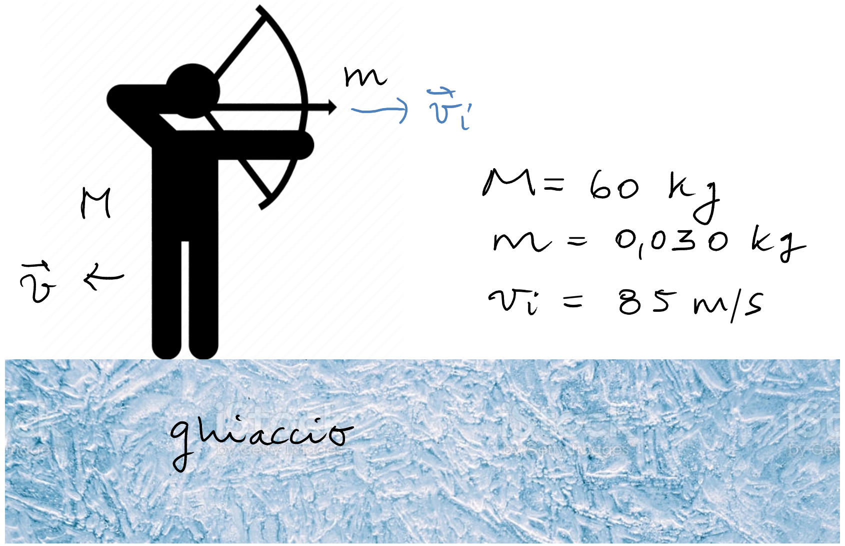
QUANTITÀ DI MOTO



$M = 60\ kg$

$m = 0{,}030\ kg$

$v_i = 85\ m/s$

$\sum \vec{F} = \vec{0}$ III Newton : $\vec{F}_{12} = -\vec{F}_{21}$

$\sum \vec{F} = \vec{0}$ II Newton

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{0}$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = \vec{0}$$

$$\frac{d}{dt}(m_1 \vec{v}_1) + \frac{d}{dt}(m_2 \vec{v}_2) = \vec{0}$$

$$\frac{d}{dt}(\underbrace{m_1 \vec{v}_1} + \underbrace{m_2 \vec{v}_2}) = \vec{0} \rightarrow$$ legge di conservazione "costante del moto"

$\vec{p} \equiv m\vec{v}$ **quantità di moto**

$[|\vec{p}|] = \frac{ML}{T}$ SI: $kg \frac{m}{s}$

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \vec{0} \Rightarrow \frac{d\vec{p}}{dt} = \vec{0}$$

$|\vec{p}_1| = p_1$

$|\vec{p}_2| = p_2$

$E_{c1} = E_{c2}$

?

$p_1 < p_2$

$p_1 = p_2$

$p_1 > p_2$

$$\cancel{\frac{1}{2}} m_1 |\vec{v}_1|^2 = \cancel{\frac{1}{2}} m_2 |\vec{v}_2|^2$$

$$\underbrace{m_1 |\vec{v}_1|} \cdot |\vec{v}_1| = \underbrace{m_2 |\vec{v}_2|} |\vec{v}_2|$$

$$p_1 |\vec{v}_1| = p_2 |\vec{v}_2| \Rightarrow p_1 = \frac{|\vec{v}_2|}{|\vec{v}_1|} p_2 \Rightarrow$$ dipende da $|\vec{v}_2| / |\vec{v}_1|$

!!

- massa costante II Newton : $\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

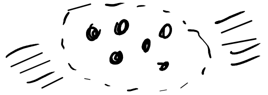
- massa non costante $m = m(t)$: $\sum \vec{F} = \frac{d\vec{p}}{dt}$ → $\vec{p}$ totale generalizzazione della II Newton

es: stella cadente, razzo a propulsione

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\cdot\vec{v} + m\frac{d\vec{v}}{dt}$$

Leggi di conservazione

Sistema isolato : $\sum \vec{F}_e = \vec{0}$



non interagisce con l'esterno

$\vec{F}_i$ interne

$\vec{F}_e$ esterne → 0

- $\vec{F}_i$ forze conservative ⇒ $\Delta E = 0$
- III Newton $\sum \vec{F}_i = 0$ ⇒ $\Delta\vec{p} = \vec{0}$

totali

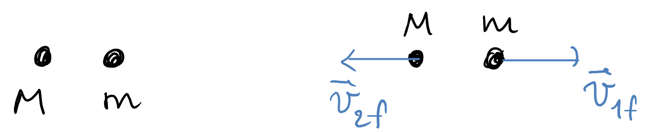
vettoriale

Sistema non isolato

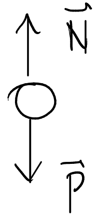
- $\vec{F}_e$ conservative ⇒ $\Delta E = 0$
- $\sum \vec{F}_e = \vec{0}$ ⇒ $\Delta\vec{p} = \vec{0}$

contro-esempi: attrito, forza esterna variabile nel tempo

Arciere:



iniziale finale

$\rightarrow$ 

$\sum \vec{F}_e = \vec{0}$, $\sum \vec{F}_i = \vec{0}$ $\Rightarrow$ $\sum \vec{F} = \vec{0}$

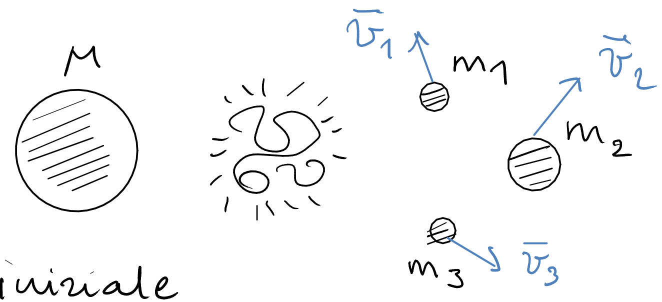
III Newton

Legge conservazione q. moto

$$\vec{p}_i = \vec{p}_f \rightarrow \vec{0} = m\vec{v}_{1f} + M\vec{v}_{2f} \Rightarrow m v_{1f}\,\vec{e}_x + M v_{2f}\,\vec{e}_x = \vec{0}$$

$$m v_{1f} + M v_{2f} = 0 \Rightarrow v_{2f} = -\frac{m}{M} v_{1f} = -\frac{0{,}03\ \text{kg}}{60\ \text{kg}} \times 85\ \frac{\text{m}}{\text{s}} = -0{,}042\ \frac{\text{m}}{\text{s}} \checkmark$$

Disintegrazione, decadimento:

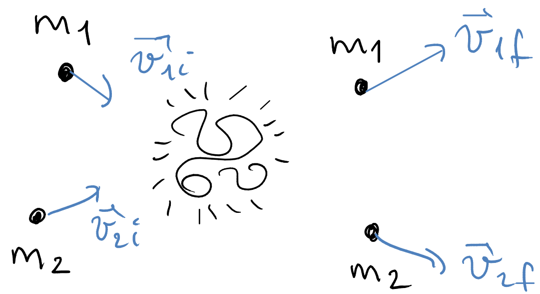


iniziale finale

$\sum \vec{F}_e = \vec{0}$

$\vec{p}_i = \vec{p}_f$ $\qquad \vec{0} = \sum_{j=1}^{3} m_j \vec{v}_j$

Urti:



iniziale finale

$\sum \vec{F}_e = \vec{0}$

$$\vec{p}_i = \vec{p}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$m_1(\vec{v}_{1f} - \vec{v}_{1i}) = m_2(\vec{v}_{2f} - \vec{v}_{2i})$$

$$m_1 \Delta\vec{v}_1 = m_2 \Delta\vec{v}_2$$

Sistema isolato

- $\sum \vec{F} = \vec{0} \Rightarrow \qquad \vec{p}_i = \vec{p}_f$
- F_i conservative $\qquad E_i = E_f \rightarrow E_{ci} = E_{cf}$ (in i e f interazioni tra corpi trascurabili)

elastici : $\quad E_{ci} = E_{cf} \qquad \Delta E_c = 0$

anelastici : $\quad E_{ci} > E_{cf} \qquad \Delta E_c < 0 \qquad$ ⚠

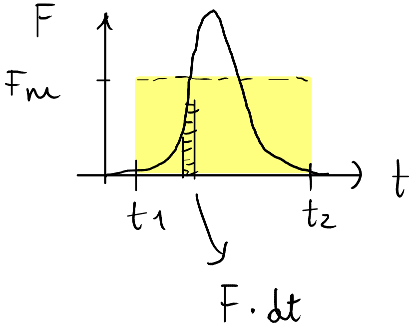
$$\begin{cases} dE_c = (\sum \vec{F}) \cdot d\vec{r} = \delta W \\ d\vec{p} = \underbrace{(\sum \vec{F})\, dt}_{\text{impulso elementare}} \equiv \delta I \end{cases}$$

$\rightarrow \quad dE = dE_c + dE_p = \delta W [\sum \vec{F}_{nc}]$

$$\frac{d\vec{p}}{dt} = \sum \vec{F}$$

$[I] = [F] \cdot [dt]$

SI: $N \cdot s$



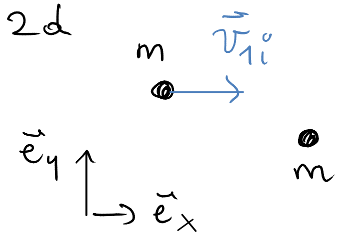
$$\int_{t_1}^{t_2} F\, dt = I = F_m \cdot \Delta t = \boxed{\Delta \vec{p}}$$

$\Delta t = t_2 - t_1$

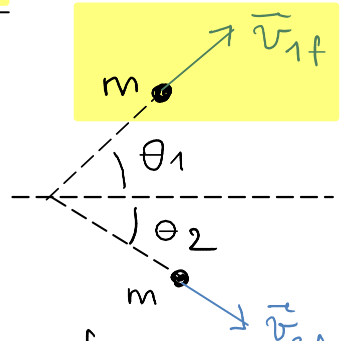
Es.: Airbag

Es.: urto tra 2 protoni

2d



iniziale



finale

$|\vec{v}_{1i}| = 3.5 \times 10^5 \ m/s$

$|\vec{v}_{1f}| = 2.8 \times 10^5 \ m/s$

$\theta_1 = 37°$

isolato

$\Rightarrow \quad \theta_2 = ? \ ; \quad |\vec{v}_{2f}| = ? \quad$ Elastico ?

$$\Sigma \vec{F} = \vec{0} \Rightarrow \quad \vec{p}_i = \vec{p}_f$$

$$m\vec{v}_{1i} + \vec{0} = m\vec{v}_{1f} + m\vec{v}_{2f}$$

$$\cancel{m} \, |\vec{v}_{1i}| \, \vec{e}_x = \cancel{m} |\vec{v}_{1f}| \cos\theta_1 \, \vec{e}_x + \cancel{m} |\vec{v}_{1f}| \sin\theta_1 \, \vec{e}_y + \cancel{m} |\vec{v}_{2f}| \cos\theta_2 \, \vec{e}_x - \cancel{m} |\vec{v}_{2f}| \sin\theta_2 \, \vec{e}_y$$

$$\begin{cases} |\vec{v}_{1i}| = |\vec{v}_{1f}| \cos\theta_1 + |\vec{v}_{2f}| \cos\theta_2 \\ 0 = |\vec{v}_{1f}| \sin\theta_1 - |\vec{v}_{2f}| \sin\theta_2 \end{cases} \Rightarrow \quad |\vec{v}_{2f}| = \frac{\sin\theta_1}{\sin\theta_2} |\vec{v}_{1f}|$$

$$E_{ci} = \frac{1}{2} m |\vec{v}_{1i}|^2$$

$$E_{cf} = \frac{1}{2} m |\vec{v}_{1f}|^2 + \frac{1}{2} m |\vec{v}_{2f}|^2$$

$$|\vec{v}_{1i}| = |\vec{v}_{1f}| \cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2 \, |\vec{v}_{1f}|$$

$$|\vec{v}_{1i}| - |\vec{v}_{1f}| \cos\theta_1 = \sin\theta_1 |\vec{v}_{1f}| \frac{1}{\tan\theta_2} \rightarrow \frac{|\vec{v}_{1f}| \sin\theta_1}{|\vec{v}_{1i}| - |\vec{v}_{1f}| \cos\theta_1} = \tan\theta_2$$

$$\theta_2 = \arctan\left(\frac{|\vec{v}_{1f}| \sin\theta_1}{|\vec{v}_{1i}| - |\vec{v}_{1f}| \cos\theta_1} \right) = 53°$$

$$\Rightarrow \quad |\vec{v}_{2f}| = \frac{\sin\theta_1}{\sin\theta_2} |\vec{v}_{1f}| = 2.1 \times 10^5 \ m/s$$