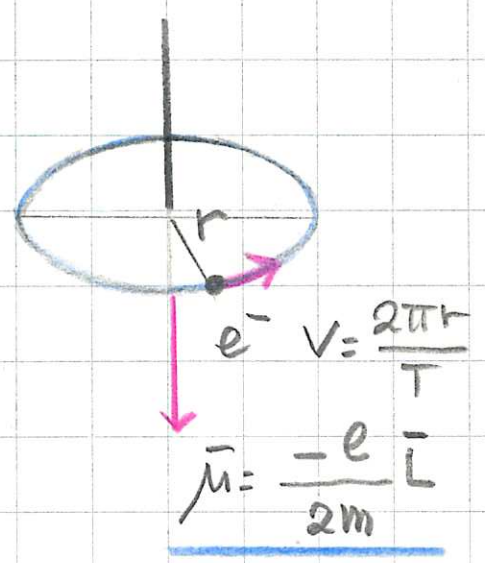
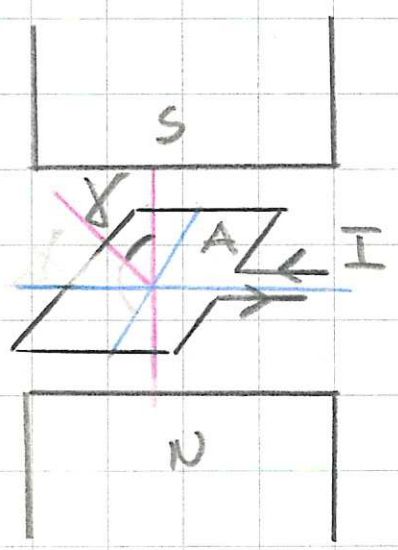


$|\vec{\mu}| = I \cdot A$  [ $A m^2$ ] WHERE  $I$  IS THE CURRENT AND  $A = \Delta \text{AREA}$ . AS WE KNOW FROM D. GRIFFITH - INTRO DUC. ... -  $\vec{\mu} \perp$  TO THE PLANE OF THE LOOP. WE CAN ALSO DEFINE THE TORQUE  $\vec{\tau} = \vec{\mu} \times \vec{B}$ . THE MAGNETIC POTENTIAL ENERGY OF THE DIPOLE IS

(45)  $V_{MAG} = -\vec{\mu} \cdot \vec{B} = \int_{\pi/2}^{\alpha} \tau d\alpha = -|\vec{\mu}| |\vec{B}| \cos \alpha$

WHERE  $\alpha$  IS THE ANGLE BETWEEN  $\vec{\mu}$  AND  $\vec{B}$  FOR



IN ATOMIC AND NUCLEAR PHYSICS THE  $\vec{\mu}$  IS OFTEN DEFINED AS THE TORQUE IN A UNIFORM AND STATIONARY  $|\vec{H}|$  FIELD (NOT  $|\vec{B}|$ )  $\Rightarrow$

(46)  $\vec{\tau} = \vec{\mu}' \times \vec{H}$   $|\mu'| = \mu_0 I A$ .

THIS CLASSICAL DESCRIPTION, ALTHOUGH  $\vec{L}$  IN THE QUANTUM FORMALISM SHOULD BECOME A QUANTUM OPERATOR  $\hat{L}$  (WE WILL SEE THIS SOON), IS MISSING AN ESSENTIAL PART THAT CANNOT BE DERIVED BY

ANY CLASSICAL MODEL: THE SPIN. WE WILL DISCUSS THIS QUANTUM PROPERTIES OF THE PARTICLES IN GENERAL (REMEMBER THAT THE SPIN IS ALSO A PROPERTIES OF NEUTRAL PARTICLES SUCH AS THE NEUTRON, ATOMS ETC. AND IT CAN BE AN INTEGER: 0, 1, 2... OR FRACTIONAL  $\frac{1}{2}, \frac{3}{2} \dots$ ). THE SPIN IS ORIGINATED BY AN INTRINSIC ANGULAR MOMENTUM TO WHICH CORRESPOND AN INTRINSIC MAGNETIC MOMENT.

SINCE  $\hat{L} \approx \hbar$  SCALE OF  $\mu$  SET BY THE BOHR MAGNETON  $\mu_B = \frac{e\hbar}{2m_e}$  ( $\mu = \frac{e\hbar}{2m_e c}$  IN GAUSS) (47)

OBSERVATION: (REMEMBER THAT IN ATOMIC PHYSICS  $\mu_B$  IS A PHYSICAL CONSTANT ASSUMED AS THE NATURAL UNITS FOR EXPRESSING THE MAGNETIC MOMENT OF AN  $e^-$  CAUSED BY, EITHER ITS ORBITAL OR SPIN ANGULAR MOMENTUM.)

SO FOR THE  $e^-$  OF AN HYDROGEN ATOM IN A UNIFORM  $\vec{B} = B\hat{z}$  IS  $\hat{H} = \hat{H}_0 + \frac{e}{2m} B \hat{L}_z$ , WHERE  $\hat{H}_0 =$

$$\frac{\hat{p}^2}{2m} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad (48)$$

SINCE  $[\hat{H}_0, \hat{L}_z] = 0$ , THE EIGENSTATES OF THE UNPERTURB HAMILTONIAN ( $\hat{H}_0$ ) OF  $\hat{H}$  DEFINED BY  $\psi_{n, l, m}(r)$  REMAIN EIGENSTATES OF  $\hat{H}$ .

WITH EIGENVALUES

$$E_{n, \ell, m} = -\frac{1}{n^2 R_y} + \hbar \omega_L m$$

(49)

WHERE  $\omega_L = \frac{eB}{2m}$  IS THE LARMOR FREQUENCY

AND  $R_y$  IS THE RYDBERG UNIT ENERGY

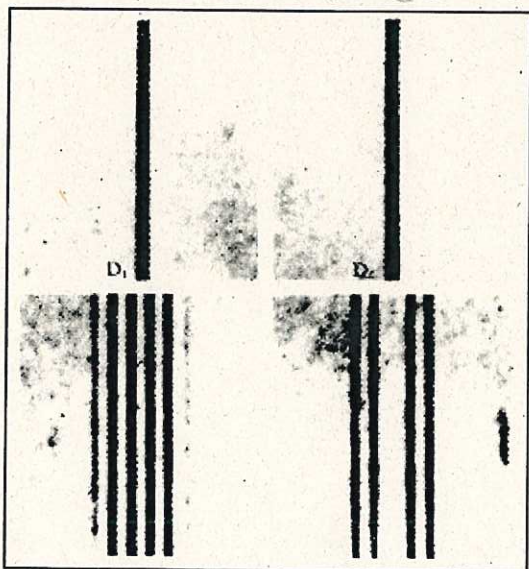
• OBSERVATION  $1 R_y = h c R_{\infty} = \frac{m e^4}{4 \epsilon_0^2 \hbar^2}$

(FOR H ATOM  $R_{\infty} \approx R_H$  SINCE  $R = R_{\infty} \frac{m_p}{m_e + m_p}$

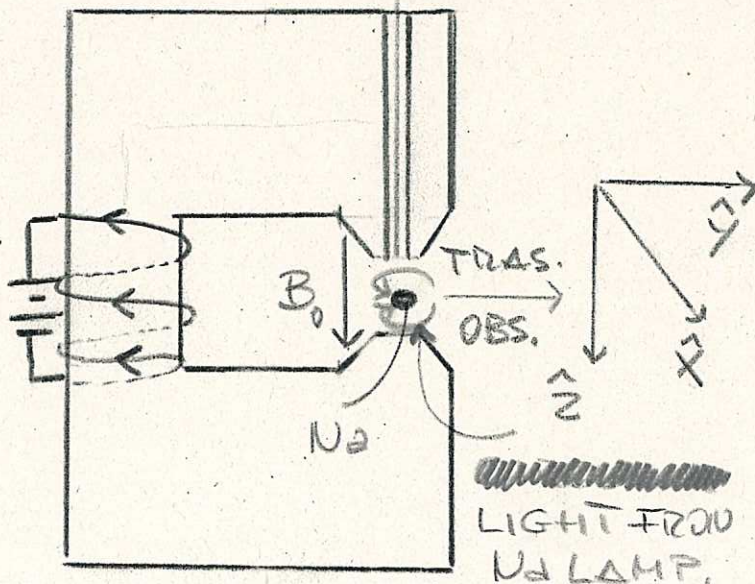
BEING  $m_p$  THE PROTON MASS).

WITHOUT SPIN CONTRIBUTION AN ATOM IN A UNIFORM MAGNETIC FIELD, THE  $(2\ell + 1)$ -FOLD DEGENERACY IS LIFTED AND THE ELECTRONIC LEVELS SPLIT (SEPARATE) BY AN ENERGY  $\hbar \omega_L$

Na ABSORPTION  
Na VAPOUR LAMP



LONGITUDINAL  
OBSERVATION



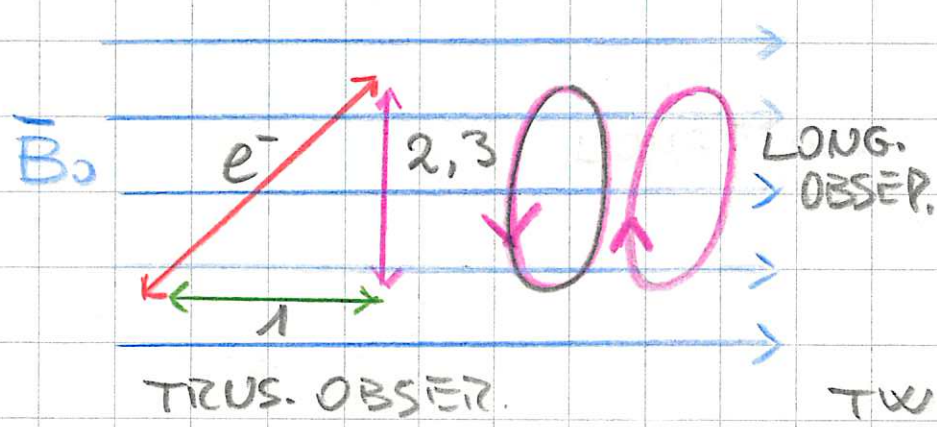
BUT THE MODEL WE DEVELOPED SO FAR CAN NOT EXPLAIN THE ZEEAMAN EXPERIMENT FOR THE SPIN IS MISSING.

WHAT IS THE SPIN? WE WILL SEE THIS LATER IN THE COURSE.

HOWEVER HERE WE CAN TRY TO GIVE AN EXPLANATION USING CLASSICAL E.D. OF THE PHENOMENOLOGY DESCRIBING WHAT IS KNOWN AS ORDINARY ZEEAMAN EFFECT. IT IS IMPORTANT TO CLARIFY THAT THERE IS ALSO ANOMALOUS ZEEAMAN EFFECT (ACTUALLY MOST OF THE ZEEAMAN EFFECTS ARE ANOMALOUS). THIS HAPPEN WHEN THE ANGULAR MOMENTUM AND THE MAGNETIC MOMENT OF THE ORBIT AND SPIN ( $J = L + S$  OR  $J = L + S$ ) CANNOT JUST BE DESCRIBED BY ONE OF THE TWO QUANTUM NUMBER  $L$  OR  $S$  BUT ARE DETERMINED BY BOTH. THIS IS THE GENERAL CASE IN WHICH ATOMIC MAGNETISM IS DUE TO THE SUPERPOSITION OF SPIN AND ORBITAL MAGNETISM. OTHERWISE THE ZEEAMAN EFFECT IS ORDINARY, THIS IS THE CASE CONSIDERED AND EXPLAINED BY LORENTZ WITH CLASSICAL E.D.

IN THIS MODEL WE DISCUSS THE EMISSION OF LIGHT BY AN ELECTRON WHOSE MOTION ABOUT THE NUCLEUS IS INTERPRETED AS AN OSCILLATION SO WE CAN CONSIDER THE PROJECTION

# OF THE MOTION ON THE DIRECTIONS OF A SUITABLE REFERENCE FRAME



ANY CIRCULAR MOTION CAN BE REDUCED TO THE COMBINATION OF TWO OSCILLATIONS

THEREFORE, IN THE MODEL WE HAVE THE

- COMPONENT 1  $\parallel \vec{B}_0$ . THIS EXPERIENCES NO FORCES ( $\vec{v} \parallel \vec{B}_0$ )  $\Rightarrow$  ITS FREQUENCY REMAINS UNCHANGED.
- THE CIRCULARLY OSCILLATING COMPONENTS 2, 3 ARE ACCELERATED OR SLOWED DOWN BY THE EFFECT OF THE MAGNETIC INDUCTION TURNING ON AND OFF  $\vec{B}_0$  DEPENDING ON THEIR DIRECTION OF MOTION. THE  $\omega$  FREQUENCIES ARE INCREASED OR DECREASED BY

(50) 
$$\Delta\omega = \pm \frac{1}{2} \left( \frac{e}{m_0} \right) B_0 \quad (\text{Q.M. TERMS})$$

$$\Delta\omega = \pm \left( \mu_B / \hbar \right) B_0.$$
 CLASSICALLY WE CAN

CALCULATE THE FREQUENCY SHIFT  $\Delta\omega$  FOR THE OSCILLATOR COMPONENTS AS FOLLOWS. WITH OUT THE MAGNETIC FIELD,  $\omega$  OF THE  $e^-$  IS  $\omega_0$  (NO SHIFT). THE COULOMB FORCE AND THE CENTRIFUGAL FORCE ARE BALANCE OF COURSE IN THE CLASSICAL MODEL WE

NEGLECT THE RADIATION EFFECTS, OTHERWISE THE  $e^-$  WILL HAVE A SPIRAL MOTION COLLAPSING ON THE NUCLEUS  $\Rightarrow$

$$m \omega_0^2 r = \left( \frac{Z e^2}{4\pi \epsilon_0 r^2} \right) r \quad (51)$$

IN A HOMOGENEOUS  $\vec{B}_0 = B_0 \hat{z}$  THE LORENTZ FORCE ACTS IN ADDITION ACCORDINGLY WITH THE FOLLOWING MAXWELL EQS.

$$\begin{aligned}
 (52) \quad & m \ddot{x} + m \omega_0^2 x - e \dot{y} B_0 = 0 \quad (3) \text{ FROM THE} \\
 & m \ddot{y} + m \omega_0^2 y + e \dot{x} B_0 = 0 \quad (2) \text{ MOTION} \\
 & m \ddot{z} + m \omega_0^2 z = 0 \quad (1) \text{ DECOMP.}
 \end{aligned}$$

FROM 1  $z = z_0 e^{i\omega_0 t} \Rightarrow \omega_0$  REMAINS UNCHANGED, (2) AND (3) CAN BE SOLVED BY SUBSTITUTING  $u = x + iy$ ;  $v = x - iy$ , THE EQS, WITH THE APPROXIMATION  $e B_0 / 2m \ll \omega_0$ , GIVE THE SOLUTIONS

$$\begin{cases}
 u = u_0 \exp [i(\omega_0 - e B_0 / 2m)t] \\
 v = v_0 \exp [i(\omega_0 + e B_0 / 2m)t]
 \end{cases} \quad (53)$$

THESE ARE THE EQS OF MOTION FOR A LEFT HAND AND RIGHT HAND CIRCULAR MOTION WITH FREQUENCIES  $\omega_0 \pm \delta\omega$  WITH  $\delta\omega = e B_0 / 2m$ . THE  $e^-$  OSCILLATOR COMPONENT

(2) AND (3) THUS EMIT OR ABSORB CIRCULARLY POLARIZED LIGHT WITH  $\omega = \omega_0 \pm \delta\omega$ . THE SPLITTING OF THE SPECTRAL LINES

OBSERVED IN THE ORDINARY ZEEHMAN EXPERIMENT ARE ALSO PREDICTED BY THE CLASSICAL E.D. FOR EXAMPLE WITH  $B_0 = 1 \text{ T}$   $\bar{\nu} \approx 1.4 \cdot 10^{10} \text{ s}^{-1}$   
 $\Rightarrow \approx 0.465 \text{ cm}^{-1} \approx 2.15 \cdot 10^7 \text{ nm}$  IN THE FAR IR REGION  $\approx 5.76 \cdot 10^{-5} \text{ eV}$ . IN THIS CASE THE CLASSICAL ED PREDICTIONS AGREE WITH THE EXPERIMENTAL PHENOMENOLOGY IN PARTICULAR  $\rightarrow$  LINEAR POLARIZED COMPONENT (1) HAS THE CHARACTERISTIC OF A HERTZIAN DIPOLE OSCILLATOR  $\parallel \bar{B}_0$ . THE  $\bar{E}$  VECTOR OF THE EMITTED RADIATION IS  $\parallel \bar{B}_0$  AND THE INTENSITY OF THE RADIATION IS ZERO IN THE  $\bar{B}_0$  DIRECTION. THIS UNSHIFTED ZEEHMAN COMPONENT IS CALLED  $\pi$  ( $\pi$  FOR PARALLEL). THIS IS OBSERVED TRANSVERSALLY. COMPONENTS FROM (2) AND (3) ARE OBSERVED IN THE DIRECTION OF  $\bar{B}_0$  AND ARE CIRCULARLY POLARIZED, BUT LINEARLY POLARIZED OBSERVED  $\perp \bar{B}_0$ . THE CIRCULARLY POLARIZED RADIATIONS ARE CALLED  $\sigma^+$ ,  $\sigma^-$  FOR RIGHT AND LEFT POLARIZATION.

