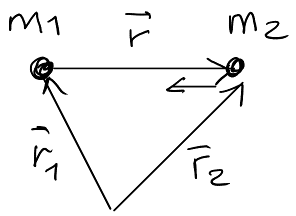
Modello di massa puntiforme.

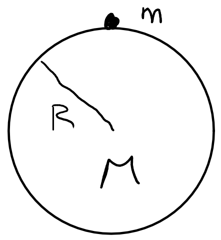
Legge di gravitazione universale



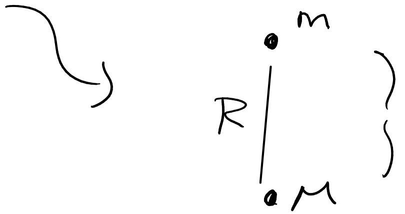
$\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\vec{F}_g^{(2)} = -G \frac{m_1 m_2}{|\vec{r}|^2} \underbrace{\frac{\vec{r}}{|\vec{r}|}}_{\text{versore}}$$

$$\vec{F}_g^{(1)} = -\vec{F}_g^{(2)}$$



Teor. Gauss → interazione equivalente a quella con una massa M concentrata nel centro della sfera



→ modello di massa puntiforme (punto materiale)

Quale tipo di moto non riusciremo a descrivere con qs. modello? → rotazione!

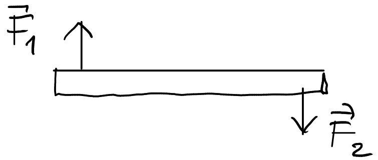


diagramma delle forze

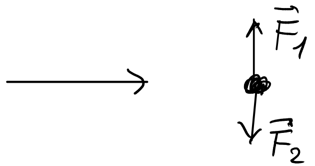
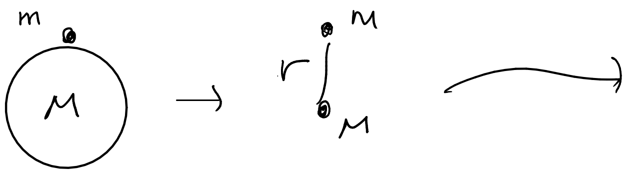
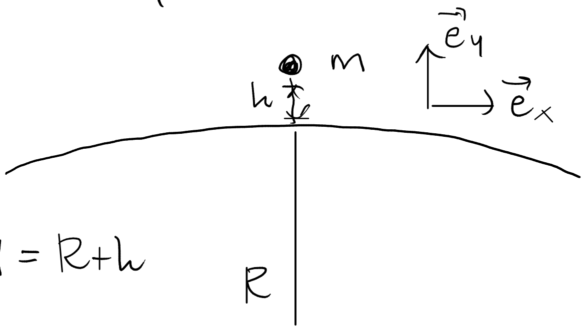


diagramma di corpo libero

Accelerazione di gravità → dipende dall' altitudine e dalla latitudine





$|\vec{r}| = R+h$

$$\vec{F}_g = -G\frac{mM}{(R+h)^2}\vec{e}_y$$

1. $h = 0$

$$\vec{F}_g = -G\frac{mM}{R^2}\vec{e}_y \qquad \text{II Newton: } \boxed{\Sigma\vec{F}} = m\vec{a} \rightarrow \boxed{\vec{F}_g = -m\underbrace{\frac{GM}{R^2}}_{g}\vec{e}_y} = m\vec{a}$$

$$\vec{a} = \vec{g} = -g\vec{e}_y$$

Es: stima della densità della terra (media)

$$g = \frac{GM}{R^2} \Rightarrow M = \frac{gR^2}{G} \Rightarrow \rho = \frac{M}{V} = \frac{gR^2}{\frac{4}{3}\pi R^3 G} = \frac{3g\cancel{R^2}}{4\pi R^{\cancel{3}} G} = \frac{3g}{4\pi R G}$$

$$\rho_{\text{granito}} \sim 3\times10^3\frac{kg}{m^3} \qquad = \frac{3\times9.8}{4\times3{,}14\times6\times10^6\times6\times10^{-11}}\frac{kg}{m^3} = 6.5\times10^3\frac{kg}{m^3}$$

2. $h \ll R$

$$g = g(h) \rightarrow g = \frac{GM}{(R+h)^2} = \underbrace{\frac{GM}{R^2}}_{g}\frac{1}{(1+\frac{h}{R})^2} = g\left(1+\frac{h}{R}\right)^{-2} \approx g\left(1-2\frac{h}{R}\right) = g(h)$$

$$(1+x)^n \approx 1+nx$$

$$\frac{|\Delta g|}{g} \approx 2\frac{h}{R} \qquad h = 10^3\,m\ ,\ R \sim 6\times 10^6\,m \qquad \frac{\Delta g}{g} \approx \frac{1}{3}\times 10^{-3} \approx 3\times 10^{-4} = 0.03\,\%$$

$$\Delta g = g(h) - g(0) = g \cdot \left(-2\frac{h}{R}\right)$$

Es.: stimare la differenza percentuale $\frac{\Delta g}{g}$ tra l'accelerazione di gravità terrestre ai poli e quella all'equatore

$R_0 = 6{,}378 \times 10^6\ m$ equatore

$R_1 = 6{,}356 \times 10^6\ m$ poli

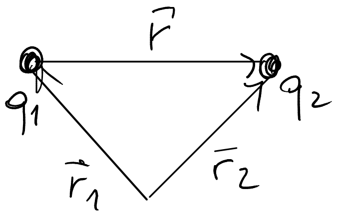
2. Interazioni elettrostatiche

~1600 fenomeno generale

~1700 Franklin → 2 tipi di cariche elettriche +, − ⇒ interazioni repulsive / attrattive

Coulomb → $|\vec{F}_e| \sim \frac{1}{r^2}$

Legge di coulomb



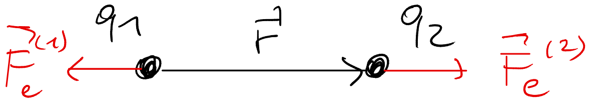
$\vec{r} = \vec{r}_2 - \vec{r}_1$

$$|\vec{F}_e| = k_e \frac{|q_1||q_2|}{|\vec{r}|^2}$$

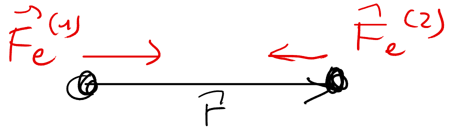
↑ costante di Coulomb

$$k_e = 8.99 \times 10^9 \; \frac{N m^2}{C^2}$$

$[q] = Q$ SI: C, Coulomb



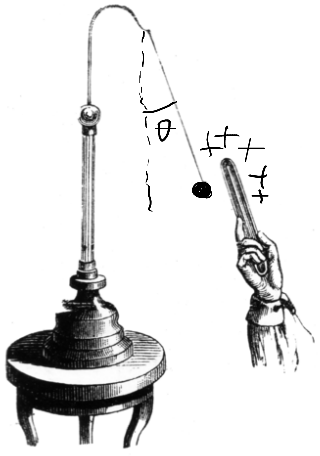
$q_1 \cdot q_2 > 0$



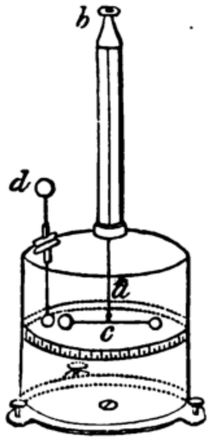
$q_1 \cdot q_2 < 0$

$$\vec{F}_e^{(2)} = + k_e \frac{q_1 \, q_2}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{F}_e^{(1)} = - \vec{F}_e^{(2)}$$



Elettroscopio



Elettrometro di Coulomb

Protone : $q = +1.602 \times 10^{-19}\ C$ $\quad m = 1.672 \times 10^{-27}\ kg$ $\quad\rightarrow$ carica elementare $\equiv e$

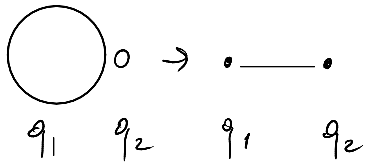
Elettrone : $q = -1.602 \times 10^{-19}\ C$ $\quad m = 9{,}109 \times 10^{-31}\ kg$

Neutrone : $q = 0\ C$ $\quad m = 1.674 \times 10^{-27}\ kg$

2 protoni a distanza r :

$$\frac{|\vec{F}_e|}{|\vec{F}_g|} = \frac{k_e q^2}{G m^2} = \frac{\cancel{9}\times 10^{9}\ \frac{N m^2}{C^2} \cdot (\cancel{1{,}6}\times 10^{-19})^2\ C^2}{\cancel{6}\times 10^{-11}\ \frac{N m^2}{kg^2} \cdot (\cancel{1{,}7}\times 10^{-27})^2\ kg^2} \approx \underbrace{10^{9}\times 10^{11}}_{10^{20}} \times \underbrace{10^{-38}\times 10^{54}}_{10^{16}} \approx \underline{\underline{10^{36}}}$$

Anche qui : modello di <u>carica puntiforme</u>



Campi di forze

Problema : forza a distanza !

~1800 Faraday → forze elettrostatiche → linee di forze

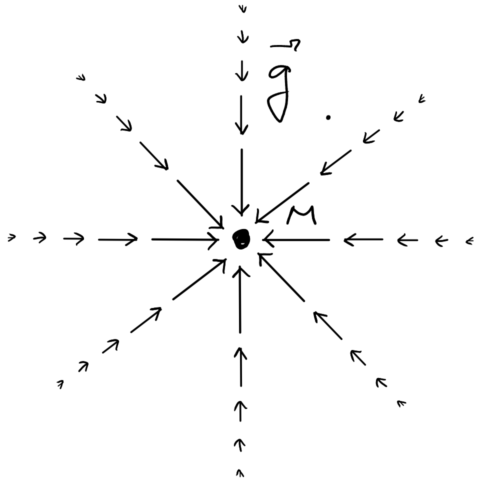
Maxwell → campi elettromagnetici → formalismo matematico

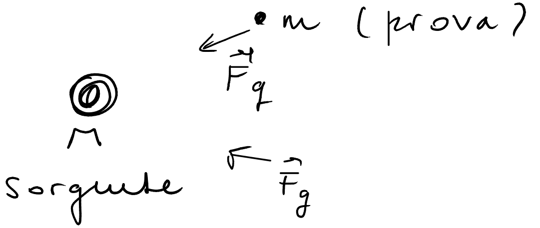
Campo : grandezza fisica che varia nello spazio e/o nel tempo

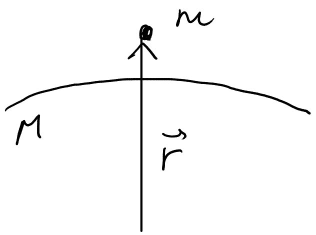
→ scalari

→ vettoriale

Campo gravitazionale : $\boxed{\vec{g} \equiv \frac{\vec{F}_g}{m}}$ $\vec{F}_g = m\vec{g}$







$$\vec{F}_g = -G\,\frac{mM}{|\vec{r}|^2}\,\frac{\vec{r}}{|\vec{r}|}$$

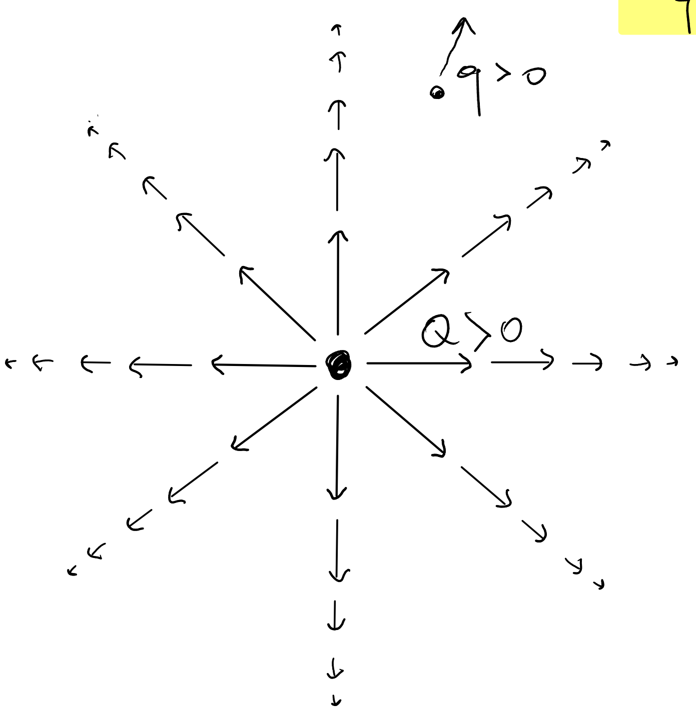
$$\vec{g} = -\underbrace{\frac{GM}{|\vec{r}|^2}}_{g}\,\frac{\vec{r}}{|\vec{r}|} = \vec{a} \Rightarrow$$

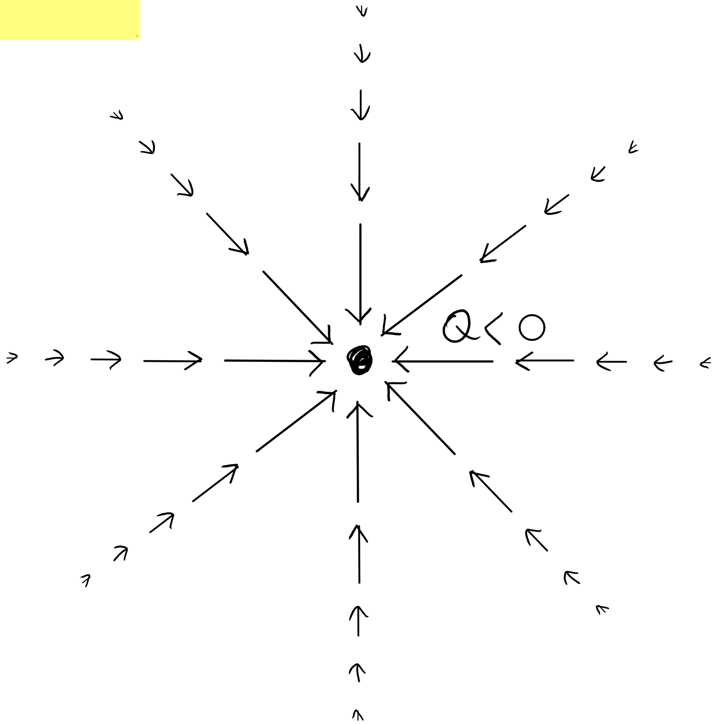
campo gravitazionale = accelerazione di gravità

Campo elettrico

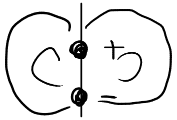
carica di prova q : $\boxed{\dfrac{\vec{F}_e}{q} = \vec{E}}$

SI : $|\vec{E}| \to \dfrac{N}{C}$ $\quad |\vec{E}| \sim \dfrac{1}{|\vec{r}|^2}$ carica puntiforme





Dipolo elettrico :



$|\vec{E}| \sim \dfrac{1}{|\vec{r}|^3}$

Es: $\vec{E} = \vec{E}_1 + \vec{E}_2$