Moto di un proiettile

Osservazioni empiriche:

- massa non influisce sulla traiettoria
- in presenza di attrito, c'è effetto di massa o diametro
- attrito ha un effetto, ma non cruciale
- gittata aumenta più che proporzionalmente alla velocità iniziale
- gittata massima è per 45°
- altezza massima per 90°

} velocità iniziale fissa

Modello: moto di un punto sul piano con accelerazione costante

- $\vec{a} = \text{cost}$
- oggetto $\equiv$ punto
- trascuro attrito

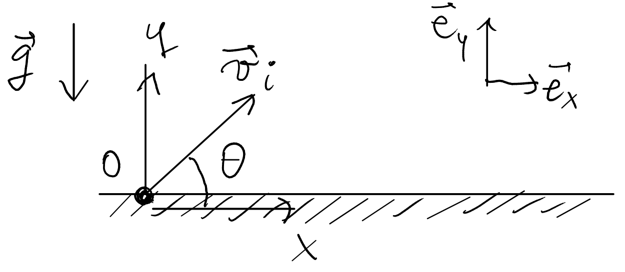
$$\vec{a} = \frac{d^2x}{dt^2}\vec{e}_x + \frac{d^2y}{dt^2}\vec{e}_y = a_x\vec{e}_x + a_y\vec{e}_y = \text{cost}$$

$$\vec{v}_i = v_{ix}\vec{e}_x + v_{iy}\vec{e} \qquad \vec{r}_i = x_i\vec{e}_x + y_i\vec{e}_x$$

$$\begin{cases} \dfrac{d^2x}{dt^2} = a_x \longrightarrow x = \frac{1}{2}a_x(t-t_i)^2 + v_{ix}(t-t_i) + x_i \\ \dfrac{d^2y}{dt^2} = a_y \longrightarrow y = \frac{1}{2}a_y(t-t_i)^2 + v_{iy}(t-t_i) + y_i \end{cases}$$

$$\Rightarrow \vec{r} = \frac{1}{2}a_x(t-t_i)^2\vec{e}_x + v_{ix}(t-t_i)\vec{e}_x + x_i\vec{e}_x + \frac{1}{2}a_y(t-t_i)^2\vec{e}_y + v_{iy}(t-t_i)\vec{e}_y + y_i\vec{e}_y$$

legge oraria del moto $\leftarrow$ $\boxed{\vec{r} = \frac{1}{2}\vec{a}\cdot(t-t_i)^2 + \vec{v}_i\cdot(t-t_i) + \vec{r}_i}$



$\vec a = \vec g = \text{cost} \qquad t_i = 0$

Sistema coordinate, base cartesiana $(\vec e_x, \vec e_y)$

$\vec r = x\,\vec e_x + y\,\vec e_y$, $\vec v_i = v_{ix}\,\vec e_x + v_{iy}\,\vec e_y = |\vec v_i|\cos\theta\,\vec e_x + |\vec v_i|\sin\theta\,\vec e_y$

$\vec r_i = \vec 0 = 0\,\vec e_x + 0\,\vec e_y$, $\vec a = 0\,\vec e_x - g\,\vec e_y$ $\qquad (g = 9{,}81\ m/s^2)$

$$\vec r = -\frac{1}{2} g t^2\,\vec e_y + |\vec v_i|\cos\theta\, t\,\vec e_x + |\vec v_i|\sin\theta\, t\,\vec e_y = x\,\vec e_x + y\,\vec e_y$$

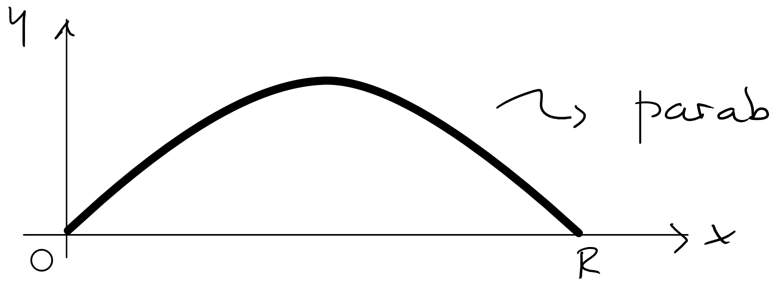
$$\begin{cases} x = |\vec v_i|\cos\theta\, t & \rightarrow \text{ moto rettilineo unif.} \\ y = -\frac{1}{2} g t^2 + |\vec v_i|\sin\theta\, t & \rightarrow \text{ moto uniform. accelerato} \end{cases}$$

Traiettoria: $y = y(x) \rightarrow$ elimino t

$$t = \frac{x}{|\vec v_i|\cos\theta} \rightarrow y = -\frac{1}{2} g \left(\frac{x}{|\vec v_i|\cos\theta}\right)^2 + \cancel{|\vec v_i|}\sin\theta \cdot \frac{x}{\cancel{|\vec v_i|}\cos\theta} = -\frac{1}{2}\frac{g}{|\vec v_i|^2\cos^2\theta}x^2 + \tan\theta\, x$$

$$y = -\frac{1}{2}\frac{g}{|\vec v_i|^2\cos^2\theta}x^2 + \tan\theta\, x$$

$$= x\left(\tan\theta - \frac{1}{2}\frac{g}{|\vec v_i|^2\cos^2\theta}x\right)$$



$\leadsto$ parabola

Gittata

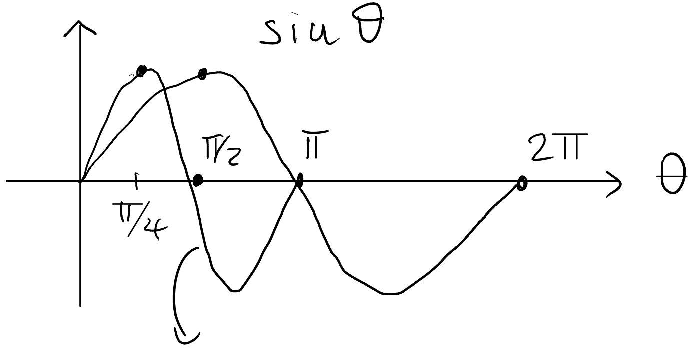
$y = y(x)$ $\qquad$ $\underline{0 = y(R)}$

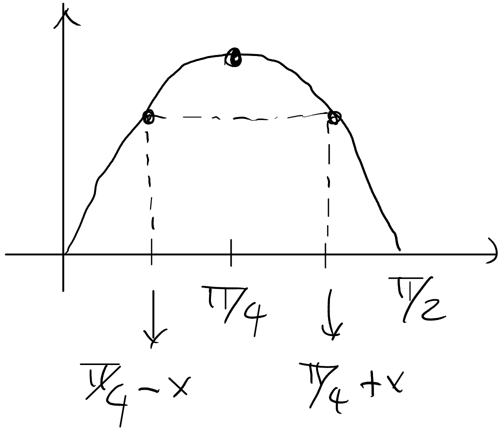
$\cancel{x=0}$, $\tan\theta = \frac{1}{2}\frac{g}{|\vec{v}_i|^2\cos^2\theta}R \Rightarrow R = \frac{2\sin\theta\,|\vec{v}_i|^2\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}\cdot g} = \frac{\overbrace{2\sin\theta\cos\theta}^{\sin(2\theta)}\,|\vec{v}_i|^2}{g}$

$R \sim |\vec{v}_i|^2$ $\boxed{✓}$, $R \sim \frac{1}{g}$, $R \sim \sin(2\theta)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$





- R_{max} se $\theta = \pi/4 = 45°$ $\boxed{✓}$
- R ha una simmetria rispetto a $\theta = \pi/4 = 45°$

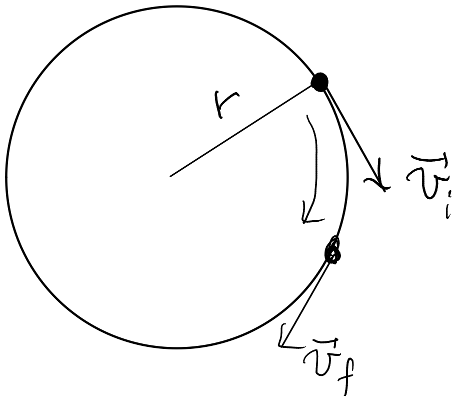
Altezza massima

$v_y = -gt + |\vec{v}_i|\sin\theta$

$v_y(t) \longrightarrow v_y(t^*) = 0\ ? \rightarrow t^* = \frac{|\vec{v}_i|\sin\theta}{g}$

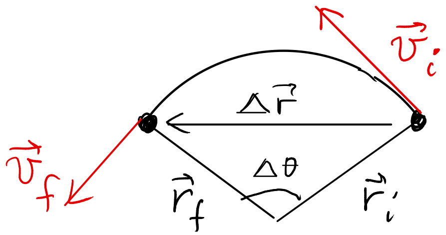
$H = y(t^*) = -\frac{1}{2}\cancel{g}\frac{|\vec{v}_i|^2\sin^2\theta}{g^{\cancel{2}}} + \frac{|\vec{v}_i|^2\sin^2\theta}{g} = \frac{1}{2}\frac{|\vec{v}_i|^2\sin^2\theta}{g}$, $H \sim |\vec{v}_i|^2$, H_{max} se $90°$

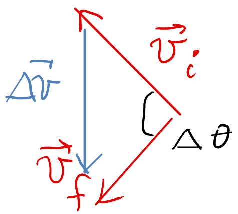
Moto circolare uniforme



Ipotesi: $r = \text{cost}$, $|\vec{v}| = \text{cost}$

Accelerazione media $\vec{a}_m \equiv \dfrac{\Delta \vec{v}}{\Delta t}$





$$\frac{|\Delta \vec{v}|}{|\vec{v}|} = \frac{|\Delta \vec{r}|}{|\vec{r}|}$$

$$\frac{|\Delta \vec{v}|}{\Delta t} = \frac{|\Delta \vec{r}|}{\Delta t} \frac{|\vec{v}|}{r}$$

$$|\vec{a}_m| = \frac{|\Delta \vec{r}|}{\Delta t} \cdot \frac{|\vec{v}|}{r}$$

Limite $\Delta t \to 0$, $\vec{a}_m \to \vec{a}$ $\Rightarrow$ $|\vec{a}| = |\vec{v}| \cdot \dfrac{|\vec{v}|}{r} = \dfrac{|\vec{v}|^2}{r}$ $\leftarrow$ accelerazione centripeta

$$a_c = \frac{v^2}{r}$$

Periodo dell'orbita: $2\pi r = |\vec{v}| \cdot \tau \Rightarrow \tau = \dfrac{2\pi r}{|\vec{v}|}$

Strategia:
1. diagramma
2. modello
3. eq. vettoriale
4. sist. coord.
5. calcolo grandezza
6. interpretazione