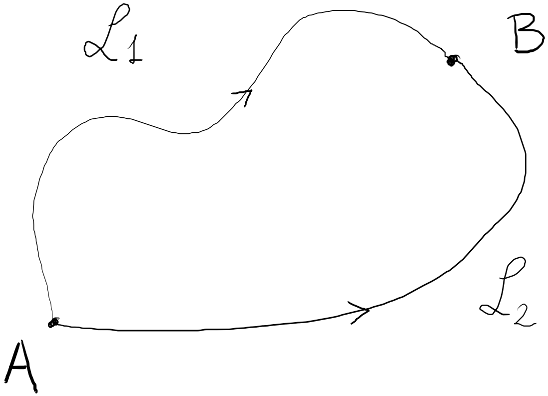
FORZE CONSERVATIVE

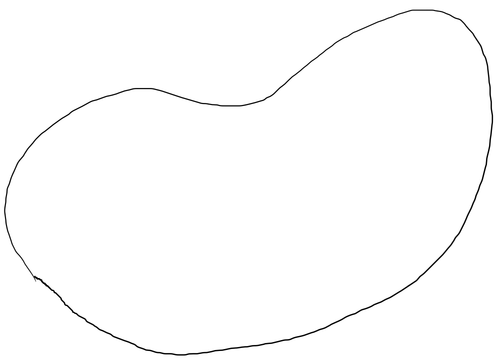
I)



$$\mathcal{L}_1 = \mathcal{L}_2$$

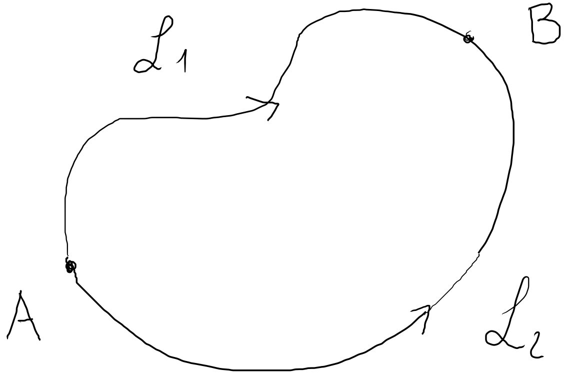
I) $\Leftrightarrow$ II)

II)

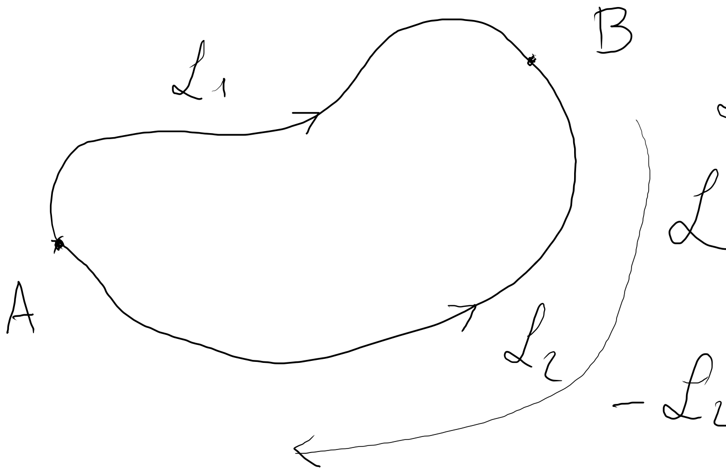


$$\mathcal{L} = 0$$

Assumiamo I) $\Rightarrow$ II)



$$\mathcal{L}_1 = \mathcal{L}_2$$



$$\mathcal{L}_1 = \mathcal{L}_2$$

$$\mathcal{L}_{ABA} = \mathcal{L}_1 + (-\mathcal{L}_2)$$
$$\overset{!}{=} \mathcal{L}_1 - \mathcal{L}_1 = 0$$

ENERGIA POTENZIALE (solo per forze conservative)

Definisco $U(\vec{r})$ tale che

$$\mathcal{L}_{AB} = U(\vec{r}_A) - U(\vec{r}_B)$$

$\vec{r}_A$ e $\vec{r}_B$ sono vettori posizione definiti in un opportuno sistema di riferimento

$$\mathcal{L}_{AB} = U_A - U_B = -\Delta U$$

Ammettono en. potenziale:

→ $P = m\bar{g}$ (forza costante)

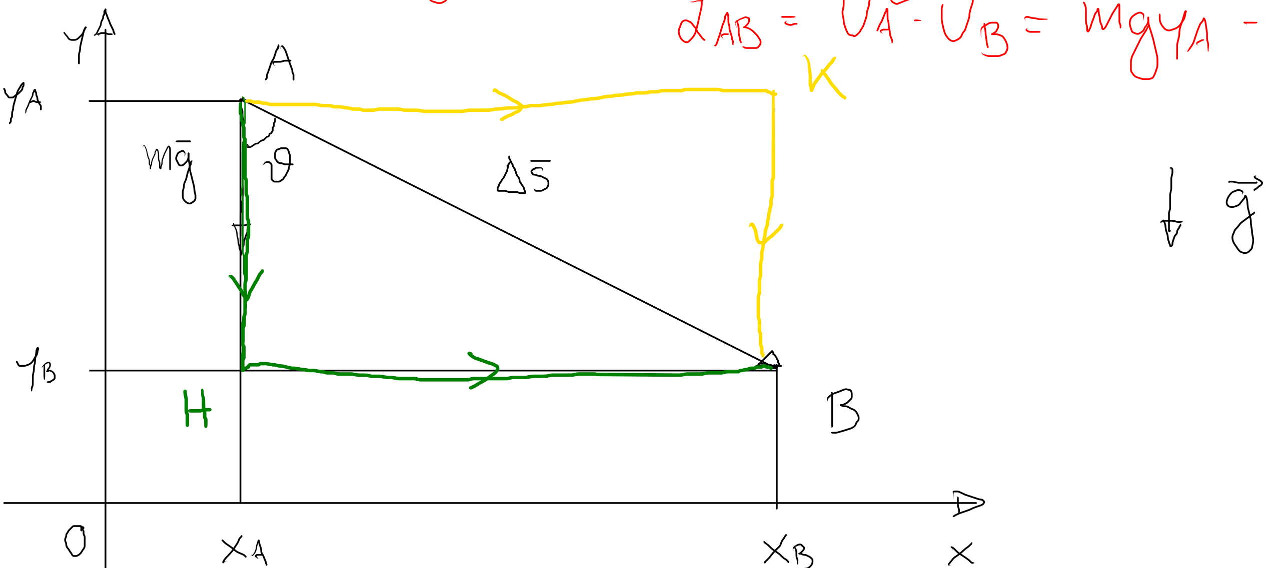
→ $\vec{F} = -K\bar{x}$

→ $F_g = -\dfrac{G m_1 m_2}{r^2}\hat{r}_{12}$ (forza radiale con intensità $\propto \frac{1}{r^2}$)

ESEMPIO : $\bar{P} = m\bar{g}$

$U = mgy$

$\mathcal{L}_{AB} = U_A - U_B = mgy_A - mgy_B$



$$\mathcal{L}_{AB} = \bar{P} \cdot \Delta\bar{s} = m\bar{g} \cdot \Delta\bar{s} = mg\,\Delta s \cos\vartheta$$

$$= mg\,(y_A - y_B)$$

$$\mathcal{L}_{AB} = \mathcal{L}_{AH} + \mathcal{L}_{HB} = mg\,(y_A - y_B) + 0 = mg\,(y_A - y_B)$$

$$\mathcal{L}_{AB} = \mathcal{L}_{AK} + \mathcal{L}_{KB} = 0 + mg\,(y_A - y_B) = mg\,(y_A - y_B)$$

TEOREMA LAVORO ENERGIA PER FORZE CONSERVATIVE

$$\left.\begin{aligned} \mathcal{L} &= \Delta K \\ \mathcal{L} &= -\Delta U \end{aligned}\right\} \quad \Delta K = -\Delta U \qquad \Delta K + \Delta U = 0$$

$$\Delta(K+U) = 0 \qquad \Delta E_{mecc} = 0$$

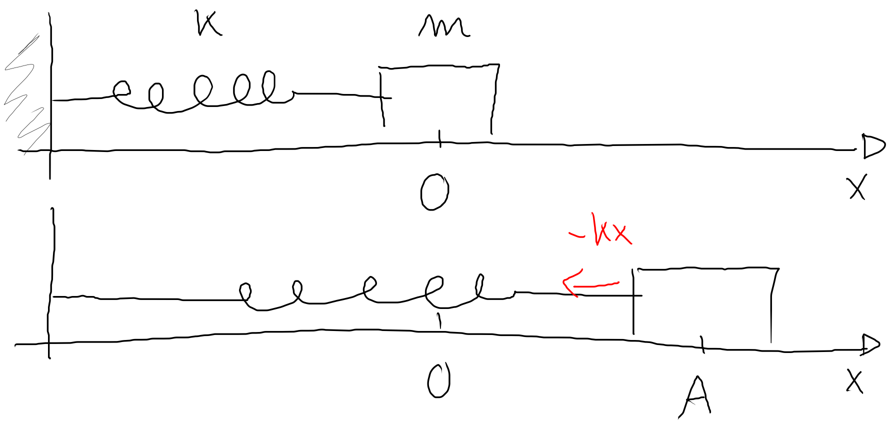
$$K_B - K_A = U_A - U_B \qquad K_A + U_A = K_B + U_B$$

$\to$ $E_{mecc} = K + U$

$$E_{mecc\,A} = E_{mecc\,B}$$

$\to$ Conservazione energia meccanica (in un sistema conservativo)

Esempio



$$x(t) = A\cos(\omega t) \qquad \omega = \frac{2\pi}{T} = 2\pi\nu$$

$$v(t) = A\omega \,\mathrm{sen}(\omega t)$$

$$a(t) = -A\omega^2 \cos(\omega t) = -\omega^2 x(t)$$

$$\left.\begin{array}{l} F = -kx = ma \\ a = -\frac{k}{m}x \end{array}\right\} \omega^2 = \frac{k}{m}$$

$$U = \frac{1}{2}kx^2$$

$$\begin{aligned} E_{mecc} &= K + U \\ &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}m\left(A\omega \,\mathrm{sen}(\omega t)\right)^2 + \frac{1}{2}k\left(A\cos(\omega t)\right)^2 \\ &= \frac{1}{2}\underbrace{m A^2 \omega^2}_{k} \mathrm{sen}^2(\omega t) + \frac{1}{2}kA^2\cos^2(\omega t) \\ &= \frac{1}{2}kA^2 \underbrace{\left[\mathrm{sen}^2(\omega t) + \cos^2(\omega t)\right]}_{1} = \frac{1}{2}kA^2 \end{aligned}$$

Emecc si conserva!

$$E_{mecc} = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \qquad \omega^2 = \frac{k}{m}$$

$$x = \pm A \Rightarrow \frac{1}{2}mv^2 = 0 \qquad v = 0$$

$$x = 0 \Rightarrow \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$$

$$v_{max}^2 = \frac{k}{m}A^2 = \omega^2 A^2 \qquad v_{max} = \pm \omega A$$

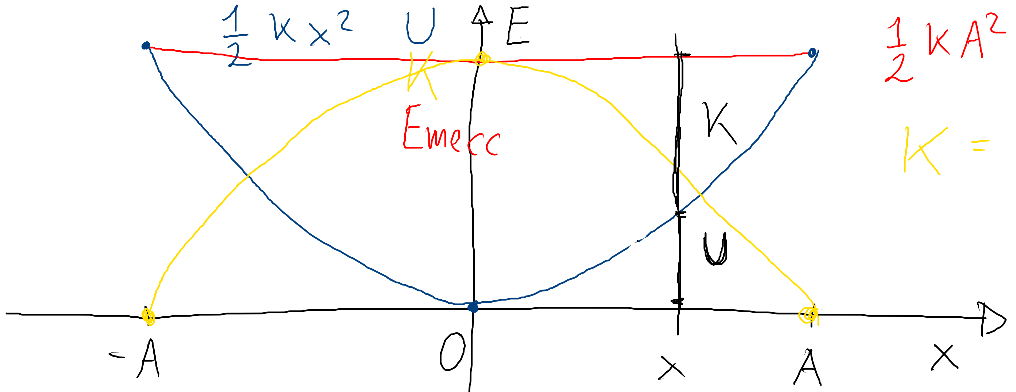
$$x \text{ qualsiasi} \Rightarrow \frac{\cancel{1}}{\cancel{2}}mv^2 + \frac{\cancel{1}}{\cancel{2}}kx^2 = \frac{\cancel{1}}{\cancel{2}}kA^2$$

$$v^2 + \frac{k}{m}x^2 = \frac{k}{m}A^2$$

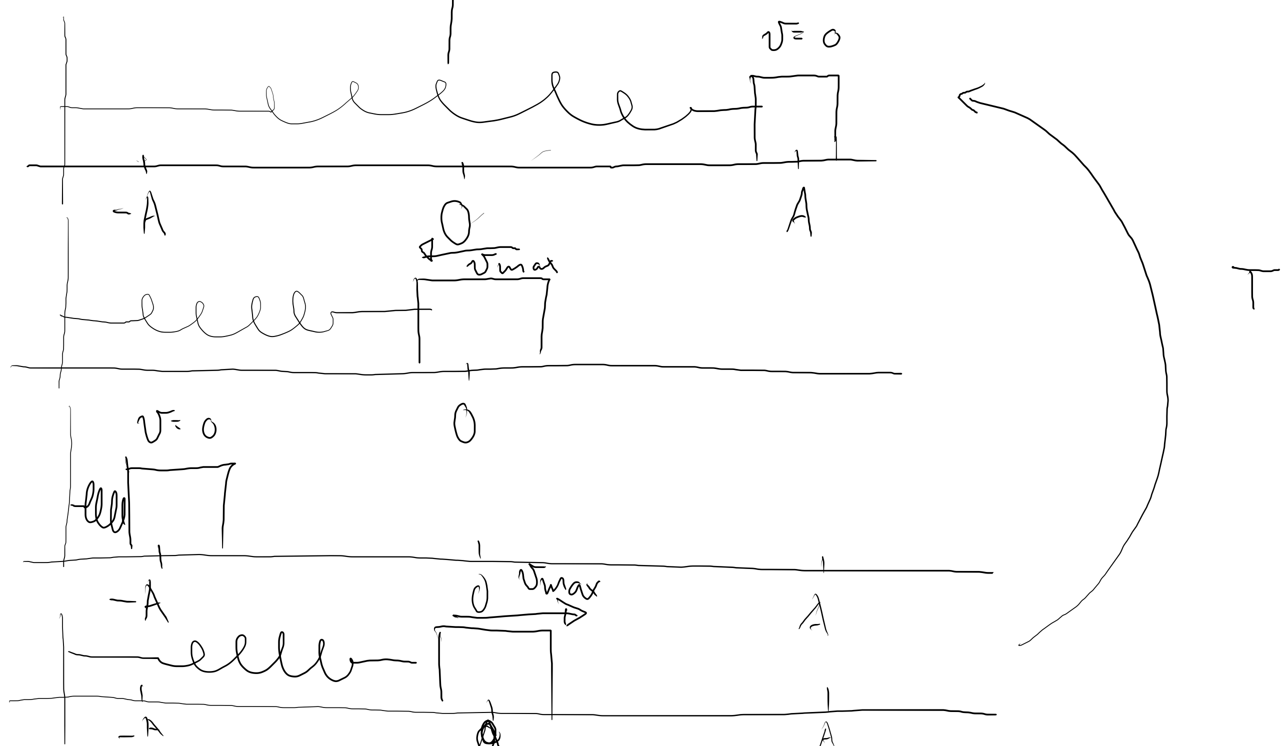
$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v = \pm \omega \cdot \sqrt{A^2 - x^2}$$



$$K = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$



FORZE DISSIPATIVE

$\mathcal{L} = \Delta K$ $\quad$ $\mathcal{L}$ è la somma dei $\mathcal{L}$ fatti dalle varie forze

$\mathcal{L}_C + \mathcal{L}_D = \Delta K$

$-\Delta U + \mathcal{L}_D = \Delta K$

$\mathcal{L}_D = \Delta K + \Delta U = \Delta(K+U) = \Delta E_{mecc}$

$\mathcal{L}_D < 0$ $\quad$ $\Delta E_{mecc} < 0$ $\quad$ E_{mecc} NON si conserva

$*$

$-\Delta E_{mecc} = \Delta E_{int}$ $\quad$ E_{int} = E interna del sistema

$\Delta K + \Delta U = \Delta E_{mecc}$

$\Delta K + \Delta U - \Delta E_{mecc} = 0$ $\quad *$

$\Delta K + \Delta U + \Delta E_{int} = 0$

$\Delta(\underbrace{K + U + E_{int}}_{E_{TOT}}) = 0$

$E_{TOT} = K + U + E_{int}$

$\Delta E_{TOT} = 0$ $\quad$ E_{TOT} si conserva