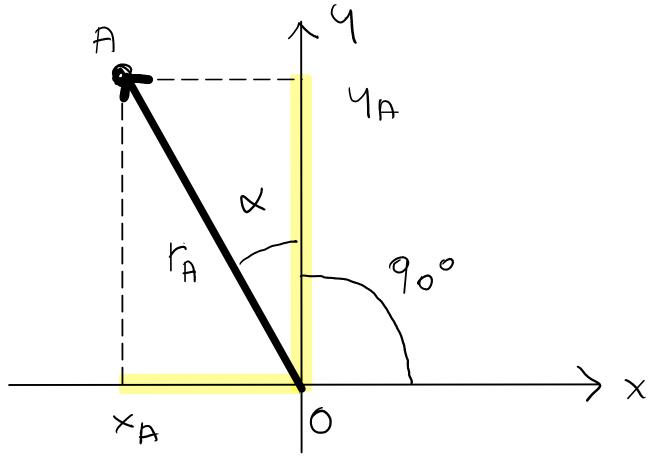
30°, 45°, 60°, 90°

π/6, π/4, π/3, π/2

	0°	30°	60°	90°
sen	0	$1/2$	$\sqrt{3}/2$	$1/\sqrt{2}$
cos	1	$\sqrt{3}/2$	$1/2$	$1/\sqrt{2}$
tan	0	$1/\sqrt{3}$	$\sqrt{3}$	1

Esempi:

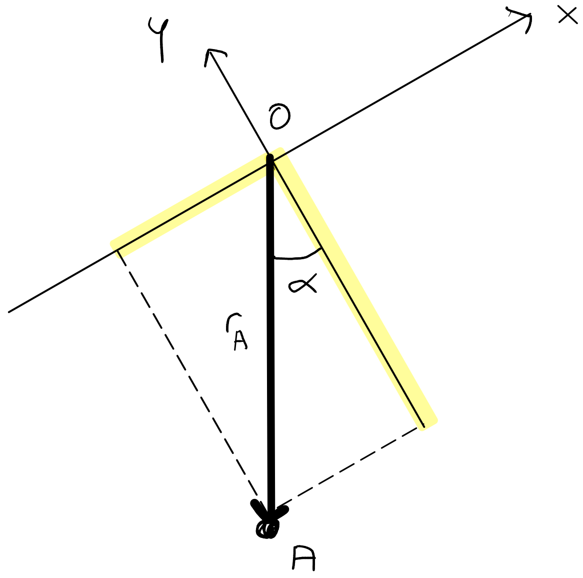
a)



$$\begin{cases} x_A = r_A \cos(90° + \alpha) \\ y_A = r_A \sin(90° + \alpha) \end{cases}$$

$$\begin{cases} x_A = -r_A \sin\alpha \\ y_A = r_A \cos\alpha \end{cases}$$

b)



$$\begin{cases} x_A = r_A \cos(270° - \alpha) \\ y_A = r_A \sin(270° - \alpha) \end{cases}$$

$$\begin{cases} x_A = -r_A \sin\alpha \\ y_A = -r_A \cos\alpha \end{cases}$$

2) Vettori

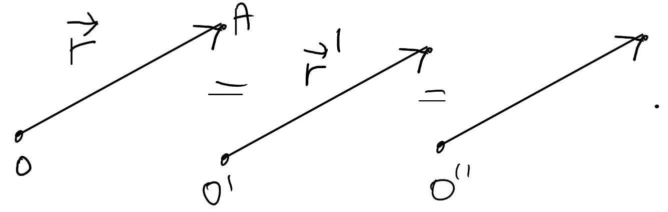
Grandezze fisiche descritte da numero reale in unità appropriate → distanza, massa, densità
→ SCALARI

Posizione punto sul piano: $(x,y) \in \mathbb{R}^2$, più in generale $\mathbb{R}^M$
→ VETTORI

$[\,\bar{r},\ \underline{r},\ \underset{\sim}{r},\ \mathbf{r}\,]$

Definizione geometrica

- direzione → $// \ \overline{OA}$
- verso → freccia
- modulo → lunghezza segmento $\overline{OA}$



punto di applicazione non conta!

modulo $|\vec r|$ $[\,\|\vec r\|\,]$

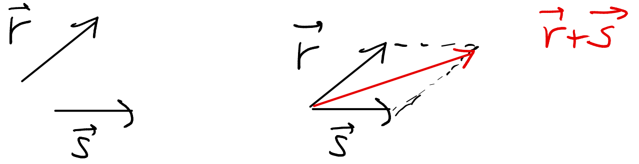
* Uguaglianza: $\vec r = \vec r\,'$

* Opposto: $-\vec r$ → invertendo il verso di $\vec r$

Operazioni

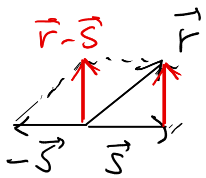
- moltiplicazione per scalare: $\vec r\,' = a\vec r$, $a \in \mathbb{R}$

- somma tra vettori: $\vec r, \vec s$



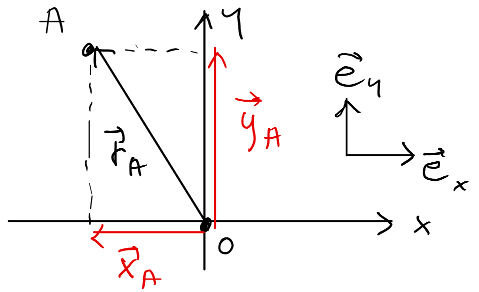
regola del parallelogramma

- differenza : $\vec{r} - \vec{s}$



- commutativa : $\vec{r} + \vec{s} = \vec{s} + \vec{r}$
- associativa : $\vec{r} + (\vec{s} + \vec{t}) = (\vec{r} + \vec{s}) + \vec{t}$
- distributiva : $a(\vec{r} + \vec{s}) = a\vec{r} + a\vec{s}$, $a \in \mathbb{R}$

Definizione in componenti



$\vec{r}_A = \vec{x}_A + \vec{y}_A$ ← vettori componenti

VERSORI : 2 vettori orientati come gli assi cartesiani di modulo unitario

$|\vec{e}_x| = 1$, $|\vec{e}_y| = 1$

$\vec{x}_A = x_A \vec{e}_x$, $\vec{y}_A = y_A \vec{e}_y$

$x_A, y_A \in \mathbb{R}$ componenti cartesiane

$\vec{r}_A = x_A \vec{e}_x + y_A \vec{e}_y$ → espresso nella base $(\vec{e}_x, \vec{e}_y)$

$[\vec{A} = A_x \vec{e}_x + A_y \vec{e}_y]$

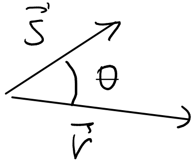
Modulo : $|\vec{r}_A| = \sqrt{x_A^2 + y_A^2}$

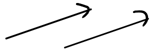
Somma: $\vec{r}_A = x_A \vec{e}_x + y_A \vec{e}_y$, $\vec{r}_B = x_B \vec{e}_x + y_B \vec{e}_y$

$\vec{r}_C = \vec{r}_A + \vec{r}_B = (x_A + x_B)\vec{e}_x + (y_A + y_B)\vec{e}_y$

$x_C = x_A + x_B$, $y_C = y_A + y_B$

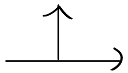
- Prodotto scalare : $\vec{r} \cdot \vec{s} = |\vec{r}| \cdot |\vec{s}| \cdot \cos\theta \rightarrow \cos\theta = \dfrac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|}$





$\theta = 0$

$\vec{r} \cdot \vec{s} = |\vec{r}||\vec{s}|$



$\theta = 90°$

$\vec{r} \cdot \vec{s} = 0$

$\vec{r} \cdot \vec{s} = r_x s_x + r_y s_y$

$|\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}} = \sqrt{r_x^2 + r_y^2}$

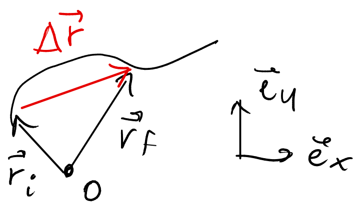
<u>Analisi dimensionale</u> : ha senso parlarne per un vettore?

$\vec{r} = x\,\vec{e}_x + y\,\vec{e}_y \qquad \vec{e}_x = 1 \cdot \vec{e}_x + 0 \cdot \vec{e}_y$

$[x] = L \quad [y] = L$

adimensionali

Cinematica sul piano



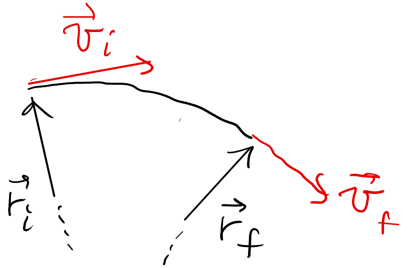
Vettore posizione: $\vec{r} = x\,\vec{e}_x + y\,\vec{e}_y$

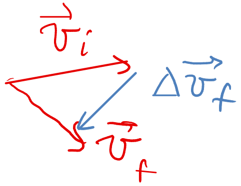
Spostamento: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

Velocità media: $\vec{v}_m \equiv \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{1}{\Delta t}\cdot\Delta\vec{r} = \dfrac{1}{\Delta t}\left(\Delta x\,\vec{e}_x + \Delta y\,\vec{e}_y\right) = \dfrac{\Delta x}{\Delta t}\vec{e}_x + \dfrac{\Delta y}{\Delta t}\vec{e}_y$

Velocità istantanea: $\Delta t \to 0$

$$\vec{v} \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y$$





Accelerazione media

$$\vec{a}_m \equiv \frac{\Delta\vec{v}}{\Delta t}$$

Accelerazione istantanea

$$\vec{a} \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\vec{e}_x + \frac{d^2y}{dt^2}\vec{e}_y$$

Moto rettilineo uniforme

$$\vec{v} = \text{cost} = v_x\vec{e}_x + v_y\vec{e}_y = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y$$

$$\begin{cases}\dfrac{dx}{dt} = v_x \\[2mm] \dfrac{dy}{dt} = v_y\end{cases} \Rightarrow \begin{cases} x = v_x(t-t_i) + v_{xi} \\ y = v_y(t-t_i) + v_{yi}\end{cases} \Rightarrow \vec{r} = \vec{v}(t-t_i) + \vec{r}_i$$

Moto unif accelerato

$\vec{a} = \text{cost}$ ....

$$\vec{r} = \frac{1}{2}\vec{a}(t-t_i)^2 + \vec{v}_i(t-t_i) + \vec{r}_i$$