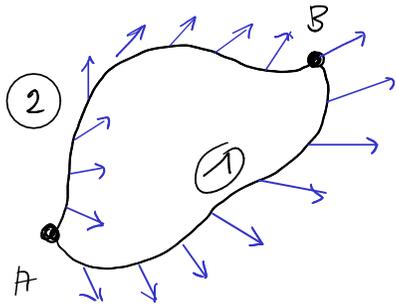


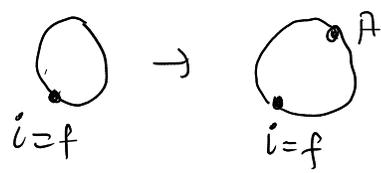
Energia potenziale

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$



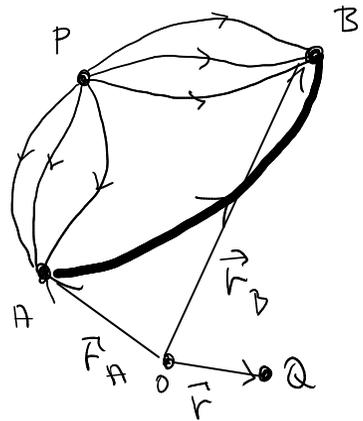
Forze conservative: W_{AB} non dipende dal percorso tra AB.

$$\oint \vec{F} \cdot d\vec{r} = 0 \quad \forall \text{ percorso chiuso}$$



$$\oint \vec{F} \cdot d\vec{r} = \int_i^A \vec{F} \cdot d\vec{r} + \int_A^i \vec{F} \cdot d\vec{r} = \int_i^A \vec{F} \cdot d\vec{r} - \int_i^A \vec{F} \cdot d\vec{r} = 0$$

Fissiamo P di riferimento



$$W_{PB} = \int_P^B \vec{F} \cdot d\vec{r} \equiv -E_P(\vec{r}_B)$$

$$W_{PA} = \int_P^A \vec{F} \cdot d\vec{r} \equiv -E_P(\vec{r}_A)$$

Definisco l'energia potenziale

$$E_P(\vec{r}) \equiv -W_{PA}$$

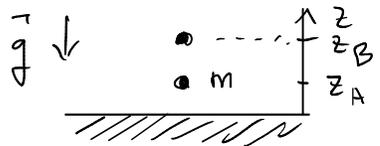
$Q \rightarrow \vec{r}$

$$W_{AB} = W_{AP} + W_{PB} = W_{AP} - W_{BP}$$

$$= -W_{PA} + W_{PB} = E_P(\vec{r}_A) - E_P(\vec{r}_B) = -\Delta E_P$$

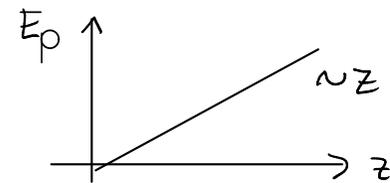
E_P è definita a meno di una costante \rightarrow scelta del punto P

1. Gravitazione terrestre



$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = -mg \int_{z_A}^{z_B} dz = -mg(z_B - z_A)$$

$$E_p = mgz \quad (+ \text{cost})$$

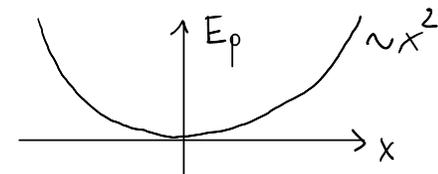


2. Elasticità



$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = -k \int_{x_A}^{x_B} x dx = -\frac{1}{2}k(x_B^2 - x_A^2)$$

$$E_p = \frac{1}{2}kx^2$$

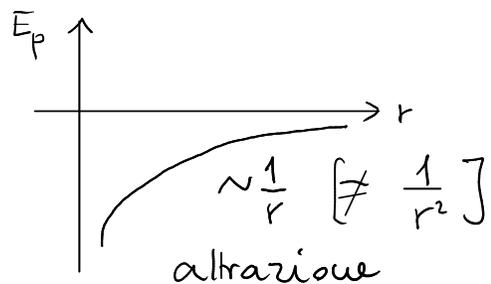


3. Gravitazione universale

A diagram showing two masses, m and M , separated by a distance r .

$$E_p = -G \frac{mM}{r}$$

es.:



$$F_x = -\frac{dE_p}{dx}$$

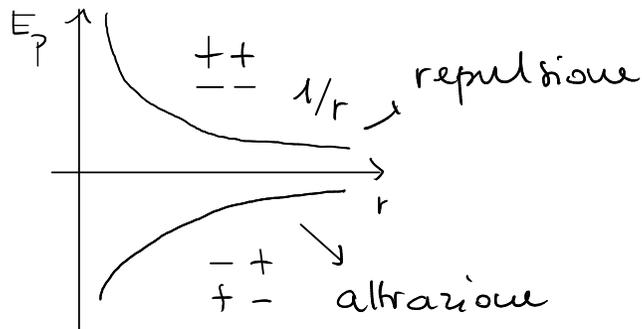
$$F_y = -\frac{dE_p}{dy}$$

4. Elettrostatiche

A diagram showing two charges, q and Q , separated by a distance r .

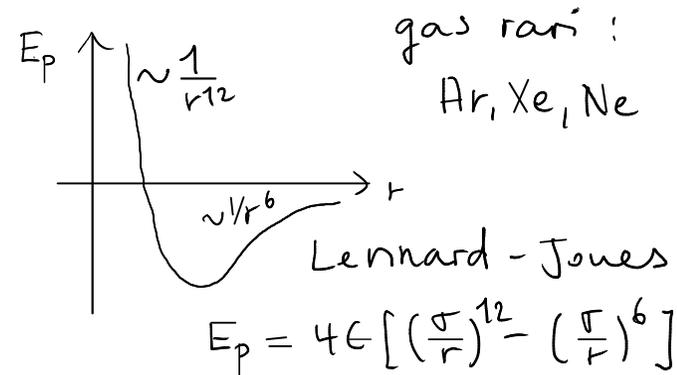
$$E_p = k_c \frac{qQ}{r}$$

es.



5. Intermolecolari

piccoli $r \rightarrow$ repulsione
grandi $r \rightarrow$ attrazione



Conservazione dell'energia meccanica

$$\begin{cases} W = \Delta E_c & W = W[\Sigma \vec{F}] \quad \text{teorema energia cinetica} \\ W = -\Delta E_p & \text{forze conservative } \Sigma \vec{F}_c \end{cases}$$

$$\Delta E_c = -\Delta E_p \rightarrow \Delta E_c + \Delta E_p = 0 \rightarrow \Delta(E_c + E_p) = 0 \quad \text{legge di conservazione di } E$$

$$E \equiv E_c + E_p \quad \text{energia meccanica (totale)} \Rightarrow \Delta E = 0 \Rightarrow E_i = E_f$$

$\rightarrow E$ è una "costante del moto"

Forze fondamentali sono conservative (Δ forze magnetiche)

Forze macroscopiche --- dipende

peso

attrito dinamico

reazione normale

attrito viscoso

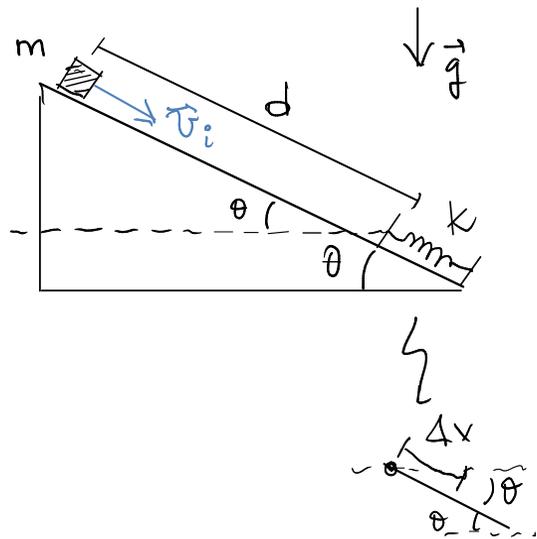
elastica

NON conservative

$$\Delta E_c = \underbrace{W[\Sigma \vec{F}_c]}_{-\Delta E_p} + W[\Sigma \vec{F}_{nc}] \rightarrow \Delta E_c + \Delta E_p = W[\Sigma \vec{F}_{nc}]$$

$$\Delta E = W[\Sigma \vec{F}_{nc}] \rightarrow \text{teori energia meccanica}$$

Esempio: blocco-molla



$$\rightarrow \Delta x = ?$$

Sistema: { blocco, molla }

Attrito trascurabile

$$\Delta E = 0 \Rightarrow E_i = E_f$$

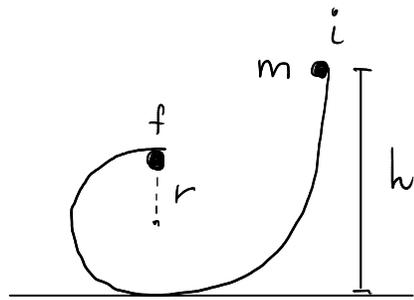
$$E_{ci} + E_{pi} = E_{cf} + E_{pf} \rightarrow \frac{1}{2} m |\vec{v}_i|^2 + mg d \sin \theta = -mg \Delta x \sin \theta + \frac{1}{2} k \Delta x^2$$

$\swarrow \searrow$ grav. molla $\swarrow \searrow$ grav. molla

$$\frac{1}{2} k \Delta x^2 - mg \sin \theta \Delta x - \left(\frac{1}{2} m |\vec{v}_i|^2 + mg d \sin \theta \right) = 0$$

$$\Delta x_{1,2} = \frac{mg \sin \theta \pm \sqrt{m^2 g^2 \sin^2 \theta + 2k(\dots)}}{k} \rightarrow \Delta x = \frac{\dots + \sqrt{\dots}}{k}$$

ES: skateboard



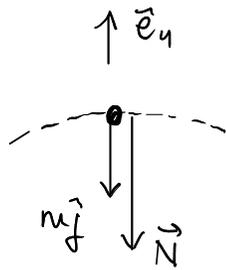
1. conservazione energia (no attrito) $\rightarrow v_f$

2. reazione normale $N > 0$ nel punto finale

$$\vec{v}_i = \vec{0}$$

$$E_{ci} + E_{pi} = E_{cf} + E_{pf}$$

$$0 + mgh = \frac{1}{2} m |\vec{v}_f|^2 + mg 2r \rightarrow |\vec{v}_f|^2 = \frac{2g(h-2r)}{1} \\ |\vec{v}_f| = \sqrt{2g(h-2r)}$$



II Newton: $\Sigma \vec{F} = m\vec{a}$

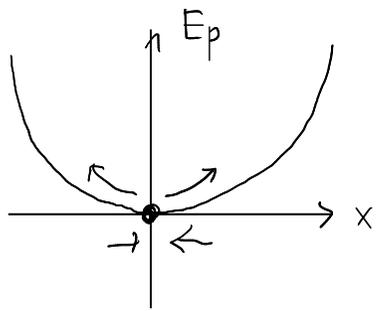
$$m\vec{g} + \vec{N} = m\vec{a}_c \rightarrow -mg\vec{e}_y - N\vec{e}_y = -m \frac{|\vec{v}_t|^2}{r} \vec{e}_y$$

$$mg + N = m \frac{|\vec{v}_t|^2}{r}$$

$$0 \leq \frac{N}{m} = \frac{|\vec{v}_t|^2}{r} - g = \frac{2g(h-2r)}{r} - g = 2g \frac{h}{r} - 4g - g$$

$$2 \frac{h}{r} - 5 \geq 0 \Rightarrow h \geq \frac{5}{2} r = \frac{5}{4} D$$

Equilibrio statico: $\vec{F} = \text{cost immobile} \Leftarrow \Sigma \vec{F} = \vec{0} + \vec{v} = \vec{0}$



molla $\frac{1}{2} kx^2$

$$E_p = \frac{1}{2} kx^2$$

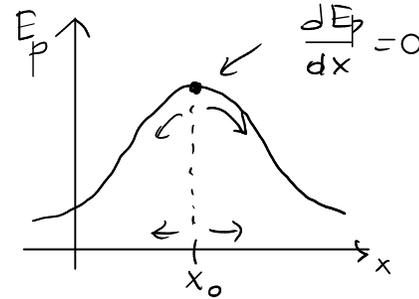
$$F_x = - \frac{dE_p}{dx}$$

$\frac{dE_p}{dx} = 0 \rightarrow$ punto stazionario

$$x > 0 \Rightarrow F_x < 0$$

$$x < 0 \Rightarrow F_x > 0$$

\rightarrow minimo \rightarrow equilibrio
STABILE



$$\Delta x = (x - x_0) > 0 \quad F_x > 0$$

$$\Delta x < 0 \quad F_x < 0$$

\rightarrow massimo \rightarrow equilibrio
INSTABILE

$\frac{d^2 E_p}{dx^2} > 0$ stabile

$\frac{d^2 E_p}{dx^2} < 0$ instabile

$\frac{d^2 E_p}{dx^2} = 0$ "indifferente"

