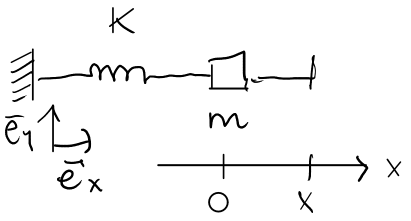
OSCILLAZIONI

- Forza costante → moto uniform. accelerato → $m \frac{d^2x}{dt^2} = \text{cost}$
- Forza attrito viscoso → moto smorzato → $m \frac{d^2x}{dt} = -\xi \frac{dx}{dt}$
- Forza di richiamo → moto armonico → $m \frac{d^2x}{dt^2} = -kx$



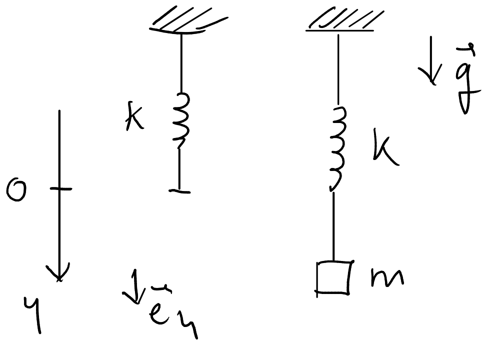
Legge di Hooke : $\vec{F} = -kx\,\vec{e}_x$

II Newton : $-kx\,\vec{e}_x = m\vec{a} = m\frac{d^2x}{dt^2}\vec{e}_x + \underbrace{m\frac{d^2y}{dt^2}}_{=0}\vec{e}_y$

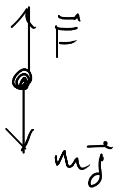
$$m\frac{d^2x}{dt^2} = -kx$$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$ oscillatore armonico

Ruolo della gravità:



II Newton : $\underbrace{m\frac{d^2x}{dt}}_{=0}\vec{e}_x + m\frac{d^2y}{dt^2} = -ky\,\vec{e}_y + mg\,\vec{e}_y$



$$m\frac{d^2y}{dt^2} = -ky + mg$$

$\sum\vec{F} = \vec{0}$ → $ky_{eq} = mg$ → $y_{eq} = \frac{mg}{k}$ posizione di equilibrio

$$\boxed{k = \frac{mg}{y_{eq}}}$$ SI: $\frac{N}{m}$

Nuova coordinata : $y' = y - y_{eq}$ (cambio di variabile)

$\frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2}$, $y = y' + y_{eq}$ $\Rightarrow$ $m\frac{d^2y'}{dt^2} = -K\left(y' + \frac{mg}{k}\right) + mg = -ky'$

$$\frac{d^2y'}{dt^2} = -\frac{k}{m}y'$$

PHET : misurare periodo τ per k, m fissati

Soluzione generale : combinazione lineare di sin( ) e cos( ) oppure :

$y' = A\cos(\omega t + \phi)$

$\frac{dy'}{dt} = -A\omega\sin(\omega t + \phi)$

$\frac{d^2y'}{dt^2} = -A\omega^2\cos(\omega t + \phi)$ $\Rightarrow$ $\omega^2 = \frac{k}{m}$ $\Rightarrow$ $\omega = \sqrt{\frac{k}{m}}$ <u>frequenza angolare</u>

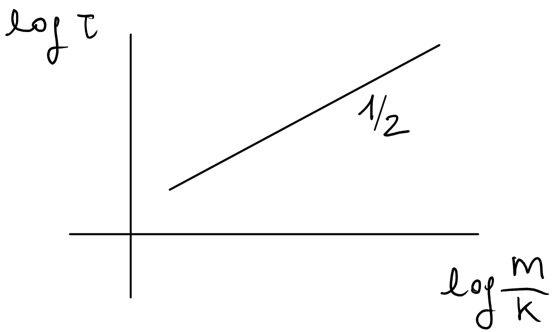
$\rightarrow$ $\underline{y' = A\cos\left(\sqrt{\frac{k}{m}}\,t + \phi\right)}$ $\rightarrow$ legge oraria del moto $\rightarrow$ oscillazioni armoniche

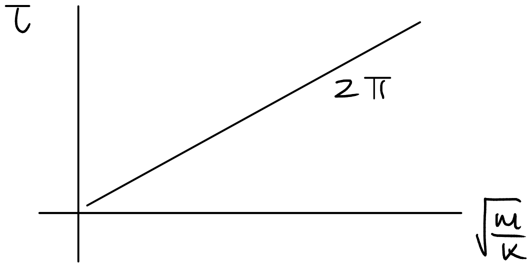
<u>Periodo</u> oscillazione : $\cos(\omega t + \phi) = \cos(\omega(t+\tau) + \phi)$

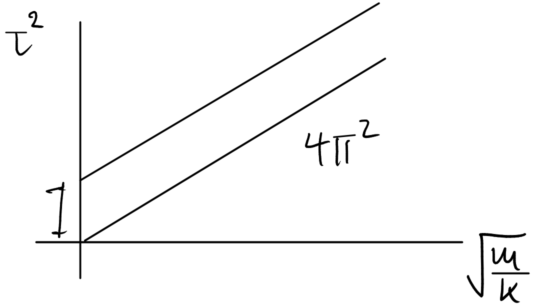
$\cancel{\omega t} + \cancel{\phi} + 2\pi = \cancel{\omega t} + \omega\tau + \cancel{\phi}$ $\Rightarrow$ $\omega\tau = 2\pi$ $\rightarrow$ $\omega = \frac{2\pi}{\tau}$ SI: $\frac{rad}{s}$

<u>Frequenza</u> : $f = \frac{\omega}{2\pi} = \frac{1}{\tau}$ $\tau = 2\pi\sqrt{\frac{m}{k}}$ $\rightarrow$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $\nearrow m \nearrow \tau$

$\nearrow k \searrow \tau$







molla reale :

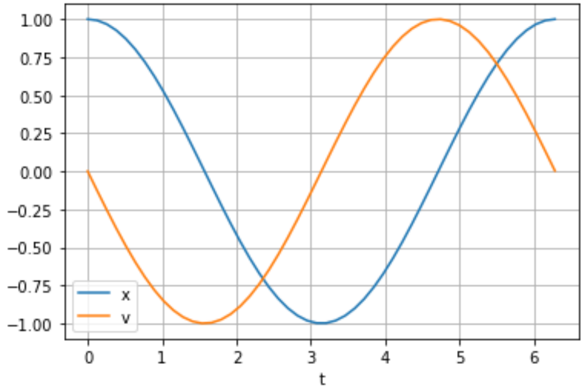
$$\tau = 2\pi \sqrt{\frac{m + m_0}{k}}$$

$$\tau^2 = 4\pi^2 \left(\frac{m}{k} + \frac{m_0}{k} \right)$$

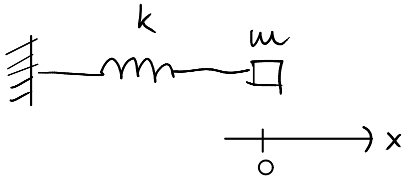
$$\tau = 2\pi \left(\frac{m}{k} \right)^{1/2}$$

Condizioni iniziali : $y'_i = y'_0$, $v_{y'_i} = 0$

$$\begin{cases} y'_0 = A \cos(\phi) \Rightarrow A = y_0 \\ 0 = -A\omega \sin(\phi) \Rightarrow \phi = 0 \end{cases} \Rightarrow \underline{y' = y'_0 \cos(\omega t)}$$



Energia meccanica oscillatore armonico

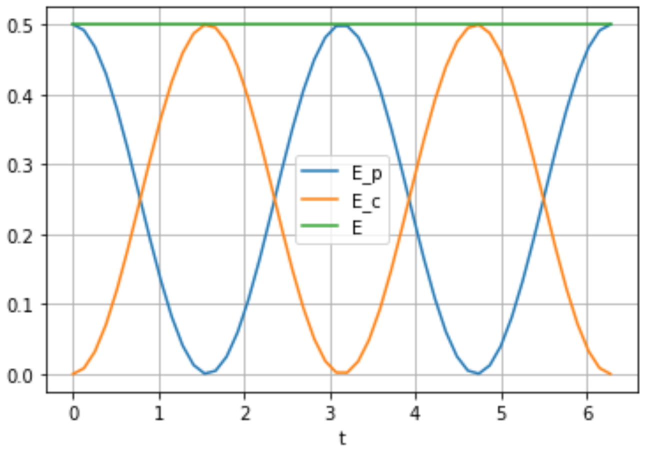


$$E = E_c + E_p = \frac{1}{2} m |\vec{v}|^2 + \frac{1}{2} k x^2 = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$\omega^2 = \frac{k}{m}$$

$$\begin{cases} x = A \cos(\omega t + \phi) \\ v_x = -A\omega \sin(\omega t + \phi) \end{cases}$$

$$E = \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} A^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi) = \frac{1}{2} k A^2 [\underbrace{\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)}_{=1}] = \frac{1}{2} k A^2$$

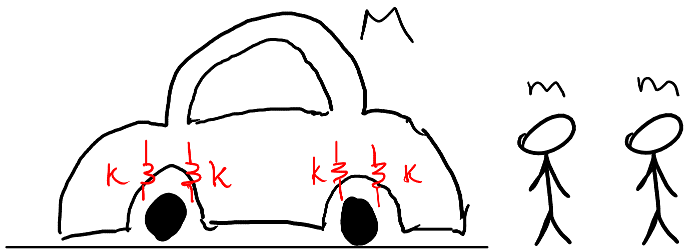


$\Rightarrow$ E è costante

(conservazione energia meccanica)

$E \sim A^2$

Esempio: oscillazioni di un automobile



$M = 1300\ kg$

$m = 80\ kg$

$k = 20000\ N/m$

f = frequenza delle oscillazioni verticali = ?