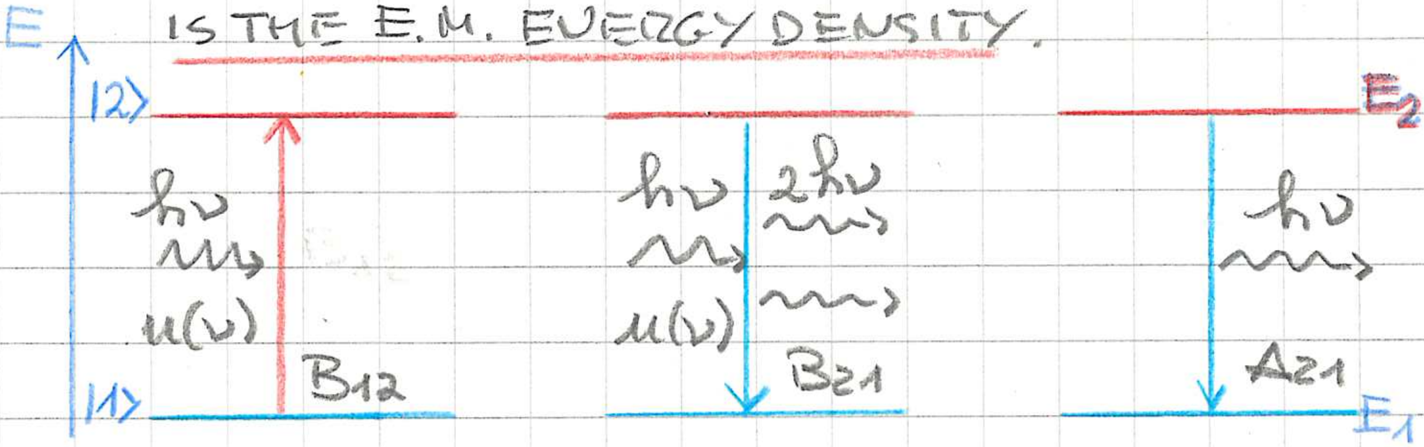
QUANTUM THEORY OF RADIATIVE ABSORPTION AND EMISSION

EINSTEIN'S TWO LEVELS MODEL AND TRANSITION PROBABILITY COEFFICIENTS

THE QUANTUM THEORY OF RADIATION ASSUMES THAT LIGHT IS EMITTED OR ABSORBED WHENEVER A QUANTUM SYSTEM (ATOM, MOLECULE OR SOLID) MAKES A JUMP BETWEEN TWO QUANTUM STATES. THE ABSORPTION PROCESS TAKES PLACE WHEN A QUANTUM OF ENERGY IS ABSORBED BY A SYSTEM THAT GET EXCITED. INSTEAD THE EMISSION PROCESS IS MORE COMPLEX. THIS PROCESS STARTS FROM AN EXCITED QUANTUM SYSTEM THAT CAN RELEASE A QUANTUM OF ENERGY (PHOTON) UNDER THE INTERACTION WITH A RADIATION FIELD (STIMULATED EMISSION) OR AS A SPONTANEOUS ACTION (SPONTANEOUS EMISSION). IN COMMON THESE PROCESSES HAVE THE ENERGY CONSERVATION SO THAT $\Delta E = h\nu$. A UNIFIED PICTURE OF THESE PROCESSES WAS FIRST GIVEN BY EINSTEIN INTRODUCING THE CONCEPT OF TRANSITION PROBABILITY FROM AN INITIAL STATE TO A FINAL STATE. ALTHOUGH, NOT NECESSARY TO EXPLAIN THE ABSORPTION AND STIMULATED EMISSION EISTEIN MADE THE ASSUMPTION THE THE E.M. FIELD ENERGY IS QUANTIZED,

ASSUMPTION THAT HOWEVER IS A "CONDITIO SINE QUA NON" TO ACCOUNT FOR THE SPONTANEOUS EMISSION. TO MAKE MORE EXPLICIT THE MECHANISMS BEHIND THIS MODEL LET'S USE THE FOLLOWING PICTURE, WHERE $u(\nu)$ IS THE E.M. ENERGY DENSITY.



ABSORPTION — STIMULATED EMISSION — SPONTANEOUS EMISSION

COMBINING A STATISTICAL ENSAMBLE OF N SYSTEMS WITH N_1 IN $|1\rangle$ AND N_2 IN $|2\rangle$ SO BEING $N = N_1 + N_2$ AND CONSIDERING B_{12}, B_{21} AND A_{21} THE ABSORPTION, STIMULATED EMISSION AND SPONTANEOUS EMISSION PROBABILITIES RESPECTIVELY WE CAN NOW WRITE THE RATE EQUATIONS FOR SUCH PROCESSES

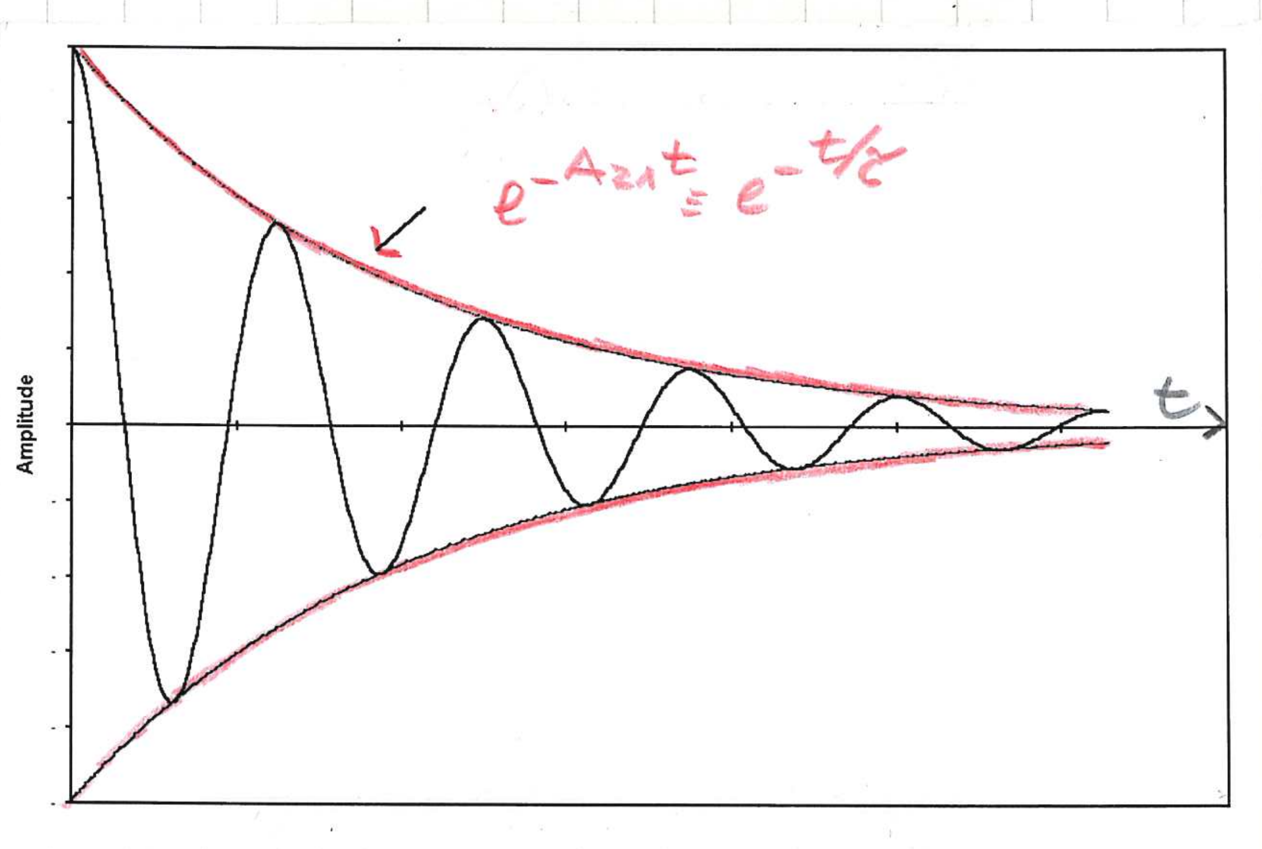
- ABSORPTION
$$\frac{dN_1(t)}{dt} = -B_{12} N_1(0)\, u(\nu) \qquad (125)$$

- STIMULATED EMISSION
$$\frac{dN_2(t)}{dt} = -B_{21} N_2(0)\, u(\nu) \qquad (126)$$

- SPONTANEOUS EMISSION $\frac{dN_2(t)}{dt} = -A_{21} N_2(0)$ (127)

- <u>OBSERVATION</u>: THIS LAST DIFF. EQ CAN BE SOLVED FOR $N_2(t)$ TO GIVE $N_2(t) = N_2(0)\exp(-A_{21}t)$ BY MAKING $A_{21} = \frac{1}{\tau} \Rightarrow N_2(t) = N_2(0)\exp-\left(\frac{t}{\tau}\right)$ (128)

$\Rightarrow$ A_{21} HAS THE DIMENSION OF t^{-1} (FREQUENCY) AND IS THE DEMPING TIME OF AN HARMONIC OSCILLATOR THAT RELAXES FROM AN EXICITED STATE TO A GROUND STATE, HOWEVER IN THE QUANTUM MODEL IS THE TRANSITION PROBABILITY OF THE SPONTANEOUS EMISSION



WE CAN GO EVEN FURTHER. IN THE CLASSICAL DESCRIPTION THE POWER OUTPUT OF THE DAMPED H.O. WILL DECAY EXPONENTIALLY FOR $W \propto E_0^2$ BEING E_0 THE AMPLITUDE OF THE $\bar{E}$ FIELD $\Rightarrow W(t) = W(0)\, e^{-t/\tau}$.
IF WE REPRESENT $\bar{E} = \bar{E}_0 \cos(\omega_0 t)$, IF E_0 DECAYS EXPONENTIALY THE COMPLEX REPRESENTATION OF THE $\bar{E}(t) = E_0 \exp\left[-t\left(\frac{1}{2\tau} + i\omega_0\right)\right]$
THE FREQUENCY DEPENDENCE OF THE $E_0(\omega)$ IS GIVEN BY THE F.T.

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} E(t)\, e^{-i\omega t}\, dt$$

$$= \frac{E_0}{\sqrt{2\pi}} \left[\frac{1}{i(\omega_0 - \omega) + 1/2\tau}\right] \quad (129)$$

THE IRRADIANCE AS A FUNCTION OF FREQUENCY IS

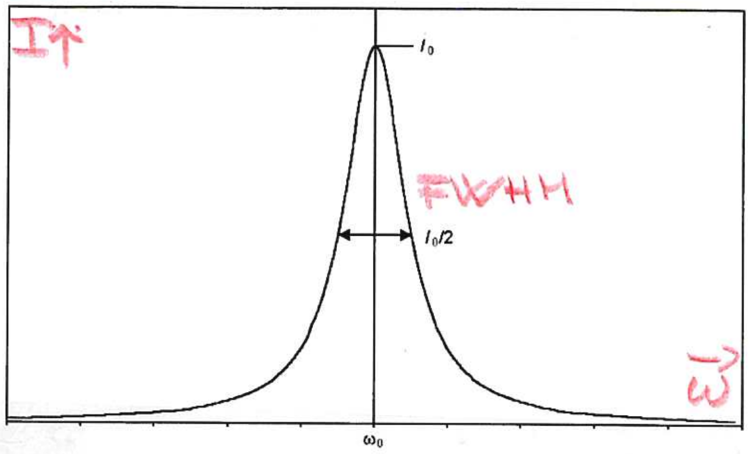
$$I(\omega) = \tilde{E}(\omega) \cdot \tilde{E}^*(\omega) = \frac{E_0^2}{\sqrt{2\pi}} \left[\frac{1}{(\omega - \omega_0)^2 + 1/4T^2}\right]$$

$$\Rightarrow I(\omega) = I_0 \left[\frac{1/4T^2}{(\omega_0 - \omega)^2 + 1/4T^2}\right] \quad \text{WHICH IS} \quad (130)$$

A LORENTIAN FUNCTION WITH A FWHM (FULL WIDTH HALF MAXIMUM; $\Delta\omega$ AT $I_0/2$) OF $\Delta\omega = 1/\tau$). GOING TO THE EINSTEIN'S MODEL THE FWHM $\Delta\omega$ IN THE FREQUENCY

DOMAIN IS NOTHING BUT THE TRANSITION PROBABILITY A_{21}.

INTERESTING TO NOTE THAT FOR A MONOCROMATIC WAVE $\Delta\omega = 0 \Rightarrow A_{21} = 0 \Rightarrow$ THE PROBABILITY TO GENERATE A MONOCROMATIC WAVE BY SPONTANEOUS EMISSION IS ZERO. WE MUST ALSO REMEMBER THAT $1/2 = \gamma$ WHERE γ IS THE DAMPING RATE $\Rightarrow$ FOR AN ISOLATED ATOM $\gamma \equiv A_{21} \Rightarrow$ IN Q.M. THE DAMPING RATE MUST BE INTERPRETED AS THE SPONT. EMISS. TRANS. PROBAB.



AT EQUILIBRIUM THE $N_1 \to N_2$ TRANSITIONS MUST EQUAL THE $N_2 \to N_1$ TRANSITIONS PER UNIT OF TIME $\Rightarrow$

$$\underset{\text{ABS}}{\frac{dN_1(t)}{dt}} = \underset{\text{STIM EMIS}}{\frac{dN_2(t)}{dt}} + \underset{\text{SPONT. EMISSION}}{\frac{dN_2^A(t)}{dt}} \quad (131)$$

$$\Rightarrow B_{12} N_1(0) u(\nu) = B_{21} N_2(0) u(\nu) + A_{21} N_2(0) \quad (132)$$

SINCE WE DEAL WITH A STATISTICAL SYSTEM THE BOLTZMANN PARTITION FUNCTION APPLIES $\Rightarrow$

$$\frac{N_2}{N_1} = e^{-\Delta E / K_B T} \quad (133)$$

SO FROM (132) WE OBTAIN

$$\frac{N_1}{N_2} = e^{\Delta E / K_B T} = \frac{B_{21} u(\nu) + A_{21}}{B_{12} u(\nu)} \Rightarrow (134)$$

$$u(\nu) = \frac{A_{21}}{B_{12} e^{\Delta E / K_B T} - B_{21}} \quad (135) \quad \text{BUT } \Delta E = E_2 - E_1 = h\nu$$

OBSERVATION ΔE IS THE ENERGY GAP BETWEEN THE STATES $|1\rangle$ AND $|2\rangle$ THAT AT RESONANCE COINCIDES WITH THE PHOTON ENERGY $h\nu$.

(134) MUST HOLD FOR ALL THE ν AND T.

AT LOW FREQUENCY THE RAYLEIGH-JEANS LAW DERIVED FROM THE CLASSICAL E.D. IS CONSISTENT WITH THE B.B. RADIATION, IN MORE QUANTITATIVE TERMS THE R-J HOLDS WHEN $h\nu \ll K_B T \Rightarrow u(\nu) = \frac{8\pi\nu^2 K_B T}{c^3}$

$$\Rightarrow e^{h\nu / K_B T} \simeq 1 + h\nu / K_B T + \dots \quad (136)$$

INSTEAD FOR $T \to \infty \Rightarrow u(\nu) \to \infty$ AND $e^{h\nu / K_B T} \to 1$ SO (135) BECOMES

$$u(\nu) \to \infty \simeq \frac{A_{21}}{B_{12} - B_{21}} \Rightarrow B_{12} = B_{21} \equiv B$$

$$\Rightarrow (135) \quad u(\nu) = \frac{A_{21}}{B} \cdot \frac{1}{(e^{h\nu / K_B T} - 1)} \quad (137)$$

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UNDER THE CONDITION $h\nu \ll K_B T$ (137) BECOMES

$$u(\nu) \simeq \frac{A_{21}}{B} \frac{1}{1+\frac{h\nu}{K_B T}-1}$$

$$\simeq \frac{A_{21}}{B} \frac{K_B T}{h\nu}$$

BUT FOR $h\nu \ll K_B T$ THE R-J HOLDS $\Rightarrow$

$$u(\nu) = \frac{8\pi\nu^2 K_B T}{c^3} \Rightarrow \frac{A_{21}}{B}\frac{K_B T}{h\nu} \simeq \frac{8\pi\nu^2 K_B T}{c^3}$$

$$\Rightarrow \frac{A_{21}}{B} \simeq \frac{8\pi h\nu^3}{c^3} \Rightarrow \text{(137) BECOMES}$$

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/K_B T}-1}$$

WHICH IS THE PLANCK'S LAW, AS EXPECTED SINCE THE SYSTEM RADIATION-MATTER IS AT EQUILIBRIUM HENCE IS A BB.

OBSERVATION IT IS INTERESTING TO NOTE FROM THE AVERAGE ENERGY EQ. OF THE MODE $\langle E \rangle = \frac{h\nu}{e^{h\nu/K_B T}-1}$ IS POSSIBLE TO OBTAIN THE AVERAGE NUMBER OF PHOTONS IN THE MODE $\langle n \rangle = \frac{\langle E \rangle}{h\nu} = \frac{1}{e^{h\nu/K_B T}-1}$ (138)

WE CAN NOW CONSIDER A GENERIC MODE

OF THE BB. FOR EXAMPLE $h\nu \simeq 1$ eV $\Rightarrow$ $\nu \simeq 2.4 \times 10^{14}$ Hz. WITH $K_B \simeq 8.617 \times 10^{-5}$ eV/K $\Rightarrow K_B T \simeq 2.58 \times 10^{-2} \simeq 0.025$ eV $\Rightarrow \langle n \rangle \simeq e^{-40}$ $\simeq 4.25 \times 10^{-18}$ PHOTONS AT $T \simeq 300K$.

FOR A LASER THE POWER OUTPUT

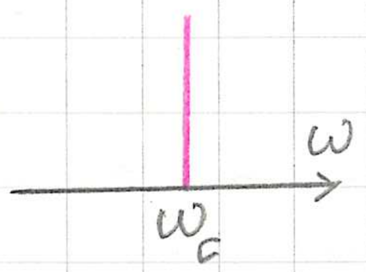
$$W_{OUT} = \frac{\gamma_2 c}{2 L_e} h\nu \langle n \rangle \quad \text{WITH } \gamma_2 = -\ln(1-T_2)$$

WITH T_2 THE TRANSMISSION OF THE OUTPUT MIRROR OF THE LASER AND L_e THE LENGTH OF THE CAVITY. IF WE CONSIDER A He-Ne LASER OF 50 cm CAVITY, $\lambda \simeq 630$ nm WITH $W_{OUT} = 10$ mW $\Rightarrow \langle n \rangle \simeq 10^{10}$ PHOTONS.

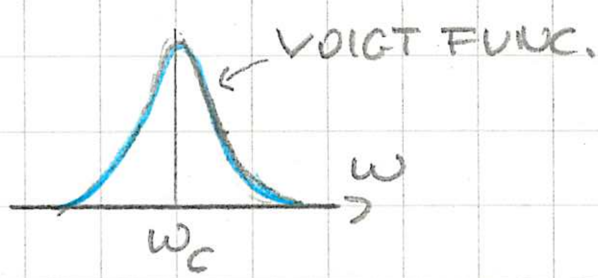
- INTERACTION OF THE E.M. FIELD WITH MATTER: THE SEMICLASSICAL MODEL

THE INTERACTION BETWEEN THE E.M. RADIATION AND THE MATTER IS DESCRIBED BY MODELS BASED ON THE ABRAHM-LORENTZ OSCILLATORS WHERE THE E.M. FIELDS INTERACT WITH A CHARGE THAT CAN BE FREE (DRUDE MODEL) OR BOUND OR WITH AN $\bar{E}$ FIELD INSIDE THE MATTER RESULTING FROM POLARIZATION EFFECTS. GIVING RISE TO THE PHOTON-PHONON INTERACTION. IN THE CLASSICAL DESCRIPTION THE MATTER-RADIATION RESONANCE IS SET WHEN THE PROPER FREQUENCY

OF THE OSCILLATOR, ω_0, METCHES THE ω OF THE E.M. WAVE, THAT IN AN IDEAL MODEL WE CONSIDER MONOCROMATIC. IN A REALISTIC EXPERIMENT THE E.M. WAVE IN THE FREQUENCY DOMAIN IS CHARACTERIZED BY A VOIGT FUNCTION (THE CONVOLUTION BETWEEN A GAUSSIAN AND A LORENTIAN FUNCTIONS) WHICH CENTRAL FREQUENCY IS REGARDED AS THE E.M. WAVE FREQUENCY



MONOCHROMATIC



WAVEPACKET

IN QUANTUM MODELS THE RESONANCE CONDITION IS ACHIEVED WHEN THE E.M. FIELD FREQUENCY MULTIPLIED BY h METCHES THE ENERGY DIFFERENCE BETWEEN TWO QUANTUM STATES WHICH SYMMETRY ALLOWS AN ELECTRICAL DIPOLE TRANSITION $\Rightarrow \Delta E = E_2 - E_1 = h\nu$.

WE NEED TO UNDERSTAND THE MEANING OF THE WORDS IN THIS SENTENCE. TO DO SO WE START BY CONSIDERING THE STATE OF MATTER QUANTIZED BUT THE E.M. WAVE, WHICH IS TREATED AS CLASSICAL.

FOR THIS REASON THIS MODEL IS KNOWN AS SEMICLASSICAL. THE STARTING POINT, AS WE LEARNED WE NEED FIRST TO BUILD THE HAMILTONIAN THAT DESCRIBE THE INTERACTION

$H = H_M + H_L + H_{LM}$ WHERE HERE, H_M = HAMIL. OF THE MATTER, H_L IS THE HAMILTONIAN OF THE LIGHT AND H_{LM} IS THE HAMILTONIAN OF THE INTEACTION

H_M IN GENERAL IS TIME-INDEPENDENT

H_{LM} IN GENERAL IS TIME-DEPENDENT

TO DESCRIBE THE LIGHT-MATTER INTERACTION, AT LEAST FOR THE SEMI-CLASSICAL MODEL, WE CAN IGNORE $H_L \Rightarrow H \simeq H_M + H_{LM}$. H_{LM} IS ALSO KNOWN AS ELECTRIC DIPOLE HAMILTONIAN. AS WE HAVE SEEN IN THE PREVIOUS LECTURES, FROM THE CLASSICAL HAMILTONIAN BY USING THE QUANTUM OPERATORS $p \to -i\hbar\hat{\nabla}$, $x \to \hat{x}$ WE OBTAIN THE SEMI-CLASSICAL $\hat{H}_{LM}$ WHERE THE E.M. IS TREATED BY THE VECTOR POTENTIA $\bar{A}$.

- RIVISITING THE E.D. FIELDS AND POTENTIALS

THE H_{LM} SHOULD BE FORMULATED USING THE E.M. POTENTIALS RATHER THAN THE E.M. FIELDS; $\bar{B} = \bar{\nabla} \times \bar{A}$; $\bar{E} = -\bar{\nabla}\varphi - \partial_t \bar{A}$. THIS LAST FOR AN E.M. WAVE REDUCES TO $\bar{E} = -\partial_t \bar{A}$ BEING $\varphi = 0$ IN THE FREE SPACE.

WE NOTE ALSO THAT AN E.M. WAVE REPPRESENTED BY $\bar{E}$ AND $\bar{B}$ REQUIRES 6 VARIABLES (x, y, z FOR EACH FIELD). THIS IS A CONDITION DERIVED BY THE MAXWELL EQS WHERE THE 6 VARIABLES ARE NOT, IN GENERAL, LINEAR INDEPENDENT.
USING THE POTENTIALS WE DEALS WITH 4 INDEPENDENT VARIABLES (A_x, A_y, A_z, φ). HOWEVER $\bar{A}$ IS NOT UNIQUELY DETERMINED AND DIFFERET GAUGES CAN BE USED.
WE CAN START FROM THE GENERAL RELATION BETWEEN THE POTENTIALS $\bar{A}$ AND φ

(139) $$\nabla^2 \bar{A} - \mu_0 \varepsilon_0 \partial_t^2 \bar{A} = -\mu_0 \bar{J} + \mu_0 \varepsilon_0 \bar{\nabla}(\partial_t \varphi)$$

IN FREE SPACE IT BECOMES ($\bar{J} = 0$, $\varphi = 0$)

$\Downarrow$

(140) $$\nabla^2 \bar{A} - \mu_0 \varepsilon_0 \partial_t^2 \bar{A} = 0 \quad \Rightarrow$$

THE LORENTZ'S GAUGE REDUCES TO THE COULOMB GAUGE. $(\bar{\nabla} \cdot \bar{A} + \underbrace{\mu_0 \varepsilon_0 \partial_t \varphi}_{=0}) = 0 \Rightarrow$

$\bar{\nabla} \cdot \bar{A} = 0$ THIS MEANS THAT THE COULOMB GAUGE HOLDS ALSO FOR TIME DEPENDENT E.M. FIELDS IN THE FREE SPACE. THIS BRINGS US TO A WAVE EQ. FOR $\bar{A}$

(141) $$\boxed{\nabla^2 \bar{A}(\bar{r}, t) - \frac{1}{c^2} \partial_t^2 \bar{A}(\bar{r}, t) = 0}$$

• OBSERVATION. TO QUANTIZE $\bar{A}$ FOR AN E.M. FIELD IN THE EMPTY SPACE IS BY FAR MORE

CONVENIENT THAT QUANTIZING BOTH $\bar{E}$ AND $\bar{B}$.

- IN THE FREE SPACE $\bar{E}$ AND $\bar{B}$ ARE BOTH DEFINED BY $\bar{A}$ AND $\bar{\nabla}\cdot\bar{A}=0$.

SOLUTION OF 141 IS

$$\bar{A}(\bar{r},t) = A_0\,\hat{\varepsilon}\left[e^{i(\bar{k}\cdot\bar{r}-\omega t)} + \underbrace{e^{-i(\bar{k}\cdot\bar{r}-\omega t)}}_{\text{c.c.}}\right] \quad (142)$$

FROM THIS EQ WE CAN DERIVE THE FIELDS

$$\bar{E} = -\partial_t\bar{A} = i\omega A_0\hat{\varepsilon}\left(e^{i(\bar{k}\cdot\bar{r}-\omega t)} + \text{c.c.}\right) = \bar{E}^{+}(\bar{r})e^{-i\omega t} + \bar{E}^{-}(\bar{r})e^{i\omega t}$$

(143)

$$\bar{B} = \bar{\nabla}\times\bar{A} = i(\bar{k}\times\hat{\varepsilon})A_0\left(e^{i(\bar{k}\cdot\bar{r}-\omega t)} + \text{c.c.}\right) = \bar{B}^{+}(\bar{r})e^{-i\omega t} + \bar{B}^{-}(\bar{r})e^{i\omega t}$$

FROM WHICH $\hat{k}\perp\hat{\varepsilon}\perp\frac{\bar{k}\times\hat{\varepsilon}}{|\bar{k}|} \Rightarrow \bar{k}\perp\bar{E},\bar{B}$; $\bar{E}\perp\bar{B}$

ALL WELL KNOWN, IT IS ALSO CLEAR THAT $\bar{A}$ OSCILLATES IN ANTIPHASE RESPECT TO THE FIELDS, $\bar{A}\parallel\bar{E}$ AND $\perp\bar{B}$. NOW REDUCING THE COMPLEX FORMALISM TO REAL FUNCTION WE OBTAIN $\frac{1}{2}E_0 = -\omega A_0$, $\frac{1}{2}B_0 = -|\bar{k}|A_0$

AND $E_0/B_0 = \omega/|k| = c$.

- THE MATRIX ELEMENT AND THE ELECTRIC DIPOLE OPTICAL TRANSITION PROBABILITY

AS WE HAVE SEEN IN CLASSICAL E.D, THE ABSORPTION COEFFICIENT FOR A SINGLE e^--LORENTZ OSCILLATOR IS GIVEN BY

$$\alpha(\omega) \simeq \frac{Ne^2\omega^2}{m\varepsilon_0 c} \frac{\gamma}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \quad (144)$$

N = OSCILLATOR DENSITY

ω_0 = OSCILLATOR PROPER FREQUENCY

γ = OSCILLATOR DAMPING FACTOR

ω = E.M. FIELDS FREQUENCY

m = MASS OF THE CHARGE (HERE e^- MASS)

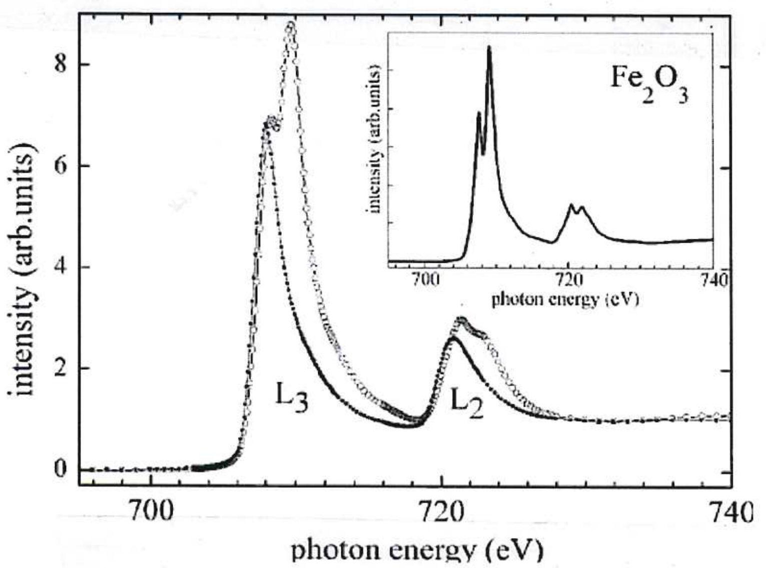
ON RESONANCE THE CROSS SECTION FOR ABSOR. IS DEFINED AS

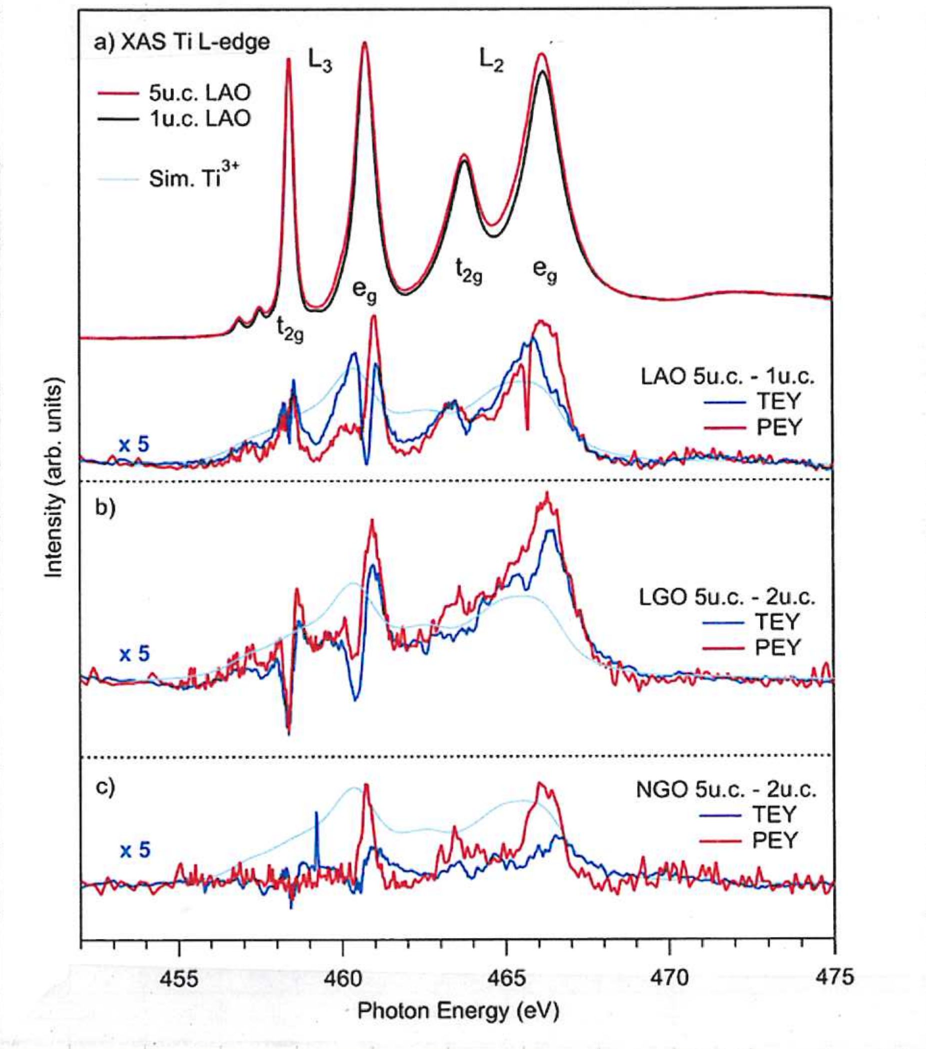
$$\alpha(\omega_0) = \sigma(\omega_0) N \quad (145)$$

FROM (144) AT RESONANCE ($\omega = \omega_0$)

$$\alpha_{CLASS}(\omega_0) = \frac{e^2 N}{m\varepsilon_0 \gamma c} \quad (146)$$

=> FOR A GIVEN OSCILLATOR THE ABSORBED RADIATION IS $\propto$ TO N. HOWEVER THE EXPERIMENTS SHOW A MUCH COMPLEX FIGURE.





THIS RESULT IS NOT SO UNEXPECTED FOR WE KNOW THAT ANY ABSORPTION TRANSITION DEPENDS ON A PROBABILITY COEFFICIENT THAT IN THE TWO LEVELS MODEL WAS MARKED AS B_{12}. THIS PROBABILITY IS A QUANTUM BEHAVIOR THAT CHARACTERIZED ANY SPECIFIC ELECTRIC DIPOLE OPTICAL TRANSITION.

TO DO THESE CALCULATIONS WE MUST START FROM THE INTERACTION HAMILTONIAN AS WE HAVE LEARNED IN THE PREVIOUS LECTURES.

FOR AN ELECTRONIC TRANSITION WE HAVE

$$\boxed{\hat{H}_{LM} = \frac{e}{2m}\left(\hat{p}\cdot\bar{A} + \bar{A}\cdot\hat{p}\right)} \quad (147)$$

WE MUST REMEMBER THAT IN THE QUANTUM FORMALISM $\hat{p}$ AND $\bar{A}$ MUST BE TREATED AS OPERATORS, WITH THE GAUGE $\bar{\nabla}\cdot\bar{A}=0$. FOR NOW WE TREAT $\bar{A}$ AS A VECTOR OPERATOR (CLASSICAL FIELD). IN STANDARD VECTORS ALGEBRA $\bar{p}\cdot\bar{A}=\bar{A}\cdot\bar{p}$ ($\bar{p}$ AND $\bar{A}$ COMMUTE). BUT THIS IS NOT GENERALLY TRUE FOR THE OPERATOR ALGEBRA. SO WE HAVE TO SHOW THAT $[\hat{p},\bar{A}]=0$. BY DEFINITION $[\hat{p},\bar{A}]=\hat{p}\cdot\bar{A}-\bar{A}\cdot\hat{p}$. FOR THE SAKE OF SIMPLICITY WE WILL PROVE THAT $\hat{p}$ AND $\bar{A}$ COMMUTE ONLY FOR THE X COORDINATE: $\underline{[p_x, A_x]} = \frac{\hbar}{i}\partial_x A_x - A_x\frac{\hbar}{i}\partial_x =$

$$(148) = \frac{\hbar}{i}(\partial_x A_x) + \frac{\hbar}{i}A_x\partial_x - A_x\frac{\hbar}{i}\partial_x = \frac{\hbar}{i}(\partial_x A_x)$$

$$\Rightarrow \underline{\hat{p}\cdot\bar{A}-\bar{A}\cdot\hat{p} = \frac{\hbar}{i}\bar{\nabla}\cdot\bar{A}} = 0 \quad \text{WITH } \bar{\nabla}\cdot\bar{A}=0$$

$\Rightarrow$ THE INTERACTING $\hat{H}_{LM} = -\frac{q}{m}\hat{p}\cdot\bar{A}$.

FOR A CLASSICAL FIELD $\bar{A}=\bar{A}_0 e^{-i(\bar{k}\cdot\bar{r}-\omega t)}$ WITH $\bar{A}_0 \equiv |\bar{A}_0|\cdot\hat{\varepsilon}$ ($\hat{\varepsilon} \equiv$ POLARIZATION UNIT VECTOR). THE SPATIAL COMPONENT OF THE VECTOR POTENTIAL FIELD $e^{i(\bar{k}\cdot\bar{r})}$, CAN BE DEVELOPED AS

$$e^{i\bar{k}\cdot\bar{r}} = 1 + i\bar{k}\cdot\bar{r} + \frac{1}{2}(i\bar{k}\cdot\bar{r})^2 + \dots$$

WHEN $|\bar{k}\cdot\bar{r}| \simeq \frac{2\pi}{\lambda}|\bar{r}| \ll 1$ THE HIGHER ORDER

TERM OF THE SERIES CAN BE DISREGARDED AND THE $\hat{H}_{LM} = -\frac{q}{m}\bar{A}\cdot\hat{p} = -\frac{q}{m}\left[A_0\hat{\varepsilon}\cdot\hat{p}\,e^{-i\omega t} + c.c.\right]$ 149

CAN BE REWRITTEN USING $A_0 = iE_0/2\omega$

AS $$\hat{H}_{LM} = \frac{-iqE_0}{2m\omega}\left[\hat{\varepsilon}\cdot\hat{p}\,e^{-i\omega t} - \hat{\varepsilon}\hat{p}\,e^{i\omega t}\right] \quad 150$$

REDUCING THE COMPLEX REPRESENTATION TO A REAL FUNCTION

$$\hat{H}_{LM} = -\frac{qE_0}{2\omega}\left(\hat{\varepsilon}\cdot\hat{p}\right)\sin\omega t. \quad 151$$

KNOWING NOW THE $\hat{H}_{LM}$, FROM THE Q.M. THE TRANSITION PROBABILITY FROM A STATE $|1\rangle$ TO A STATE $|2\rangle$ (FROM $E_1 \rightarrow E_2$ WITH $E_2 > E_1$: THIS DEFINES THE ABSORPTION PROCESS) WE IS GIVEN BY THE INTEGRAL

$$152 \quad M_{12} = \langle 2|H_{LM}|1\rangle$$ USING

$-q \equiv e$

$$153 \quad \boxed{M_{12} = \frac{e}{m}\langle 1|\hat{p}\cdot\bar{A}|2\rangle}$$. THIS INTEGRAL TAKES THE NAME OF MATRIX ELEMENT.