Fondamenti di Automatica

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Regola di Cartesio

Dato un polinomio a coefficienti REALI non tutti nulli

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

$a_i \in \mathbb{R} \quad i = 1, 2, \dots n$

allora (o) il NUMERO MASSIMO di RADICI REALI POSITIVE del polinomio $P(x)$ e' dato dal numero di variazioni di segno fra coefficienti consecutivi, trascurando eventuali coefficienti nulli

(oo) il NUMERO MASSIMO di RADICI REALI NEGATIVE del polinomio $P(x)$ e' dato dal numero di permanenze di segno fra coefficienti consecutivi, trascurando eventuali coeff. nulli

(ooo) Il NUMERO MINIMO di RADICI COMPLESSE e' pari all'ordine del polinomio ridotto del n° massimo di radici reali positive e negative

2

Il criterio di Routh-Hurwitz permette di determinare il n° effettivo di radici a parte reale positiva e negativa

③ Nell'esercizio

$$p(s) = s^3 + 2(2\mu-1)s^2 + (8\mu+1)s + 4\mu$$

Criterio di Routh-Hurwitz

$$\begin{array}{c|cc}
3 & 1 & (8\mu+1) \\
2 & 2(2\mu-1) & 4\mu \\
1 & (16\mu^2-8\mu-1)/(2\mu-1) & \\
0 & 4\mu &
\end{array}$$

$\Updownarrow$ lo studio del segno della 1ª colonna porta a:

stab. asintotica $\quad \mu > \frac{1+\sqrt{2}}{4}$

stab. semplice $\quad \mu = \frac{1+\sqrt{2}}{4}$

instabilità $\quad \mu < \frac{1+\sqrt{2}}{4}$

④ Applico la regola di Cartesio:

$$p(s) = s^3 + 2(2\mu - 1)s^2 + (8\mu + 1)s + 4\mu$$

Per avere permanenze di segno:

$$\begin{cases} 1 > 0 \\ 2\mu - 1 > 0 \\ 8\mu + 1 > 0 \\ 4\mu > 0 \end{cases} \qquad \begin{cases} \mu > \frac{1}{2} \\ \mu > -\frac{1}{8} \\ \mu > 0 \end{cases} \qquad \boxed{\mu > \frac{1}{2}}$$

NB Per $\mu > \frac{1}{2}$ per la regola di Cartesio posso SOLO dire che

(·) il massimo n° di radici REALI POSITIVE è 0

(··) il massimo n° di radici REALI NEGATIVE è 3

(···) il minimo n° di radici COMPLESSE è 0

(5) NB non so il segno delle radici complesse!

Es. scelgo $\mu = 0{,}51$

$\downarrow$

$$P(s) = s^3 + 0{,}04\,s^2 + 5{,}08\,s + 2{,}04$$

fattorizzo: (es. comando MATLAB

ROOTS([1 0,04 5,08 2,04]))

$$\boxed{\begin{aligned} p_1 &= +0{,}1755 + j\,2{,}2774 \\ p_2 &= +0{,}1755 - j\,2{,}2774 \end{aligned}}$$

NB se parte reale positiva! il criterio di Routh-Hurwitz lo aveva segnalato

$$\boxed{p_3 = -\,0{,}3910}$$

Risultato $\Rightarrow$ (i) radici REALI POSITIVE $\to 0$ OK

(ii) radici REALI NEGATIVE $\to 1$

(iii) radici complesse $\to 2$

doveva essere ≥ 0

OK

doveva essere ≤ 3

Calcolo di e^{At}

utilizzando la

forma diagonale

della matrice A

Dato il sistema LTI descritto da:

$$\begin{cases} \dot{x} = Ax + 3u \\ y = Cx \end{cases} \qquad A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$\mathrm{Tr}A > 0$

$\Downarrow$

sistema inst.

$$e^{At} = ?$$

A diagonalizzabile $\Longrightarrow$ $D = T^{-1} A T$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$T = \left[v_1 \,\middle|\, v_2 \,\middle|\, v_3 \right]$$

v_j autovettore associato a λ_j

Cerco autovalori ed autovettori corrispondenti:

$$p_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} (\lambda-1) & -1 & 0 \\ 0 & (\lambda-0) & -2 \\ -1 & 0 & (\lambda-1) \end{vmatrix} =$$

$=$ ← sviluppo il determinante lungo la 1° colonna

$$= (\lambda-1)(-1)^{1+1} \cdot \begin{vmatrix} \lambda & -2 \\ 0 & \lambda-1 \end{vmatrix} + (-1)\cdot(-1)^{3+1} \cdot \begin{vmatrix} -1 & 0 \\ \lambda & -2 \end{vmatrix}$$

$$= (\lambda-1)\left[\lambda(\lambda-1)\right] + (-1)\left|2\right| = \lambda(\lambda-1)^2 - 2$$

$$= \lambda\left[\lambda^2 - 2\lambda + 1\right] - 2 = \lambda^3 - 2\lambda^2 + \lambda - 2 = \Rightarrow$$

$$\boxed{p_A(\lambda) = (\lambda^2+1)[\lambda-2]}$$

$\lambda_1 = -j$

$\lambda_2 = +j$

$\lambda_3 = +2$

3 autovalori distinti $\Downarrow$ A è diagonalizzabile

Autovettori

v_3: $\quad A v_3 = \lambda_3 v_3 \to (A-\lambda_3 I)\, v_3 = 0$

$$v_3 \in \ker \boxed{(A-\lambda_3 I)}$$

$$(A-2I) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} =$$

$$(A-2I) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\boxed{(A-2I)v_3 = 0} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{cases} -a+b = 0 \\ 2(-b+c) = 0 \\ a - c = 0 \end{cases} \qquad \begin{cases} a = b \\ b = c \\ a = c \end{cases}$$

$$v_3 \in \left\langle \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\rangle$$

$$\lambda_1 = -j \qquad v_1 : (A - \lambda_1 I) v_1 = 0$$

$$v_1 \in \ker (A - \lambda_1 I)$$

$$(A - \lambda_1 I) = \begin{bmatrix} (1+j) & 1 & 0 \\ 0 & +j & 2 \\ 1 & 0 & (j+1) \end{bmatrix}$$

$$a, b, c \in \mathbb{C}$$

$$\boxed{(A - \lambda_1 I) v_1 = 0} \iff \begin{cases} (1+j) a + b = 0 \\ j b + 2c = 0 \\ a + (j+1) c = 0 \end{cases}$$

$$\begin{cases} (1+j)\,a + b = 0 \\ j\,b + 2c = 0 \\ a + (j+1)\,c = 0 \end{cases}$$

$$b = -(1+j)\,a$$

$$c = -\frac{j}{2}b = \frac{j(1+j)}{2}a$$

$$= \frac{-1+j}{2}a$$

Sostituisco nella 3ª equazione

$$a + (j+1)\frac{(-1+j)}{2}a = a + \left[\frac{(j)^2 - 1}{2}\right]a$$

$$= a + \left(-\frac{2}{2}\right)a = 0 \;\; \forall$$

In definitiva

$$\boxed{\begin{cases} a \text{ qualsiasi} \\ b = -(1+j)\,a \\ c = \dfrac{-1+j}{2}\,a \end{cases}}$$

$a = (1-j)$

$$v_1 \in \left\langle \begin{bmatrix} 1 \\ -(1+j) \\ \left(\dfrac{-1+j}{2}\right) \end{bmatrix} \right\rangle$$

v_2 è ovviamente il complesso coniugato di v_1

La matrice di trasformazione allora è:

$$\boxed{T = \left[\, v_1 \;\middle|\; v_2 \;\middle|\; v_3 \,\right]} = \longrightarrow$$

$$T = \begin{bmatrix} 1 & 1 & 1 \\ -(1+j) & -(1-j) & 1 \\ \left(\frac{-1+j}{2}\right) & -\left(\frac{1+j}{2}\right) & 1 \end{bmatrix} \qquad \det T = 5j$$

$$T^{-1} = -j\,\frac{1}{5} \cdot \begin{bmatrix} -\frac{1}{2}+\frac{3}{2}j & -\frac{3}{2}-\frac{1}{2}j & +2-j \\ +\frac{1}{2}+\frac{3}{2}j & +\frac{3}{2}-\frac{1}{2}j & -2-j \\ +2j & +j & +2j \end{bmatrix}$$

da cui $\qquad T^{-1} A T = D$

$$T^{-1}AT = \begin{bmatrix} -j & 0 & 0 \\ 0 & +j & 0 \\ 0 & 0 & +2 \end{bmatrix}$$

$$\boxed{A = TDT^{-1}}$$

Dra rale che:

$$\boxed{e^{At} = T\, e^{Dt}\, T^{-1}}$$

$\boxed{[\text{FA Parte 3 \#25}]}$

$$e^{At} = T \begin{bmatrix} e^{-jt}\cdot 1(t) & 0 & 0 \\ 0 & e^{jt}\cdot 1(t) & 0 \\ 0 & 0 & e^{2t}\cdot 1(t) \end{bmatrix} T^{-1} = \Rightarrow$$

$$\left[e^{At}\right]_{(1,1)} = \left[\frac{2}{5}e^{2t} + \left(\frac{3}{10}+j\frac{1}{10}\right)e^{-jt} + \left(\frac{3}{10}-j\frac{1}{10}\right)e^{jt}\right]\cdot 1(t)$$

formule di Eulero

$$\begin{cases} \operatorname{sen} x = \dfrac{e^{jx}-e^{-jx}}{2j} \\[2ex] \cos x = \dfrac{e^{jx}+e^{-jx}}{2} \end{cases}$$

$$+\frac{\cancel{6}^{\,3}}{\cancel{10}_{\,5}}\cdot\boxed{\frac{e^{jt}+e^{-jt}}{2}} \qquad +\frac{\cancel{2}^{\,1}}{\cancel{10}_{\,5}}\cdot\frac{e^{jt}-e^{-jt}}{2j} \qquad -\frac{2}{10}\left(-\frac{1}{j}\right)\cdot\frac{\quad}{2}$$

$$= \left[\frac{2}{5}e^{2t} + \frac{3}{10}\left(e^{-jt}+e^{jt}\right) - j\frac{1}{10}\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$\boxed{= \left[\frac{2}{5}e^{2t} + \frac{3}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right]\cdot 1(t)}$$

$$\operatorname{sen} x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$-j\frac{1}{10}\left(e^{jt} - e^{-jt}\right) =$$

$$-j\frac{\cancel{2}^{1}}{\cancel{10}_{5}}\left(\frac{e^{jt} - e^{-jt}}{2}\right) \qquad (j)^{-1} = \frac{1}{j} = -j$$

$$-j\frac{1}{5} \cdot \frac{e^{jt} - e^{-jt}}{2} \; \frac{\boxed{j}}{j}$$

$$\frac{1}{5}\;\frac{e^{jt}-e^{-jt}}{2j}\quad\boxed{(-j)\cdot j}$$

$$+1$$

$$\frac{1}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$e^{At}\Big|_{(1,2)} = \left[\frac{1}{5}e^{2t} - e^{-jt}\left(\frac{1}{10}-\frac{3}{10}j\right) - e^{jt}\left(\frac{1}{10}+\frac{3}{10}j\right)\right]\cdot 1(t)$$

$$= \left[\frac{1}{5}e^{2t} - \frac{1}{10}\left(e^{jt}+e^{-jt}\right) - \frac{3}{10}j\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$-\frac{1}{5}\cdot\frac{e^{jt}+e^{-jt}}{2} \qquad + \frac{3}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$= \boxed{\left[\frac{1}{5}e^{2t} - \frac{1}{5}\cos t + \frac{3}{5}\sin t\right]\cdot 1(t)}$$

$$e^{At}\Big|_{(1,3)} = \left[\frac{2}{5}e^{2t} - \frac{1}{5}(1+2j)e^{-jt} - \frac{1}{5}(1-2j)\,e^{jt}\right]\cdot 1(t)$$

$$= \left[\frac{2}{5}e^{2t} - \frac{1}{5}\left(e^{-jt} + e^{jt}\right) + \frac{2}{5}j\left(e^{jt} - e^{-jt}\right)\right]\cdot 1(t)$$

$$-\frac{2}{5}\cdot\frac{e^{jt}+e^{-jt}}{2} \qquad -\frac{4}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t - \frac{4}{5}\,\mathrm{sen}\,t\right]\cdot 1(t)$$

$$e^{At}\Big|_{(2,1)} = \left[\frac{2}{5}e^{2t} - \frac{1}{5}(1+2j)e^{-jt} - \frac{1}{5}(1-2j)e^{+jt}\right]\cdot 1(t)$$

cfr. elemento (1,3)

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t - \frac{4}{5}\operatorname{sen} t\right]\cdot 1(t)$$

$$e^{At}\Big|_{(2,2)} = \left[\frac{1}{5}e^{2t} + \frac{1}{5}(2-j)e^{-jt} + \frac{1}{5}(2+j)e^{jt}\right]\cdot 1(t)$$

$$= \left[\frac{1}{5}e^{2t} + \frac{2}{5}\left(e^{jt}+e^{-jt}\right) + \frac{1}{5j}\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$\left.e^{At}\right|_{(1,2)} = \left[\frac{1}{5}e^{2t} + \frac{2}{5}\left(e^{jt}+e^{-jt}\right) + \frac{1}{5j}\cdot\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$+\frac{4}{5}\cdot\frac{e^{jt}+e^{-jt}}{2} \qquad -\frac{2}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$= \left[\frac{1}{5}e^{2t} + \frac{4}{5}\cos t - \frac{2}{5}\,\mathrm{sen}\,t\right]\cdot 1(t)$$

$$\left.e^{At}\right|_{(1,3)} = \left[\frac{2}{5}e^{2t} - \frac{1}{5}(1-3j)e^{-jt} - \frac{1}{5}(1+3j)e^{jt}\right]\cdot 1(t)$$

$$= \left[\frac{2}{5}e^{2t} - \frac{1}{5}\left(e^{jt}+e^{-jt}\right) - \frac{3}{5j}\cdot\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$-\frac{2}{5}\cdot\frac{e^{jt}+e^{-jt}}{2} \qquad +\frac{6}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$e^{At}\Big|_{(2,3)} = \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t + \frac{6}{5}\operatorname{sen} t\right]\cdot 1(t)$$

$$e^{At}\Big|_{(3,1)} = \left[\frac{2}{5}e^{2t} - \frac{1}{5}\left(1-\frac{1}{2}j\right)e^{-jt} - \frac{1}{5}\left(1+\frac{j}{2}\right)e^{jt}\right]\cdot 1(t)$$

$$= \left[\frac{2}{5}e^{2t} - \frac{1}{5}\left(e^{jt}+e^{-jt}\right) - \frac{1}{10}j\left(e^{jt}-e^{-jt}\right)\right]\cdot 1(t)$$

$$-\frac{2}{5}\cdot\frac{e^{jt}+e^{-jt}}{2} + \frac{\cancel{2}\,1}{\cancel{10}\,5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right]\cdot 1(t)$$

$$e^{At}\Big|_{(3,2)} = \left[\frac{1}{5}e^{2t} - \frac{1}{5}\left(\frac{1}{2}+j\right)e^{-jt} - \frac{1}{5}\left(\frac{1}{2}-j\right)e^{jt}\right]\cdot \delta(t)$$

$$= \left[\frac{1}{5}e^{2t} - \frac{1}{10}\left(e^{jt}+e^{-jt}\right) + \frac{1}{5}j\left(e^{jt}-e^{-jt}\right)\right]\cdot \delta(t)$$

$$-\frac{\cancel{2}^{\,1}}{\cancel{10}_{\,5}}\cdot\frac{e^{jt}+e^{-jt}}{2} \qquad -\frac{2}{5}\cdot\frac{e^{jt}-e^{-jt}}{2j}$$

$$= \left[\frac{1}{5}e^{2t} - \frac{1}{5}\cos t - \frac{2}{5}\,\text{sen}\,t\right]\cdot \delta(t)$$

$$e^{At}\Big|_{(3,3)} = \left[\frac{2}{5}e^{2t} + \frac{1}{10}(3+j)e^{-jt} + \frac{1}{10}(3-j)e^{jt}\right] \cdot 1(t)$$

$$= \left[\frac{2}{5}e^{2t} + \frac{3}{10}\left(e^{jt}+e^{-jt}\right) - \frac{1}{10}j\left(e^{jt}-e^{-jt}\right)\right] \cdot 1(t)$$

$$\frac{\cancel{6}\,3}{5\,\cancel{10}} \cdot \frac{e^{jt}+e^{-jt}}{2} + \frac{\cancel{2}\,1}{\cancel{10}\,5} \cdot \frac{e^{jt}-e^{-jt}}{2j}$$

$$= \left[\frac{2}{5}e^{2t} + \frac{3}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right] \cdot 1(t)$$

Calcolo di e^{At}

utilizzando la

trasformata di Laplace

Dato il sistema LTI descritto da:

$$\begin{cases} \dot{x} = Ax + 3u \\ y = Cx \end{cases} \qquad A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$e^{At} = ?$$

$$e^{At} = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$$

$tr(A) = 1 + 0 + 1 = 2 > 0 \Rightarrow$ SISTEMA INSTABILE

Tra i termini di e^{At} almeno uno deve $\to \infty$ quando $t \to +\infty$

$$(sI-A) = \begin{bmatrix} (s-1) & -1 & 0 \\ 0 & s & -2 \\ -1 & 0 & (s-1) \end{bmatrix}$$

per calcolare il determinante per es. sviluppo lungo prima riga

percorso ⇒ scegliere altra riga o colonna per il calcolo del determinante

$$(sI-A)^{-1} = \frac{1}{\det(sI-A)} \left[c_{ij} \right]^T$$

$$c_{ij} \leftrightarrow (-1)^{i+j} \det\left[\hat{A}_{ij}\right]$$

compl. algebrici

Sviluppando il calcolo lungo la riga evidenziata

$$\det(sI - A) = (-1)^{1+1}(s-1)\begin{vmatrix} s & -2 \\ 0 & s-1 \end{vmatrix} +$$

$$+(-1)^{1+2}(-1)\begin{vmatrix} 0 & -2 \\ -1 & s-1 \end{vmatrix} +$$

$$+(-1)^{1+3}\cdot 0 \cdot \begin{vmatrix} 0 & s \\ -1 & 0 \end{vmatrix}$$

sistema instabile

$$= s^3 - 2s^2 + s - 2 = (s-2)(s+j)(s-j)$$

$$c_{11} = (-1)^{1+1} \begin{vmatrix} s & -2 \\ 0 & (s-1) \end{vmatrix} = s(s-1)$$

elemento della matrice dei complementi algebrici

$$c_{12} = (-1)^{1+2} \begin{vmatrix} 0 & -2 \\ -1 & s-1 \end{vmatrix} = +2$$

$$c_{13} = (-1)^{1+3} \begin{vmatrix} 0 & s \\ -1 & 0 \end{vmatrix} = s$$

$c_{21} = s-1 \qquad c_{22} = (s-1)^2 \qquad c_{23} = +1$

$c_{31} = +2 \qquad c_{32} = 2(s-1) \qquad c_{33} = s(s-1)$

$$(sI-A)^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s(s-1) & +2 & s \\ (s-1) & (s-1)^2 & 1 \\ 2 & 2(s-1) & s(s-1) \end{bmatrix}^T$$

$$= \frac{1}{(s-2)(s-j)(s+j)} \begin{bmatrix} s(s-1) & (s-1) & +2 \\ +2 & (s-1)^2 & 2(s-1) \\ s & +1 & s(s-1) \end{bmatrix}$$

Ora si tratta di antitrasformare tutti i termini

Ad esempio

$$\left[\;\right]_{1,2} = \frac{s-1}{(s-2)(s^2+1)} \rightarrow \text{sviluppo in fratti semplici}$$

$$\frac{C_1}{s-2} + \frac{C_2}{s-j} + \frac{C_2^*}{s+j}$$

completamento dei prodotti

$$\frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

per casa: provare entrambe le tecniche

Soluzione completa di

$$\boxed{e^{At} = \mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}}$$

$f(t)$	$F(s)$
$\delta(t)$	1
$1(t)$	$\dfrac{1}{s}$
$t \cdot 1(t)$	$\dfrac{1}{s^2}$
$t^2 \cdot 1(t)$	$\dfrac{2}{s^3}$
$e^{\alpha\, t} \cdot 1(t)$	$\dfrac{1}{s-\alpha}$
$t \cdot e^{\alpha\, t} \cdot 1(t)$	$\dfrac{1}{(s-\alpha)^2}$
$\sin(\omega\, t) \cdot 1(t)$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos(\omega\, t) \cdot 1(t)$	$\dfrac{s}{s^2+\omega^2}$
$t \cdot \sin(\omega\, t) \cdot 1(t)$	$\dfrac{2\,\omega\, s}{\left(s^2+\omega^2\right)^2}$
$t \cdot \cos(\omega\, t) \cdot 1(t)$	$\dfrac{s^2-\omega^2}{\left(s^2+\omega^2\right)^2}$
$e^{\sigma\, t} \cdot \sin(\omega\, t) \cdot 1(t)$	$\dfrac{\omega}{\left(s-\sigma\right)^2+\omega^2}$
$e^{\sigma\, t} \cdot \cos(\omega\, t) \cdot 1(t)$	$\dfrac{s-\sigma}{\left(s-\sigma\right)^2+\omega^2}$
$t \cdot e^{\sigma\, t} \cdot \sin(\omega\, t) \cdot 1(t)$	$\dfrac{2\,\omega\,(s-\sigma)}{\left[\left(s-\sigma\right)^2+\omega^2\right]^2}$
$t \cdot e^{\sigma\, t} \cdot \cos(\omega\, t) \cdot 1(t)$	$\dfrac{\left(s-\sigma\right)^2-\omega^2}{\left[\left(s-\sigma\right)^2+\omega^2\right]^2}$

Tabella 1: Segnali e corrispondenti trasformate di Laplace

$$(sI-A)^{-1} = \frac{1}{(s-2)(s-j)(s+j)} \begin{bmatrix} s(s-1) & (s-1) & +2 \\ +2 & (s-1)^2 & 2(s-1) \\ s & +1 & s(s-1) \end{bmatrix}$$

elemento $(1,1)$

$$\left[e^{At}\right]_{(1,1)} = \mathcal{L}^{-1}\left[\frac{s(s-1)}{(s-2)(s^2+1)}\right]$$

sviluppo in fratti semplici

$$\frac{s(s-1)}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{B}{s-j} + \frac{B^*}{s+j}$$

$$A = \lim_{s\to 2} \frac{s(s-1)}{\cancel{(s-2)}(s^2+1)}\cancel{(s-2)} = 2/5$$

$$B = \lim_{s\to j} \frac{s(s-1)}{(s-2)\cancel{(s-j)}(s+j)}\cancel{(s-j)} = \frac{1}{10}(3-j)$$

$$B^* = \frac{1}{10}(3+j)$$

$$\begin{cases} \dfrac{e^{jt}+e^{-jt}}{2} = \cos t \\[2ex] \dfrac{e^{jt}-e^{-jt}}{2j} = \operatorname{sen} t \end{cases}$$

$$\mathcal{L}^{-1}\{\,..\,\} = \left[\frac{2}{5}e^{2t} + \frac{1}{10}(3-j)e^{jt} + \frac{1}{10}(3+j)e^{-jt}\right]\cdot 1(t)$$

$$2\cdot\frac{3}{10}\,\frac{(e^{jt}+e^{-jt})}{2} - \frac{j}{10}\,\frac{(e^{jt}-e^{-jt})}{2j}\cdot 2j$$

$$\left[e^{At}\right]_{(1,1)} = \mathcal{L}^{-1}\left\{\frac{s(s-1)}{(s-2)(s^2+1)}\right\} =$$

$$= \left[\frac{2}{5} e^{2t} + \frac{3}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right] \cdot 1(t)$$

denvento (1,2)

$$\left[e^{At}\right]_{(1,2)} = \mathcal{L}^{-1}\left\{\frac{s-1}{(s-2)(s^2+1)}\right\}$$

utilizzo il "completamento di quadrato"

$$\frac{s-1}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$$\frac{s-1}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$$\boxed{A = \lim_{s\to 2} \frac{(s-1)}{\cancel{(s-2)}(s^2+1)} \cdot \cancel{(s-2)} = \frac{1}{5}}$$

sostituisco e cerco gli altri coeff. col principio di identità dei polinomi

$$(s-1) = \frac{1}{5}(s^2+1) + (s-2)(Bs+C)$$

$$s-1 = \left(\frac{1}{5}+B\right)s^2 + (C-2B)s + \left(\frac{1}{5}-2C\right)$$

$$\begin{cases} \frac{1}{5}+B = 0 \\ C-2B = 1 \\ \frac{1}{5}-2C = -1 \end{cases} \qquad \begin{cases} B = -\frac{1}{5} \\ C = 1+2B = 3/5 \\ \frac{1}{5}-6/5 = -1 \end{cases}$$

$$\mathcal{L}^{-1}\left[\frac{(s-1)}{(s-2)(s^2+1)}\right] = \frac{1}{5}\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] - \frac{1}{5}\mathcal{L}^{-1}\left[\frac{s-3}{s^2+1}\right]$$

per la linearità di $\mathcal{L}\{\cdot\}$

$$\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = \boxed{e^{2t}\cdot 1(t)}$$

$$\mathcal{L}^{-1}\left[\frac{s-3}{s^2+1}\right] = \underbrace{\mathcal{L}^{-1}\left[\frac{s}{s^2+1^2}\right]}_{\cos t\cdot 1(t)} - 3\,\underbrace{\mathcal{L}^{-1}\left[\frac{\overset{\omega}{1}}{s^2+1^2}\right]}_{\operatorname{sen} t\cdot 1(t)}$$

$s^2+\omega^2$

$$= \left[\cos t - 3\operatorname{sen} t\right]\cdot 1(t)$$

$$\left[e^{At}\right]_{(1,2)} = \mathcal{L}^{-1}\left\{\frac{s-1}{(s-2)(s^2+1)}\right\}$$

$$= \left[\frac{1}{5}e^{2t} - \frac{1}{5}\cos t + \frac{3}{5}\,\mathrm{sen}\,t\right]\cdot 1(t)$$

elemento (1,3)

$$\left[e^{At}\right]_{(1,3)} = \mathcal{L}^{-1}\left\{\frac{2}{(s-2)(s^2+1)}\right\}$$

È l'approccio più conveniente in questo caso!

$$\mathcal{L}^{-1}\left\{\frac{2}{(s-2)(s^2+1)}\right\} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$s^2+\omega^2$ $\quad\omega = +1$

$$A = \lim_{s\to 2} \frac{2}{\cancel{(s-2)}(s^2+1)} \cancel{(s-2)} = \frac{2}{5}$$

applico il principio di identità dei polinomi ai polinomi a numeratore nell'espressione ⊛

$$2 = \frac{2}{5}s^2 + \frac{2}{5} + Bs^2 + Cs - 2Bs - 2C$$

$$= \left(\frac{2}{5} + B\right)s^2 + (C - 2B)s + \left(\frac{2}{5} - 2C\right)$$

$$\begin{cases} \frac{2}{5} + B = 0 \\ C - 2B = 0 \\ \frac{2}{5} - 2C = 2 \end{cases} \qquad \begin{cases} B = -\frac{2}{5} \\ C = 2B = -\frac{4}{5} \\ \frac{2}{5} + \frac{8}{5} = 2 \end{cases} \qquad \boxed{\begin{cases} B = -\frac{2}{5} \\ C = -\frac{4}{5} \end{cases}}$$

$$\frac{-\frac{2}{5}s - \frac{4}{5}}{s^2+1^2} = K_1 \frac{s}{s^2+1^2} + K_2 \frac{1}{s^2+1^2}$$

$$\frac{s}{s^2+\omega^2} \qquad \frac{\omega}{s^2+\omega^2}$$

$$K_1 = -\frac{2}{5} \qquad K_2 = -\frac{4}{5}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{(s-2)(s^2+1)}\right\} = \frac{1}{5}\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}} - \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + - \frac{4}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

per linearità

$$\left[e^{At}\right]_{(1,3)} = \mathcal{L}^{-1}\left\{\frac{2}{(s-2)(s^2+1)}\right\}$$

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t - \frac{4}{5}\operatorname{sen} t\right]\cdot 1(t)$$

elemento $(2,1)$

$$\left[e^{At}\right]_{(2,1)} = \mathcal{L}^{-1}\left\{\frac{2}{(s-2)(s^2+1)}\right\}$$

come elemento $(1,3)$

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t - \frac{4}{5}\operatorname{sen} t\right]\cdot 1(t)$$

elemento $(2,2)$

$$\left[e^{At}\right]_{2,2} = \mathcal{L}^{-1}\left\{ \frac{(s-1)^2}{(s-2)(s^2+1)} \right\}$$

$$\frac{(s-1)^2}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

uso ancora lo stesso approccio!

$$A = \lim_{s\to 2} \frac{(s-1)^2}{\cancel{(s-2)}(s^2+1)} \cdot \cancel{(s-2)} = \frac{1}{5}$$

$$(s-1)^2 = \frac{1}{5}(s^2+1) + (Bs+C)(s-2)$$

$$s^2-2s+1 = \left(\frac{1}{5}+B\right)s^2 + (C-2B)s + \left(\frac{1}{5}-2C\right)$$

$$\begin{cases} B+1/5 = 1 \\ C-2B = -2 \\ 1/5 - 2C = +1 \end{cases} \Rightarrow \begin{cases} B = 4/5 \\ C = -2/5 \\ 1/5 + 4/5 = 1 \end{cases} \Rightarrow \begin{array}{l} B = 4/5 \\ C = -2/5 \end{array}$$

$$\mathcal{L}^{-1}\left\{\frac{(s-1)^2}{(s-2)(s^2+1)}\right\} = \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \frac{4}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + $$

$$- \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

per linearità

$$\left[e^{At}\right]_{2,2} = \mathcal{L}^{-1}\left\{\frac{(s-1)^2}{(s-2)(s^2+1)}\right\}$$

$$= \left[\frac{1}{5}e^{2t} + \frac{4}{5}\cos t - \frac{2}{5}\operatorname{sen} t\right] \cdot 1(t)$$

Elemento (2,3)

$$\left[e^{At}\right]_{2,3} = \mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-2)(s^2+1)}\right\}$$

$$\frac{2(s-1)}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

come negli altri casi

$$A = \lim_{s\to 2} \frac{2(s-1)}{\cancel{(s-2)}(s^2+1)}\cancel{(s-2)} = 2/5$$

confronto i polinomi e numeratore ed applico il principio di identità dei polinomi per determinare i coefficienti B, C

$$2s-2 = \frac{2}{5}(s^2+1) + (Bs+C)(s-2)$$

$$2s-2 = \left(\frac{2}{5}+B\right)s^2 + (C-2B)s + \left(\frac{2}{5}-2C\right)$$

$$\begin{cases} B + \frac{2}{5} = 0 \\ C - 2B = 2 \\ \frac{2}{5} - 2C = -2 \end{cases} \Rightarrow \begin{cases} B = -\frac{2}{5} \\ C = 2 + 2B = \frac{6}{5} \\ \frac{1}{5} - \frac{6}{5} = -1 \end{cases}$$

quindi per linearità

$$\mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-2)(s^2+1)}\right\} = \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + \frac{6}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

$$\left[e^{At}\right]_{2,3} = \mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-2)(s^2+1)}\right\}$$

$$= \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t + \frac{6}{5}\operatorname{sen} t\right] \cdot 1(t)$$

elemento (3,1)

$$[e^{At}]_{3,1} = \mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s^2+1)}\right\}$$ ← stesso approccio degli altri casi

$$\frac{s}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$$A = \lim_{s\to 2} \frac{s}{\cancel{(s-2)}(s^2+1)} \cdot \cancel{(s-2)} = 2/5$$

applico il principio di identità dei polinomi ai polinomi a numeratore

$$s = \frac{2}{5}(s^2+1) + (Bs+C)(s-2)$$

$$s = \left(\frac{2}{5}+B\right)s^2 + (C-2B)s + \left(\frac{2}{5}-2C\right)$$

$$\begin{cases} B + \frac{2}{5} = 0 \\ C - 2B = 1 \\ \frac{2}{5} - 2C = 0 \end{cases} \Longrightarrow \begin{cases} B = -\frac{2}{5} \\ C = 1 + 2B = \frac{1}{5} \\ \frac{2}{5} - \frac{2}{5} = 0 \end{cases}$$

quindi per linearità:

$$\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s^2+1)}\right\} = \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

$$\left[e^{At}\right]_{3,1} = \mathcal{L}^{-1}\{\ast\} = \left[\frac{2}{5}e^{2t} - \frac{2}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right]\cdot 1(t)$$

elemento (3,2)

$$\left[e^{At}\right]_{3,2} = \mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s^2+1)}\right\}$$

provare ad utilizzare l'altro approccio, usando lo sviluppo in fratti semplici

$$\frac{1}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$$A = \lim_{s\to 2} \frac{1}{\cancel{(s-2)}(s^2+1)}\cancel{(s-2)} = 1/5$$

applico il principio di identità dei polinomi

$$1 = \tfrac{1}{5}(s^2+1) + (Bs+C)(s-2)$$

$$1 = \left(\tfrac{1}{5}+B\right)s^2 + (C-2B)s + \left(\tfrac{1}{5}-2C\right)$$

$$\begin{cases} B + \frac{1}{5} = 0 \\ C - 2B = 0 \\ \frac{1}{5} - 2C = 1 \end{cases} \Longrightarrow \begin{cases} B = -1/5 \\ C = -2/5 \\ 1/5 + 4/5 = 1 \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s^2+1)}\right\} = \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + $$

$$- 2/5\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

$$\left[e^{At}\right]_{32} = \mathcal{L}^{-1}\{\,!\,\} = \left[\frac{1}{5}e^{2t} - \frac{1}{5}\cos t - \frac{2}{5}\,\mathrm{sen}\,t\right]\cdot 1(t)$$

elemento $(3,3)$

$$\left[e^{At}\right]_{3,3} = \mathcal{L}^{-1}\left\{ \frac{s(s-1)}{(s-2)(s^2+1)} \right\}$$

Come negli altri casi trovo A con la formula dei residui, B e C con il principio di identità dei polinomi

$$\frac{s(s-1)}{(s-2)(s^2+1)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+1}$$

$$A = \lim_{s \to 2} \frac{s(s-1)}{\cancel{(s-2)}(s^2+1)} \cdot \cancel{(s-2)} = 2/5$$

$$s^2 - s = 2/5\,(s^2+1) + (Bs+C)(s-2)$$

$$s^2 - s = \left(\frac{2}{5} + B\right)s^2 + \left(C - 2B\right)s + \left(\frac{2}{5} - 2C\right)$$

$$\begin{cases} \frac{2}{5} + B = +1 \\ C - 2B = -1 \\ \frac{2}{5} - 2C = 0 \end{cases} \Rightarrow \begin{cases} B = 3/5 \\ C = 1/5 \\ \frac{2}{5} - \frac{2}{5} = 0 \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{s(s-1)}{(s-2)(s^2+1)}\right\} = \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \frac{3}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1^2}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1^2}\right\}$$

$$\left[e^{At}\right]_{3,3} = \mathcal{L}^{-1}\left\{\vdots\right\} = \left[\frac{2}{5}e^{2t} + \frac{3}{5}\cos t + \frac{1}{5}\operatorname{sen} t\right]\cdot 1(t)$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad G(s) = C(sI-A)^{-1}B$$

$$\begin{cases} \dot{x}_1 = -x_1 + u \\ \dot{x}_2 = +\frac{1}{2}x_2 \\ \dot{x}_3 = +\frac{1}{5}x_1 - \frac{1}{5}x_3 \\ y = x_1 + x_2 \end{cases}$$

$u(t) = 1(t-1)$

$x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$y(t) = ?$

$$Y(s) = \underbrace{C(sI-A)^{-1}B}_{G(s)} \cdot U(s)$$

Sistema instabile

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & +1/2 & 0 \\ +1/5 & 0 & -1/5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$t_A(s) = \det(sI - A) = (s+1)\left(s+\frac{1}{5}\right)\left(s-\frac{1}{2}\right)$$

$$(sI - A) = \begin{bmatrix} (s+1) & 0 & 0 \\ 0 & \left(s-\frac{1}{2}\right) & 0 \\ -\frac{1}{5} & 0 & \left(s+\frac{1}{5}\right) \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & \frac{2}{2s-1} & 0 \\ \frac{1}{(5s+1)(s+1)} & 0 & \frac{5}{5s+1} \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B$$

$$= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

solo 2 termini di $(sI-A)^{-1}$

$$(sI-A)^{-1} = \begin{bmatrix} x & ? & ? \\ x & ? & ? \\ ? & ? & ? \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}^T$$

$$G(s) = \boxed{\frac{1}{s+1}} \leftarrow$$ 1 solo polo!

cancellazioni nascoste

$$Y(s) = G(s) \cdot U(s) = \frac{1}{s+1} \cdot \frac{1}{s} e^{-s}$$

$$Y(s) = \frac{1}{s(s+1)} e^{-s} \qquad \hat{Y}(s) = \frac{1}{s(s+1)} = \frac{\hat{C}_1}{s} + \frac{\hat{C}_2}{s+1}$$

$$\hat{C}_1 = \lim_{s \to 0} \hat{Y}(s) \cdot s = \lim_{s \to 0} \frac{1}{s+1} = +1$$

$$\hat{C}_2 = \lim_{s \to -1} \hat{Y}(s)(s+1) = \lim_{s \to -1} \frac{1}{s} = -1$$

$$\hat{Y}(s) = \frac{1}{s} - \frac{1}{s+1} \xrightarrow{\mathcal{L}^{-1}} \hat{y}(t) = \left[1 - e^{-t}\right] \cdot 1(t)$$

$$\frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} 1(t) \qquad \frac{1}{s+1} \to e^{-t} \cdot 1(t)$$

$$y(t) = \left[1 - e^{-(t-1)}\right] 1(t-1)$$

$$\begin{cases} \dot{x}_1 = u \\ \dot{x}_2 = -x_1 - \frac{1}{2} x_2 \\ \dot{x}_3 = -\frac{3}{10} x_3 + \frac{3}{10} u \\ y = x_2 \end{cases}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{3}{10} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 3/10 \end{bmatrix} \qquad D = 0$$

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} B$$

$$= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3/10 \end{bmatrix}$$

$$G(s) = -\frac{1}{s\left(s+\frac{1}{2}\right)}$$

2 poli → 2 autovalori su 3 → cancellazione nascosta

$$U(s) = \frac{1}{s}$$

$$Y(s) = -\frac{1}{s^2\left(s+\frac{1}{2}\right)} \qquad y(t) \xrightarrow[t \to +\infty]{} \; ?$$

$$= \frac{C_{1,1}}{s} + \frac{C_{1,2}}{s^2} + \frac{C_2}{s+\frac{1}{2}}$$

$$\begin{cases} \dot{x}_1 = -x_2 \\ \dot{x}_2 = -x_1 - \frac{1}{2} x_2 \\ \dot{x}_3 = -\frac{3}{10} x_3 + \frac{3}{10} u \\ y = x_1 \end{cases}$$

$u(t) = 1(t)$

$x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$y(t) = ?$

$$Y(s) = C(sI-A)^{-1} B \cdot U(s)$$

$$G(s) = 0$$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1/2 & 0 \\ 0 & 0 & -\frac{3}{10} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 3/10 \end{bmatrix} \qquad D = 0$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_3 = -\frac{3}{10}$$

$$P_A(s) = \det(sI - A) = \begin{vmatrix} s & +1 & 0 \\ +1 & s+\frac{1}{2} & 0 \\ 0 & 0 & s+\frac{3}{10} \end{vmatrix}$$

$$p_A(s) = \left(s+\frac{3}{10}\right)\left[s\left(s+\frac{1}{2}\right)-1\right]$$

$$= \left(s+\frac{3}{10}\right)\left[s^2+\frac{1}{2}s-1\right] \longrightarrow \lambda_{1,2} = -\frac{1}{4} \pm \sqrt{\frac{1}{16}+1}$$

$$= -\frac{1}{4} \pm \frac{\sqrt{17}}{4}$$

$$\lambda_3 = -\frac{3}{10}$$

$$\lambda_1 = -\frac{1}{4}+\frac{\sqrt{17}}{4} > 0 \longrightarrow \text{sistema INSTABILE}$$

$$\lambda_2 = -\frac{1}{4}-\frac{\sqrt{17}}{4} < 0$$

$$G(s) = C(sI-A)^{-1}B =$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3/10 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{p_A(s)}\left[c_{ij}\right]^T = \frac{1}{p_A(s)} \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ c_{31} & ? & ? \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & +1 & 0 \\ +1 & s+\frac{1}{2} & 0 \\ 0 & 0 & s+\frac{3}{10} \end{bmatrix}$$

$$[c_{ij}]^T \qquad c_{31} = (-1)^{3+1} \cdot \begin{vmatrix} +1 & 0 \\ s+\frac{1}{2} & 0 \end{vmatrix} = 0$$

$G(s) = 0 \leftarrow$ non ci legame I/O

Il sistema è INSTABILE!

Dalle eq. di stato si vede $\rightarrow$ dalla FdT NO

Dato il sistema

$$\begin{cases} \dot{x} = \begin{bmatrix} -2 & 4 & 1/2 \\ 0 & -4 & 1/2 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 3 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} x \end{cases}$$

2 ingressi

2 uscite

$$G(s) = C(sI-A)^{-1}B = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$$G_{11}(s)$$

$y \swarrow \quad \searrow u$

$$(sI-A) = \begin{bmatrix} (s+2) & -4 & -\frac{1}{2} \\ 0 & (s+4) & -1/2 \\ 0 & 0 & (s+2) \end{bmatrix}$$

$$\det(sI-A) = \varphi_A(s) = (s+2)^2(s+4)$$

$$(sI-A)^{-1} = ?$$

$$G_{11}(s) = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G_{12}(s) = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$G_{21}(s) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G_{22}(s) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{P_A(s)} \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

$$c_{11} = \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} (s+4) & -1/2 \\ 0 & (s+2) \end{vmatrix} (-1)^{1+1}$$

$$= \frac{1}{(s+2)^{\cancel{2}}\cancel{(s+4)}} \cdot \cancel{(s+4)}\cancel{(s+2)} = \frac{1}{s+2}$$

$$c_{12} = (-1)^{1+2} \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} 0 & -\frac{1}{2} \\ 0 & s+2 \end{vmatrix} = 0$$

$$c_{13} = (-1)^{1+3} \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} 0 & s+4 \\ 0 & 0 \end{vmatrix} = 0$$

$$c_{31} = (-1)^{3+1} \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} -4 & -\frac{1}{2} \\ (s+4) & -\frac{1}{2} \end{vmatrix} = \frac{\frac{1}{2}(s+8)}{(s+2)^2(s+4)}$$

$$c_{32} = (-1)^{3+2} \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} s+2 & -\frac{1}{2} \\ 0 & -\frac{1}{2} \end{vmatrix} = + \frac{1/2}{(s+2)(s+4)}$$

$$C_{33} = (-1)^{3+3} \frac{1}{(s+2)^2(s+4)} \begin{vmatrix} s+2 & -4 \\ 0 & s+4 \end{vmatrix}$$

$$= \frac{1}{s+2}$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s+2} & \frac{4}{(s+2)(s+4)} & \frac{1}{2}\frac{s+8}{(s+2)^2(s+4)} \\ 0 & \frac{1}{s+4} & \frac{1}{2(s+2)(s+4)} \\ 0 & 0 & \frac{1}{s+2} \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} (sI-A)^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 3 \end{bmatrix}$$

$$G_{11}(s) = \frac{1}{s+2}$$

$$G_{12}(s) = \frac{3}{2}\frac{s+8}{(s+2)^2(s+4)} - \frac{3}{s+2}$$

$$= -\frac{3}{2} \cdot \frac{(2s^2+11s+8)}{(s+2)^2(s+4)}$$

$$G_{21}(s) = 0 \qquad G_{22}(s) = \frac{3}{2}\frac{1}{(s+2)(s+4)}$$

$$G(s) = \begin{bmatrix} \frac{1}{s+2} & -\frac{3}{2}\frac{2s^2+11s+8}{(s+2)^2(s+4)} \\ 0 & \frac{3}{2}\frac{1}{(s+2)(s+4)} \end{bmatrix}$$