THEREFORE A GENERAL SOLUTION IS

$$x = X + r\cos(\omega t + \phi)$$
$$y = Y - r\sin\omega t(\omega t + \phi) \quad (67)$$

THE PARTICLE MOVE IN A CIRCLE WHICH FOR $B>0$ IS ANTI-CLOCKWISE DIRECTION. THE CENTRE OF THE CIRCLE (X, Y) THE RADIUS OF THE CIRCLE AND THE PHASE ϕ ARE ARBITRARY.

- OBSERVATION OF COURSE THE ENERGY ASSOCIATED TO THESE MOTIONS VARIES AS A CONTINUUM FUNCTIONS AS $|\bar{B}|$ IS INCREASED OR DECREASED.

- THE QUANTUM MODEL AND THE LANDAU LEVELS. (BACK TO SI UNITS)

TO GAIN THE QUANTUM MODEL IT IS NEEDED TO START WITH THE HAMILTONIAN OPERATOR $\hat{H}$ ADOPTING THE GAUGE KNOWN AS "LANDAU GAUGE". WE HAVE ALREDY SEEN THIS GAUGE $\bar{A}(\bar{r}) \equiv (A_x, A_y, A_z) \equiv$
$= B(-y, 0, 0)$ (68)

- OBSERVATION SOMETIMES A DIFFERENT NOTATION IS ADOPTED, ALTHOUGH EQUIVALENT: $\bar{A}(\bar{r}) \equiv (A_x\ A_y\ A_z) = (-By, 0, 0)$ (69)

LET'S USE THIS NOTATION NOW $\Rightarrow$
$\bar{A}(\bar{r}) = (-By, 0, 0)$, $\bar{B} = \bar{\nabla} \times \bar{A} = B\hat{z}$.

UNDER THIS GAUGE THE QUANTUM HAMIL.
$\hat{H} = \frac{1}{2m}\left(\hat{p} - Q\bar{A}\right)^2$ WITH $Q = -e$ (REMEMBER

$\varphi(\bar{r},t) = 0$ FOR e^- IS A FREE CHARGE)

WE OBTAIN

$$\hat{H} = \frac{1}{2m}\left[(\hat{p}_x - eBy)^2 + \hat{p}_y^2 + \hat{p}_z^2\right] \quad (80)$$

THAT WITH THE LANDAU GAUGE BICOMES

$$\hat{H} = \frac{1}{2m}\left[(\hat{p}_x - eBy)^2 + \hat{p}_y^2 + \hat{p}_z^2\right] \Rightarrow \quad (81)$$

THE SCHRÖDINGER EQ. TIME INDEPEN. $\Rightarrow$

$$\hat{H}\,\psi(\bar{r}) = E\,\psi(\bar{r}) \quad (82)$$

SINCE $[\hat{H}, \hat{p}] = [\hat{H}, \hat{p}_z] = 0$ BOTH p_x AND p_z ARE CONSERVED. USING A $\psi(\bar{r}) \Rightarrow$ $\psi(\bar{r}) = e^{i(p_x x + p_z z)/\hbar} \cdot \chi(y)$ WE

OBTAIN

$$\left[\frac{\hat{p}_y^2}{2m} + \frac{1}{2} m\omega^2 (y - y_0)^2\right]\chi(y) = \left(E - \frac{p_z^2}{2m}\right)\chi(y) \quad (83)$$

WHERE $y_0 = \dfrac{p_x}{eB}$ AND $\omega_c = \dfrac{eB}{m}$ IS THE

CLASSICAL CYCLOTRON FREQUENCY

- OBSERVATION p_x DEFINES THE CENTRE OF AN HARMONIC OSCILLATOR IN y WITH FREQUENCY ω WHICH EIGENVALUE OF THE ENERGY ARE GIVEN BY

$$E_{n,p_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{p_z^2}{2m} \quad (84)$$

THIS REPRESENTS A QHO (QUANTUM HARMONIC OSCILLATOR CONFINED IN THE (x,y) PLANE

$\Rightarrow$ ONLY THE $\hat{x}, \hat{y}$ PHYSICAL COMPONENTS ARE QUANTIZED. NOT THE $\hat{z}$ COMPONENTS

FOR IN OUR CASE THE CYCLOTRON ORBITS LAY IN THE (X, Y) PLANE.
BY SETTING $p_z = 0$ EQ (84) BECOMES

$$E_{n,p_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c \quad (85)$$

• APPENDIX. IN ANALOGY WITH THE QHO WE CAN INTRODUCE NEW VARIABLES. THESE ARE RISING AND LOWERING OPERATORS, ENTIRELY ANALOGOUS TO THOSE WE USE IN THE HARMONIC OSCILLATOR. THEY ARE DEFINED BY

$$\hat{a} = \frac{1}{\sqrt{2e\hbar B}}\left(\pi_x - \pi_y\right) \quad (86)$$

$$\hat{a}^+ = \frac{1}{\sqrt{2e\hbar B}}\left(\pi_x + i\pi_y\right)$$

WHERE $\bar{\pi}$ IS THE KINETIC MOMENTUM OPERATOR OF 64bis. $[\hat{a}, \hat{a}^+] \Rightarrow$

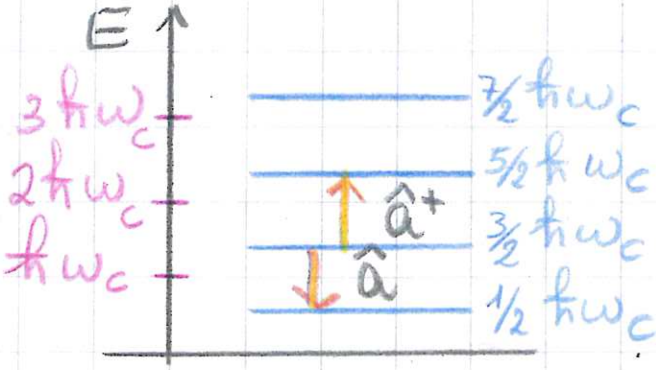
$$= \frac{c\,1}{2e\hbar B}\left[\pi_x + i\pi_y, \pi_x - i\pi_y\right] = \quad (87)$$

$$= \frac{c}{2e\hbar B}\, i\,\frac{e\hbar B}{c}(-2i) = 1$$ AND THE $\hat{H}$

$$= \frac{1}{2m}\left(\bar{p} - \frac{e}{c}\bar{A}\right)^2 = \frac{1}{2m}\bar{\pi}^2 \quad \text{(GAUSS UNITS)}$$

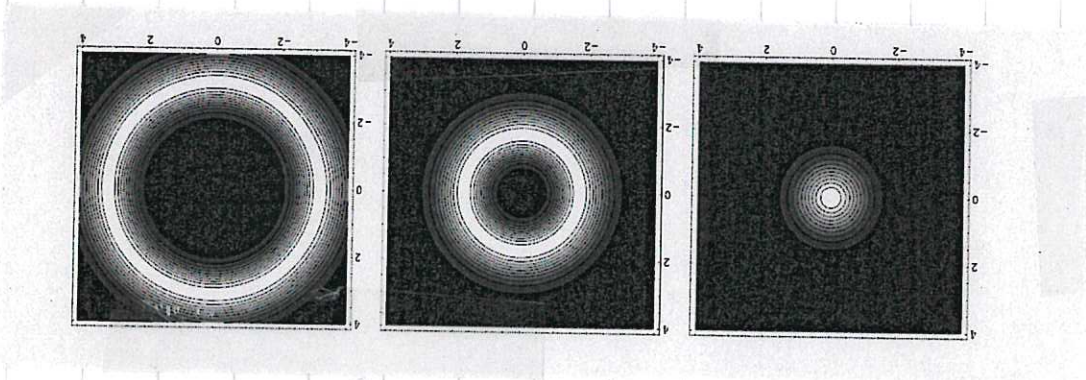
BECOMES $\hat{H}=\hbar\omega_c\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)$ AND THE (88)

EIGENVALUES (SPECTRUM) $E=\hbar\omega_c\left(n+\frac{1}{2}\right)$ (89)



BY COMPARING 88 AND 89 WE CAN ARGUE THAT $\hat{a}\hat{a}^+ = n$ WHEN n ASSUME THE ROLE OF OPERATOR ($\hat{n}$) IS KNOWN AS NUMBER OPERATOR.

• OBSERVATION THE GROUND STATE WAVEFUNCTION IS OBTAIN BY SOLVING $a|0\rangle = 0$



LANDAU LEVELS GROUND STATE WAVEFUNCTIONS FOR $n=10$, $n=3$, $n=0$

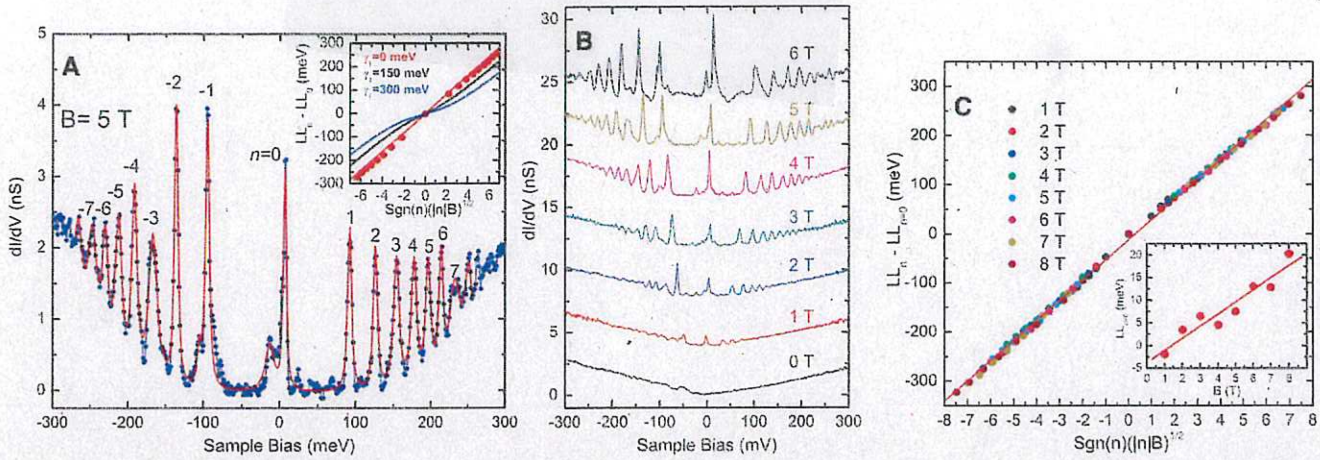
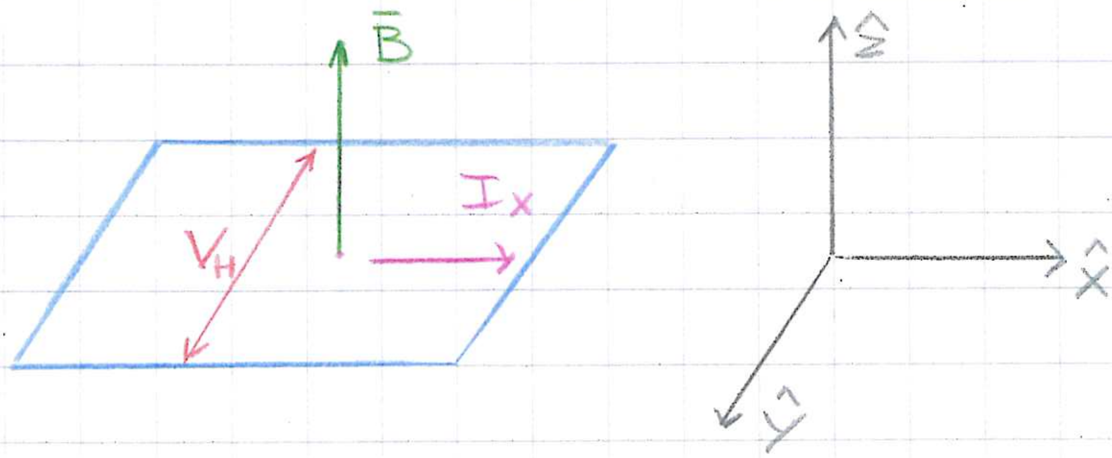


Fig. 2. Direct measurement of Landau quantization in epitaxial graphene. (**A**) Blue data points show the tunneling differential conductance spectra versus sample bias of LLs in multilayer graphene at $B = 5$ T. LL indices are marked. The red line shows a fit to a series of Voigt line shapes at the LL peak positions, which accounts for essentially all the density of states in the spectrum (tunneling set point, $V_B = 350$ mV, $I = 400$ pA). (Inset) LL peak position versus square root of LL index and applied field from the peak positions in (A). Errors in peak positions are smaller than the symbol size. Solid lines are fits to a bilayer model with interlayer coupling of zero (red), 150 meV (black), and 300 meV (blue). (**B**) LL spectra for various applied magnetic fields from 0 to 6 T. The curves are offset for clarity (tunneling set point, $V_B = 350$ mV, $I = 400$ pA). (**C**) LL peak energies for applied fields of 1 to 8 T, showing a collapse of the data when plotted versus square root of LL index and applied field. The solid line shows a linear fit yielding a characteristic velocity of $c^* = (1.128 \pm 0.004) \times 10^6\ \mathrm{ms}^{-1}$ (20). (Inset) The shift in the LL_0 peak position as a function of applied field (symbols). The error is smaller than the symbol size. The solid line is a linear fit to the data points.

- HALL EFFECT AND QUANTUM HALL EFFECT

THE ORIGINAL CLASSICAL HALL EFFECT WAS DISCOVERED IN 1879



THE ELECTRONS ARE RESTRICTED TO MOVE IN THE (X, Y) PLANE. HOW CAN WE DESCRIBE THE CLASSICAL MOTION EQ IN THIS CASE?

DRUDE MODEL WITH MAGNETIC FIELD

THE EQ. OF MOTION IS (SI UNITS)

$$m\frac{d\bar{v}}{dt} = -e\bar{E} - e\bar{v}\times B - \frac{m\bar{v}}{\tau} \quad (90)$$

IN THIS CASE THE DISSIPATIVE TERM $\frac{m\bar{v}}{\tau}$ REPRESENTS THE LOSSES DUE TO SCATTERING PROCCESES, WITH τ THE AVERAGE TIME BETWEEN TWO SUBSEQUENT SCATTERING EVENTS, THE LOSSES DUE TO RADIATIVE EFFECTS CAN BE IGNORED BECAUSE TWO SMALL.
WE ARE INTERESTED TO EQUILIBRIUM SOLUTIONS $\Rightarrow d\bar{v}/dt = \bar{a} = 0$, EQ. 90 THAN BECOMES

$$(91) \quad \bar{v} + \frac{e\tau}{m}\bar{v}\times\bar{B} = -\frac{e\tau}{m}\bar{E}$$

REMEMBERING $\bar{J} = -ne\bar{v}$ WITH n = DENSITY OF CHARGE CARRIERS. IN MATRIX NOTATION 91 BECOMES

$$(92) \quad \begin{pmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{pmatrix}\bar{J} = \frac{e^2 n\tau}{m}\bar{E}$$

WITH $\omega_B = \frac{eB}{m}$ (CYCLOTRON FREQ. PREVIOUSLY MARKED AS ω ONLY).
WE CAN INVERT EQ. 92 TO EXTRACT $\bar{J} = \sigma_{ij}\bar{E}$, WHERE σ_{ij} IS A 2x2

MATRIX $\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{pmatrix}$ (93)

THE STRUCTURE OF THE MATRIX, WITH IDENTICAL DIAGONAL COMPONENTS, AND EQUAL BUT OPPOSITE OFF-DIAGONAL COMPONENTS FOLLOWS FROM ROTATIONAL INVARIANCE. FROM THE DRUDE MODEL (SEE LECTURES OF FED) WE GET THE EXPLICIT EXPRESSION FOR THE CONDUCTIVITY

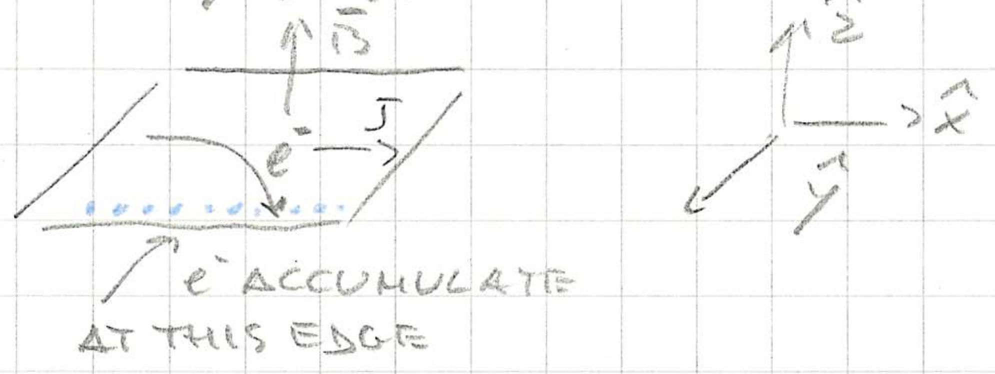
$$\sigma = \frac{\sigma_{DC}}{1+\omega_B^2\tau^2} \begin{pmatrix} 1 & -\omega_B\tau \\ \omega_B\tau & 1 \end{pmatrix} \quad (94)$$

WITH $\sigma_{DC} = \frac{ne^2\tau}{m}$ (THE CONDUCTIVITY WITHOUT $\bar{B}$)

- OBSERVATION • IN THE PRESENCE OF $\bar{B}$ σ IS NOT ANYMORE A SCALAR BUT IS A MATRIX, ALSO KNOWN AS CONDUCTIVITY TENSOR. IN THIS CASE THE FULL SYMMETRY OF THE PROBLEM WITHOUT $\bar{B}$ IS BROKEN WHEN $\bar{B}$ IS ON.

•• THE OFF-DIAGONAL TERMS IN THE MATRIX ARE RESPONSIBLE FOR THE HALL EFFECT: IN EQUILIBRIUM, A CURRENT IN THE $\hat{x}$ DIRECTION REQUIRES A FIELD WITH A COMPONENT IN THE Y-DIRECTION. IN A FINITE SYSTEM THIS MECHANISM RESULTS IN

A BUILD UP OF CHARGES ALONG THE EDGE ⊥ TO THE $\hat{y}$ DIRECTION



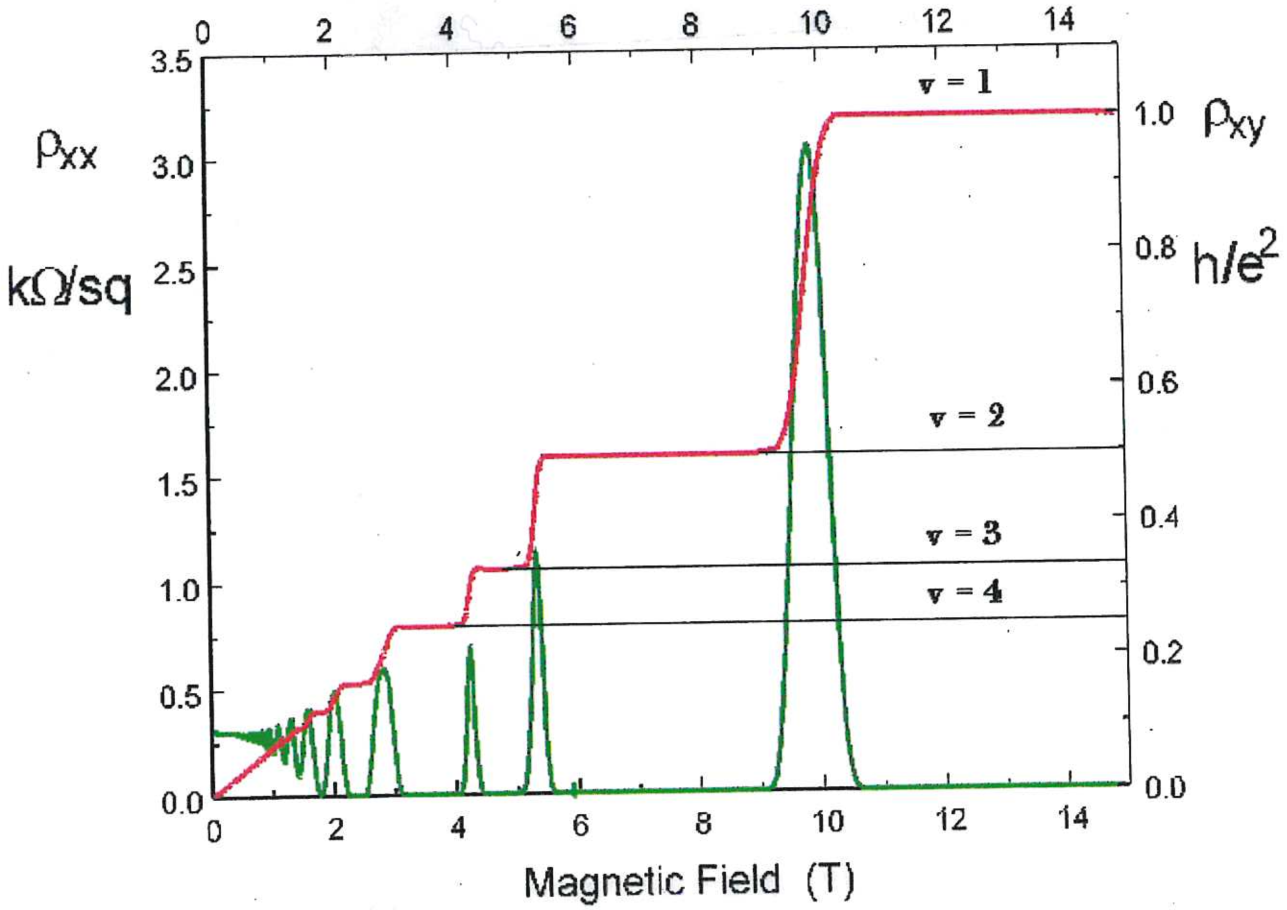
THE ACCUMULATION OF THIS CHARGES ENDS WHEN THE E_y COMPONENT BUILDS UP A POTENTIAL V_H THAT CANCEL THE LORENTZ FORCE COMPONENTS DUE TO THE $\vec{B}$ FIELD. V_H IS KNOWN AS HALL POTENTIAL.

- QUANTUM HALL EFFECTS

THIS ARGUMENT IS LEFT OPTIONAL

AN IMPORTANT MANIFESTATION OF QUANTUM MECHANICS ON MACROSCOPIC BEHAVIOR OF PHYSICAL SYSTEM CONSERNS THE TRANSPORT PROPERTIES OF A 2D ELECTRONIC GAS (FREE ELECTRONS) IN A STRONG MAGNETIC FIELD. KLITZING, DORDA AND PEPPER IN 1980 DISCOVER THE QUANTISATION OF THE TRANSPORT PROPERTIES IN A HALL SETUP WHEN A STRONG $\vec{B}$ FIELD WAS APPLIED, THIS IMPORTANT DISCOVERY, KNOWN AS INTEGER QUANTUM HALL EFFECT (IQHE) WAS RECOGNIZED WITH THE NOBEL PRIZE IN 1985.

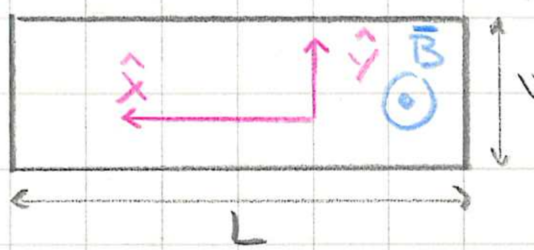
THIS DISCOVERY WAS OF PARAMOUNT IMPORTANCE IN MATERIAL SCIENCE NAMELY FOR THE FABBRICATION OF HIGH QUALITY FIELD EFFECT TRANSISTORS FOR THE REALISATION OF A 2D e^- GAS. THE IQHE OCCURS AT LOW TEMPERATURE $\Rightarrow$ WHEN THE ENERGY SCALE $K_B T$ IS $\ll$ THAN THE LANDAU-LEVELS (L-L) SEPARATION $\hbar\omega_c$. IT CONSISTS OF THE QUANTISATION OF THE HALL RESISTENCE, WHICH IS NO LONGER LINEAR IN $\vec{B}$ AS EXPECTED FROM THE CLASSICAL TREATMENT, BUT SHOWS PLATEAUS AT PARTICULAR VALUES OF THE MAGNETIC FIELD



IN THE PLATEAUS, THE HALL RESISTENCE IS GIVEN IN TERMS OF A UNIVERSAL CONSTANTS (IT IS INDEED A FRACTION OF THE INVERCE QUANTUM CONDUCTANCE e^2/h) IS GIVEN BY

$$R_H = \left(\frac{h}{e^2}\right)\frac{1}{n} \quad (95)$$

WHERE n IS AN INTEGER. THE PLATEAU IN THE HALL RESISTENCE IS ACCOMPAINED BY A VANISHING LONGITUDINAL RESISTENCE (WITH IN THE EXPERIMENTAL ERROR). HERE INSTEAD USING THE CONDUCTIVITY σ_{ij} AND RESISTIVITY ρ_{ij} IT CAN BE MORE CONVENIENT TO USE THE RESISTENCE MATRIX R_{ij} [WHERE RESISTIVITY ≡ RESISTENCE TENSOR × CROSS-SECTIONAL AREA (LENGTH 2D) ⊥ TO THE CURRENT FLOW / LENGTH OVER WHICH VOLTAGE DROP. FOR HALL RESISTENCE THINGS ARE SIMPLE WITH THE HELP OF THIS FIGURE



THE $\bar{B}$ IS OUT OF THE PAGE. R_H IS DEFINED AS THE VOLTAGE DROP ACROSS THE y DIMENSION $(W)/I_x$ WITH $I_y = 0$. IN THIS CASE $R_H = \rho_H$ WITH NO DEPENDENCE ON L OR W. BY CONTRAST $R_{xx} = (L/W)\,\rho_{xx}$

AT THE SAME TIME WE CAN DEFINE THE CONDUCTANCE TENSOR Σ'_{ij} SO THE FOLLOWING RELATIONS HOLD

$$(96)\quad V_i = \underbrace{\sum\nolimits'_j}_{\text{SUM}} R_{ij} I_j \,, \quad I_i = \underbrace{\sum\nolimits'_j}_{\text{SUM}} \underbrace{\Sigma'_{ij}}_{\text{CONDUC. MATRIX}} V_j$$

$\Rightarrow$ Σ'_{ij} IS THE INVERSE OF R_{ij}.

- OBSERVATION R_{11} (R_{xx}) MEANS THE RATIO OF V_1 TO I_1 WITH $I_2 = 0$, WHILE Σ'_{11} MEANS THE RATIO OF I_1 TO V_1 WITH $V_2 = 0$. THESE TWO QUANTITIES ARE NOT NECESSARY INVERSE. IN PARTICULAR, NOTE THAT SINCE, FOR EXAMPLE $R_{xx} = \Sigma'_{yy} / \det \Sigma'$ IT IS POSSIBLE TO HAVE R_{xx} AND Σ'_{xx} SIMULTANEOUSLY ZERO, PROVIDED THA Σ'_{xy} AND R_{xy} ARE NOT ZERO

THIS LEEDS TO THE FOLLOWING PICTURE