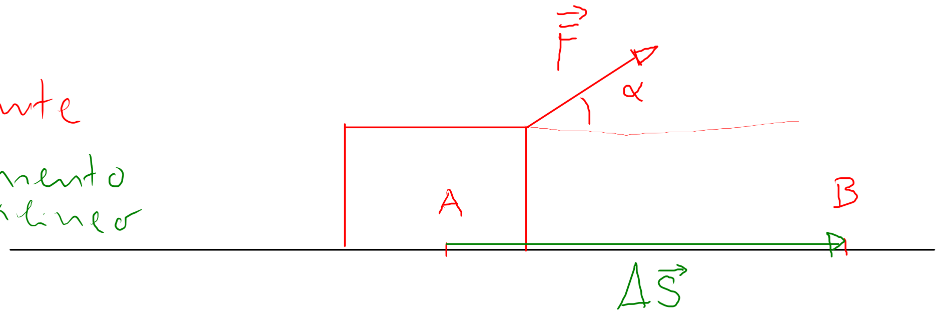
LAVORO

F costante

spostamento rettilineo



$$\vec{B}-\vec{A}=\Delta\vec{s}$$

$$\boxed{\mathcal{L} = \vec{F}\cdot\Delta\vec{s}} = |\vec{F}|\cdot|\Delta\vec{s}|\cos\alpha$$

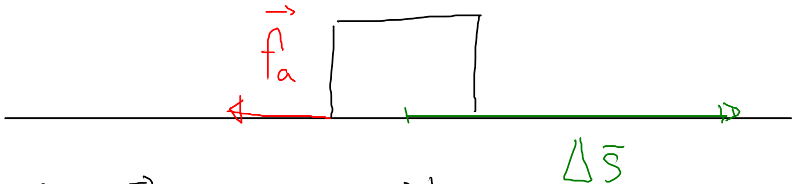
$$[\mathcal{L}] = N\cdot m = J \qquad SI$$

$$1\,J = 10^5\,\text{dyne}\cdot 10^2\,\text{cm} = 10^7\,\text{erg} \qquad c.g.s.$$

$$\cos\alpha \begin{cases} \alpha < \frac{\pi}{2} & \mathcal{L} > 0 \\ \alpha = \frac{\pi}{2} & \mathcal{L} = 0 \\ \alpha > \frac{\pi}{2} & \mathcal{L} < 0 \end{cases}$$

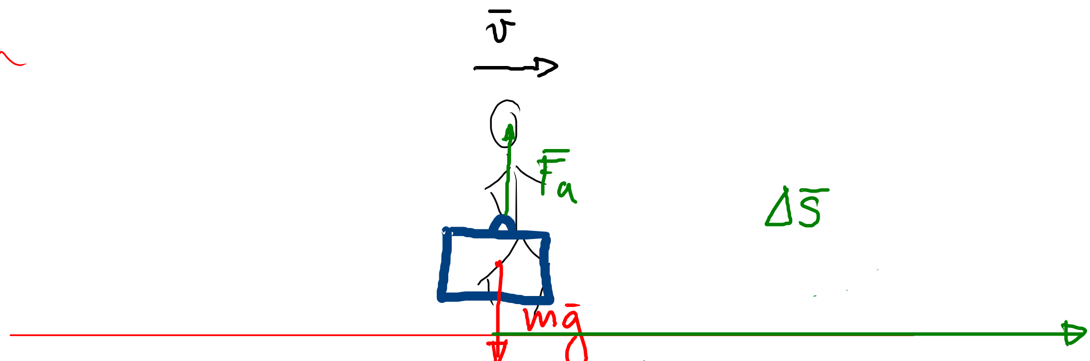
ESEMPI

1) forza d'attrito



$$\mathcal{L} = \vec{f_a}\cdot\Delta\vec{s} = |\vec{f_a}|\cdot|\Delta\vec{s}|\cdot\cos\pi = -|f_a||\Delta\vec{s}|$$

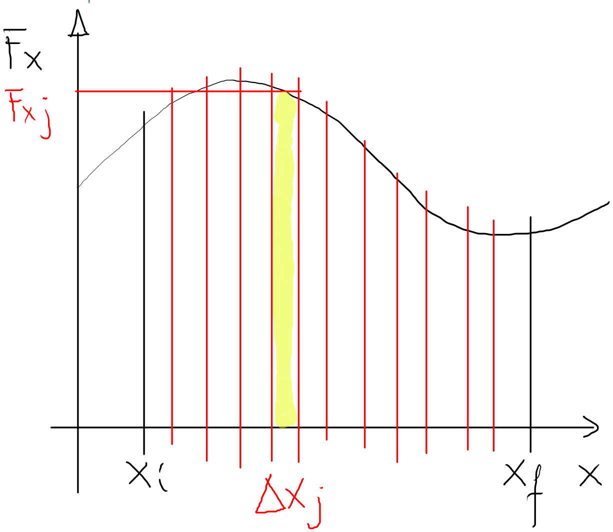
2) valigia



$$\mathcal{L} = \bar{F_a}\cdot\Delta\bar{s} = |\bar{F_a}|\cdot|\Delta\bar{s}|\cos\frac{\pi}{2} = 0$$

$|\vec{F}|$ non costante, orientata lungo x

e spostamento rettilineo (lungo x)



$$\Delta \mathcal{L}_j = F_{x_j} \cdot \Delta x_j$$

~ area del rettangolo

$$\mathcal{L} = \sum_j \Delta \mathcal{L}_j$$

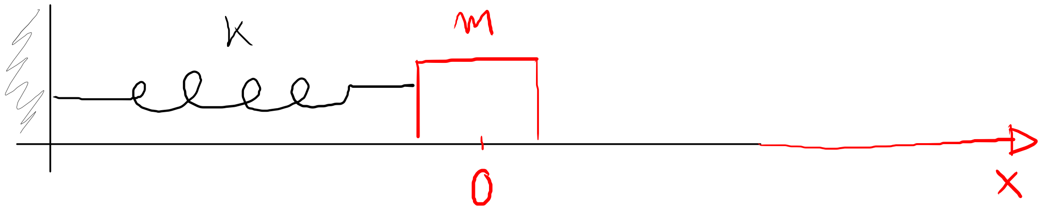
$$= \sum_j F_{x_j} \cdot \Delta x_j$$

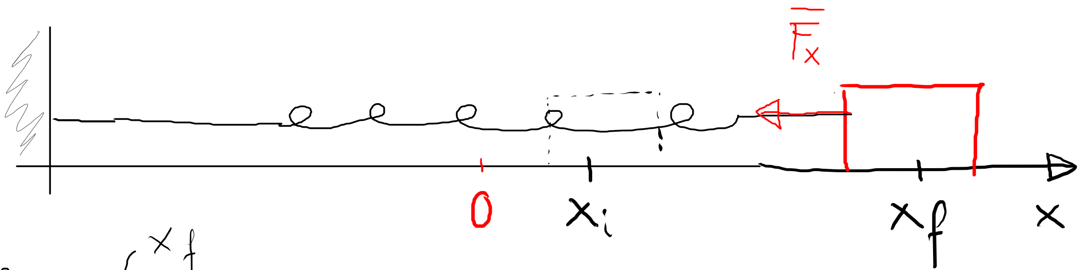
~ area sotto la curva di F_x

$$\lim_{\Delta x_j \to 0} \quad \boxed{\mathcal{L} = \int_{x_i}^{x_f} F_x \cdot dx}$$

è esattamente l'area sotto la curva

ESEMPIO





$\vec{F}_x = -k\vec{x}$

$F_x = -kx$

$$\mathcal{L} = \int_{x_i}^{x_f} F_x\,dx$$

$$= \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx = -k\left[\frac{1}{2}x^2\right]_{x_i}^{x_f}$$

$$= -k\left(\frac{1}{2}x_f^2 - \frac{1}{2}x_i^2\right) = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 < 0$$

- ENERGIA

Energia cinetica $\quad m \quad \vec{v}$

$$K = \frac{1}{2} m v^2$$

$$[K] = kg \frac{m^2}{s^2} = \overset{N}{\frac{kg\,m}{s^2}}\, m = J$$

- TEOREMA LAVORO - ENERGIA

$$\mathcal{L} = \Delta K$$

Lavoro della risultante su un corpo di massa m

OPPURE

Somma dei lavori fatti dalle forze che agiscono sul corpo di massa m

$\Delta K = K_f - K_i$

variazione di en. cin. del corpo

dimostrazione ->

$$\mathcal{L} = \sum \vec{F} \cdot \Delta \vec{s}$$

$$= \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_N \right) \cdot \Delta \vec{s}$$

$$= \vec{F}_1 \cdot \Delta \vec{s} + \vec{F}_2 \cdot \Delta \vec{s} + \vec{F}_3 \cdot \Delta \vec{s} + \ldots + \vec{F}_N \cdot \Delta \vec{s}$$

$$= \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots + \mathcal{L}_N$$

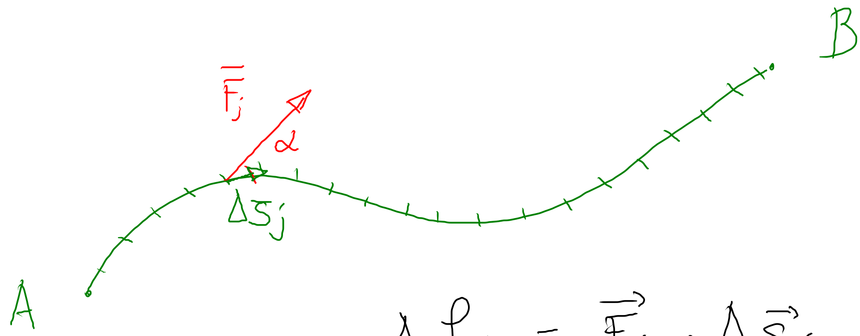
DIMOSTRAZIONE (1D)

$$\mathcal{L} = \int_{x_i}^{x_f} \sum F \cdot dx \underset{\sum F = ma}{=} \int_{x_i}^{x_f} m a \, dx = \int_{x_i}^{x_f} m \underbrace{\frac{dv}{dt}}_{a} dx = \int_i^f m \, dv \underbrace{\frac{dx}{dt}}_{v}$$

$$= \int_{v_i}^{v_f} m v \, dv = m \int_{v_i}^{v_f} v \, dv = m \left[\frac{1}{2} v^2 \right]_{v_i}^{v_f} = \underbrace{\frac{1}{2} m v_f^2}_{K_f} - \underbrace{\frac{1}{2} m v_i^2}_{K_i}$$

$$= K_f - K_i = \Delta K$$

DEFINIZIONE GENERALE DI LAVORO



$$\Delta \mathcal{L}_j = \vec{F}_j \cdot \Delta \vec{s}_j$$

$$\mathcal{L} \cong \sum_j \Delta \mathcal{L}_j = \sum_j \vec{F}_j \cdot \Delta \vec{s}_j$$

nel $\lim_{\Delta \vec{s} \to 0}$ $\quad \Delta \vec{s} \to d\vec{r}$

$$\mathcal{L} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

attenzione !
(integrale di linea)

$$\mathcal{L} = \int_{\vec{r}_i}^{\vec{r}_f} \Sigma \vec{F} \cdot d\vec{r}$$