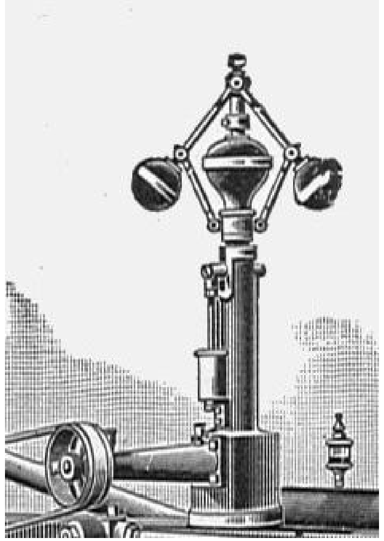
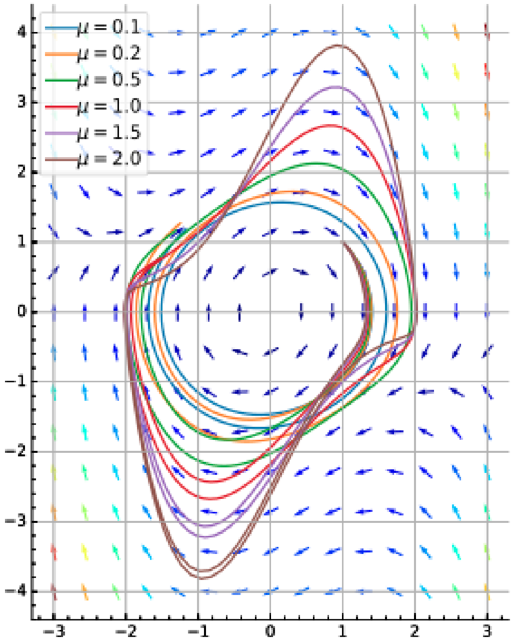
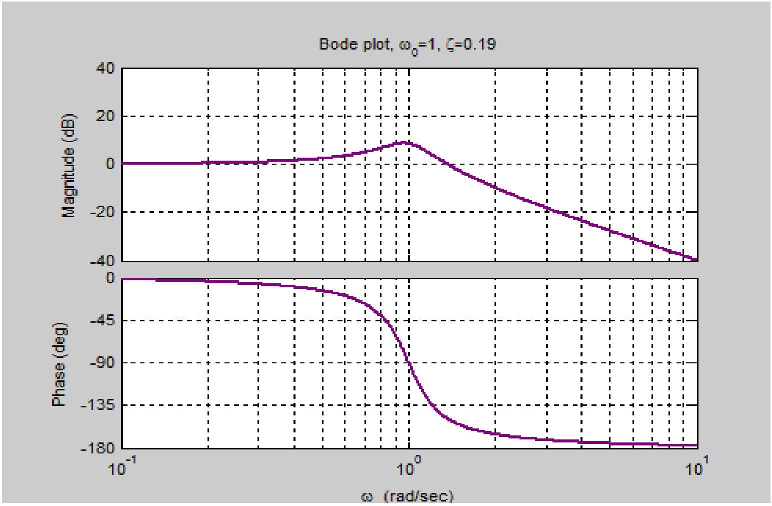


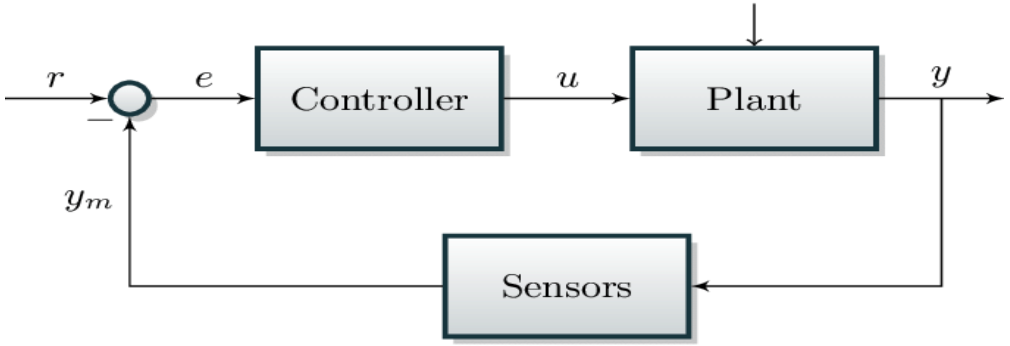



# Introduction to **Control Systems**

## Theory and applications


Enrico Regolin / Laura Nenzi

# Course Overview (1)

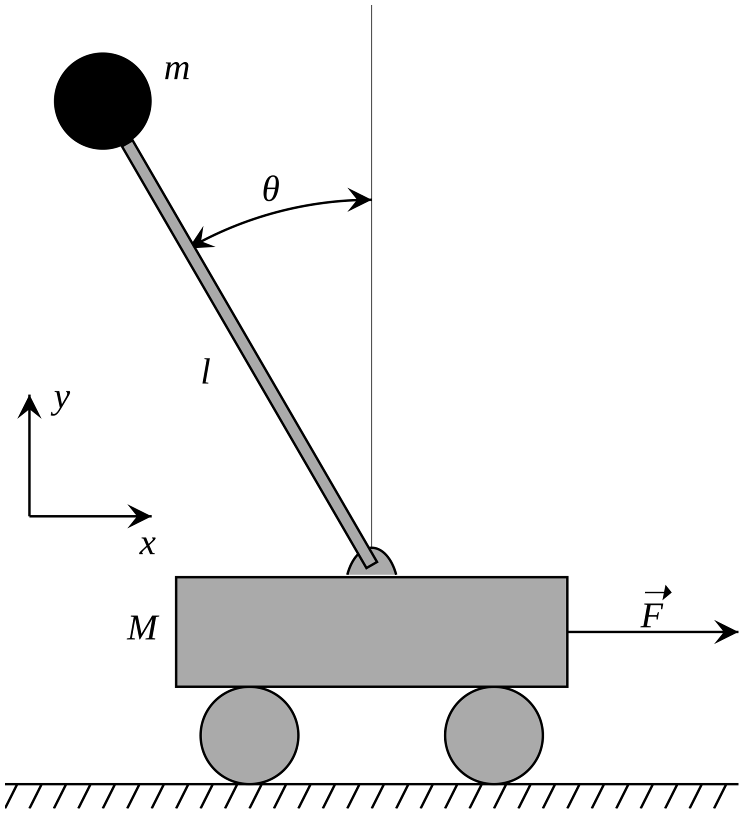
- Day 1: Linear Control (time domain)
  - Introduction
  - Dynamical Linear Systems
  - Observability & Controllability
  - PID Controllers
  - Luenberger Observer

- Day 2: Linear Control (frequency domain)
  - From State-space to Transfer Function
  - Classic Control Elements (Bode Diagram / Root Locus)
  - Introduction to Simulink.
  - Ctrl Lab (days 1,2)

# Course Overview (2)

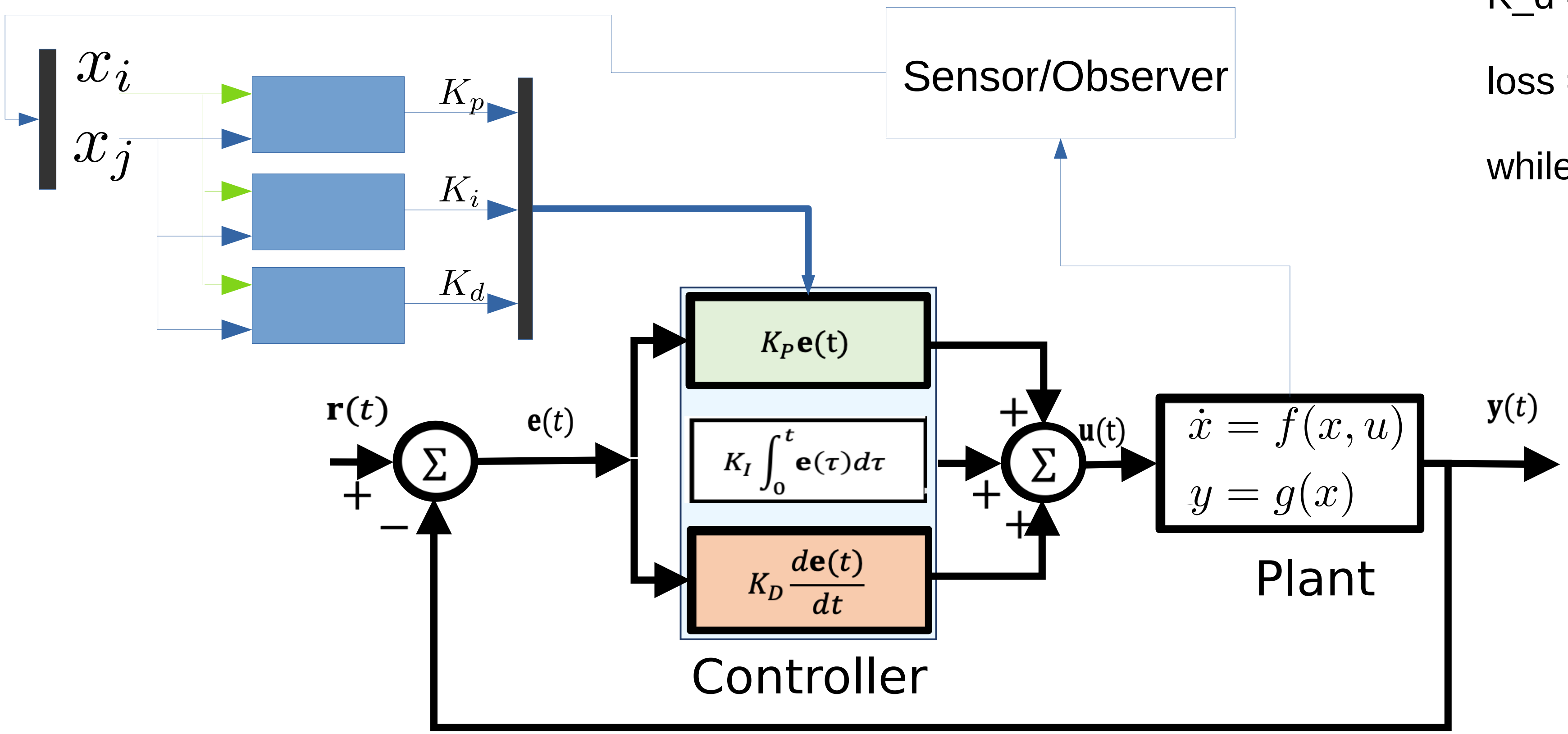
- Day 3: Optimal Control and KF Estimation
  - Optimal Control (LQR)
  - Model Predictive Control
  - Kalman Filtering
  - Sliding Mode Control (tentative)

- Day 4: Control Laboratory
  - Kalman Filtering and Optimal Control
  - Matlab/Simulink
  - Cart-pole



# Gain Scheduling Example

Used for NONLINEAR / unknown systems




<u>Calibration Routine Example</u>

K_p = f_p (state, param_set)
K_i = f_i (state, param_set)
K_d = f_d (state, param_set)

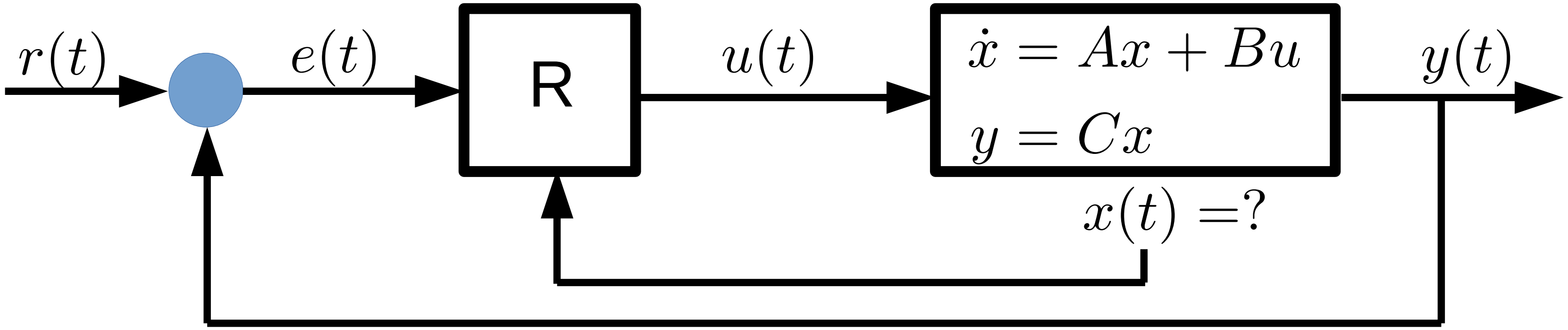
loss = g(stability, risetime, overshoot, etc.)

while not (end condition):

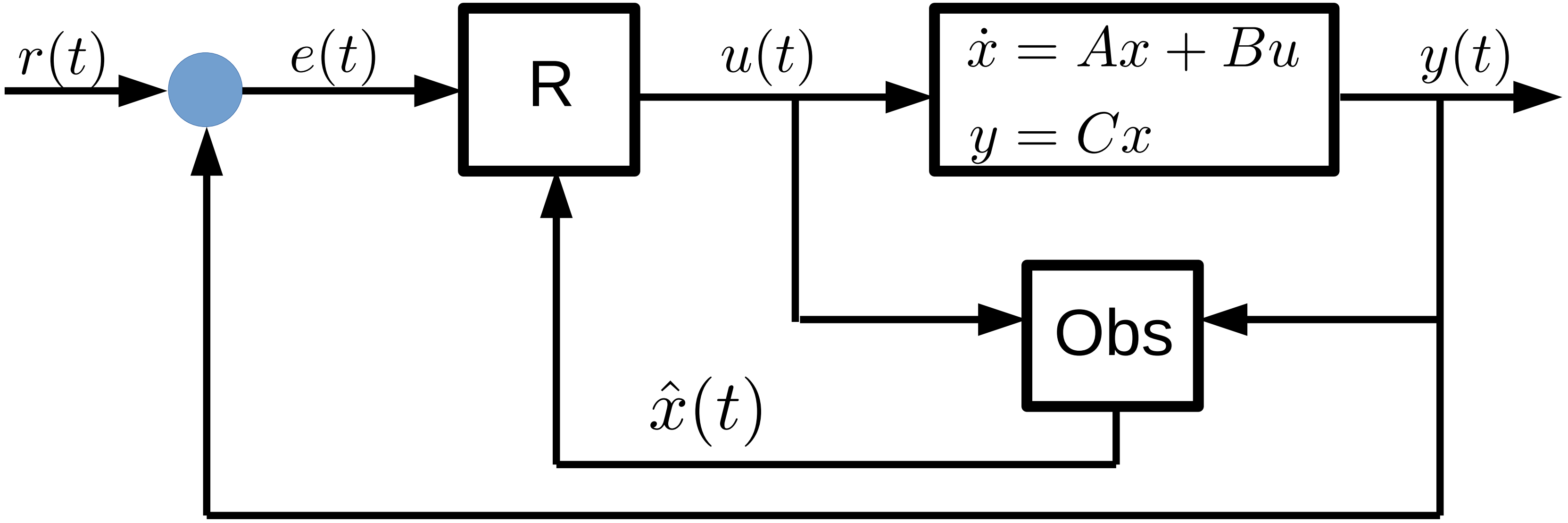
    loss = run_system (param_set)
    optimization_step(param_set)

# Observation

- Problem:
  - Control design with (partially) unknown state




- Solution:
  - Luenberger Observer



# Luenberger Observer

- State-space representation

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$
$$u = K(x_{ref} - \hat{x})$$

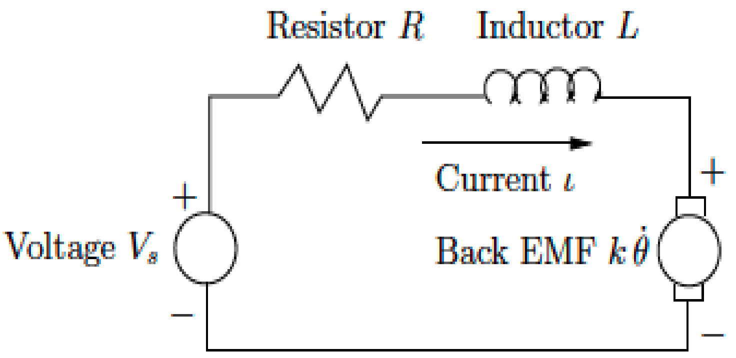
Control design parameters

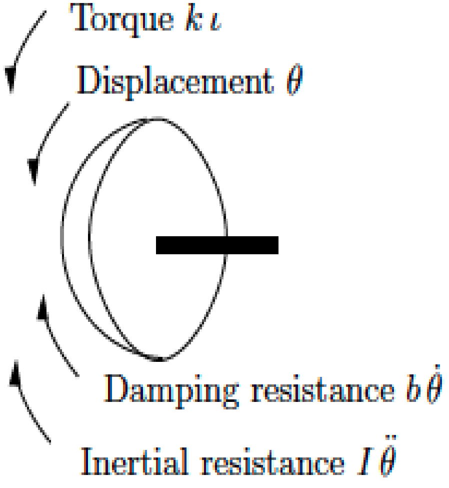
- Observer Error satisfies: $\dot{e} = (A - LC)e$
- Required: Observability, Controllability
- Pole Placement

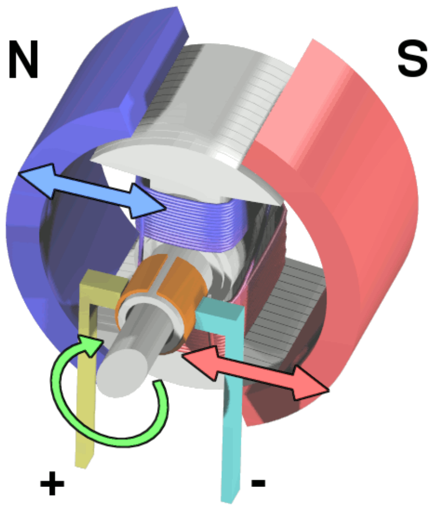
$$K : eig(A - BK) = \{\lambda_{c1}, \ldots, \lambda_{cn}\}$$
$$L : eig(A^T - LC) = \{\lambda_{o1}, \ldots, \lambda_{on}\}$$

Overall system is stable iff both observer and controller are stable

# Example - DC Motor




b = 0.1  # friction coefficient ( Nm/(rad/sec) )
I = 0.01 # mechanical inertia (Kg*m^2)
k = 0.01 # motor torque constant (Nm/A)
R = 1    # armature resistance (Ohm)
L = 0.5  # armature inductance (H)

$$V_s = Ri + L\frac{di(t)}{dt} + k\theta_v$$
$$I\frac{d\theta_v}{dt} + b\theta_v = ki$$

State-space representation
$\dot{x} = Ax + Bu$

$$x = \begin{bmatrix}\theta_v \\ i\end{bmatrix} \qquad u = V_s$$

$$A = \begin{bmatrix}-b/I & k \\ -k/L & -R\end{bmatrix} \quad B = \begin{bmatrix}0 \\ 1\end{bmatrix}$$

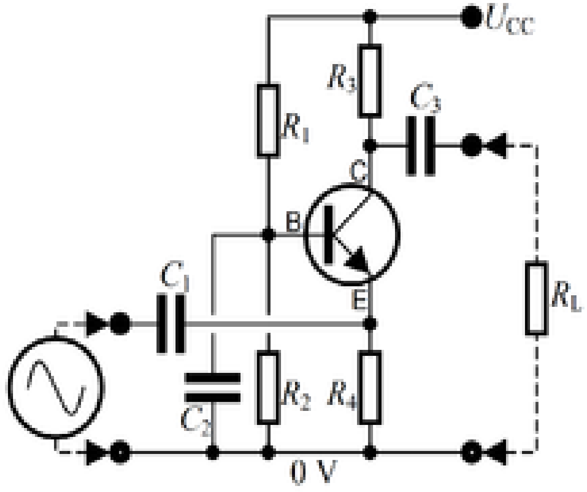
$$C = \begin{bmatrix}1 & 0\end{bmatrix}$$

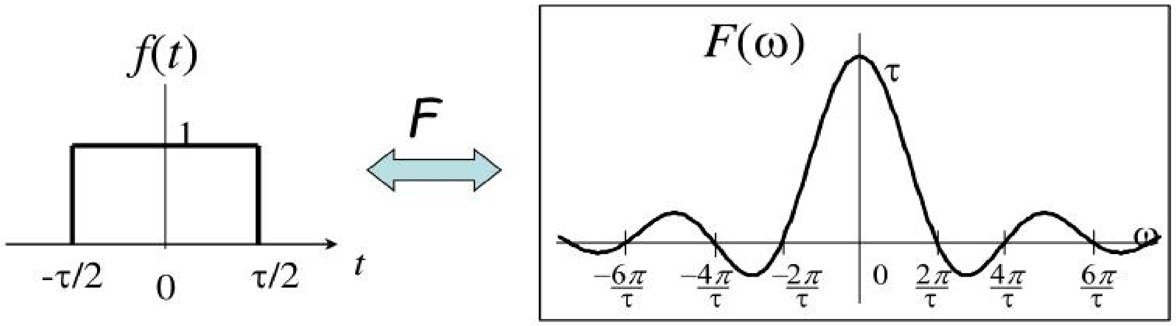
# Day 2

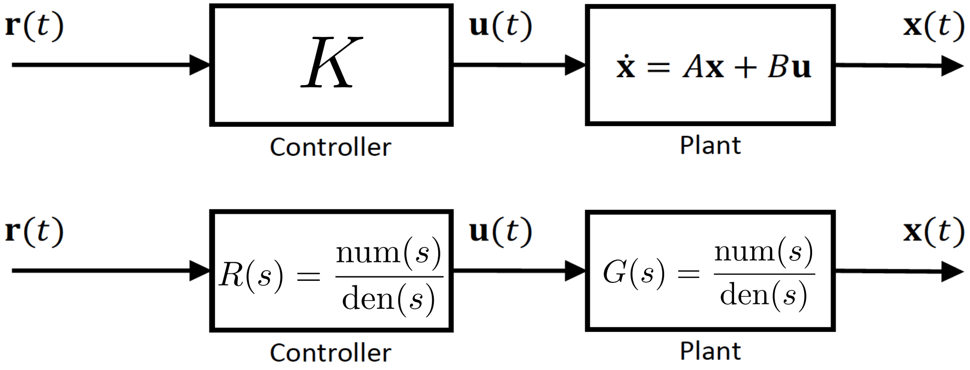
# Linear Control (frequency domain)

# Signals Theory – Frequency Analysis

- Control Theory applications precede digital computing
- Classic control theory was developed for analog electronics applications
- Signals **x(t)** can be expressed as function of frequency **X(f)** without loss of information (Fourier series, Fourier Transform, Laplace Transform)




- <u>Classical LTI Systems Control Theory</u> is frequency-domain based
- Modern tools and notation are influenced by historic development of theory

# LTI Systems in the Frequency Domain

- Classic Control Theory Approach (derived from Circuits Theory)
- Motivation: complexity in using explicit form for $x(t)$ in State-Space representation:

$$y(t) = Ce^{At}x_0 + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

- Laplace Transform of a signal $x(t)$ :

$$X(s) = \mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-st}dt \qquad s = \sigma + j\omega \in \mathbb{C}$$

- Output of L-transform is a rational function with real coefficients

$$G(s) = \frac{s(s^2+1)}{s^3+2s^2-s-1}$$

- $deg(\text{num}(s)) \leq deg(\text{den}(s))$
- Laplace Transform property:

$$H(s) = \mathcal{L}\{h(t)\}$$
$$X(s) = \mathcal{L}\{x(t)\}$$

$$\Rightarrow \quad \mathcal{L}^{-1}\{H(s)X(s)\} = (h * x)(t) := \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

# LTI Systems in the Frequency Domain

- Intuition behind the Laplace Transform of a signal

- Imaginary components of complex numbers are always accompanied by conjugate, as complex numbers are defined as square roots of negative numbers, e.g. $\sqrt{-1} = \pm j$

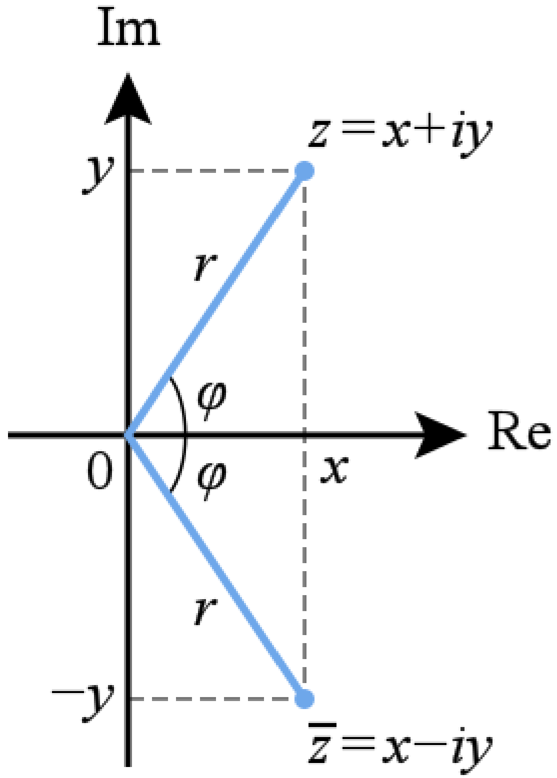
- Choose an elementary input $u(t) = e^{st}, \quad s \in \mathbb{C}$
- If $s$ is real, $u(t)$ is an exponential
- If $s$ is imaginary then the elementary has to be considered with its conjugate:

$$u(t) + u^*(t) = e^{j\omega t} + e^{-j\omega t} = 2\cos(\omega t)$$

(in this case u(t) is “half” sinusoidal signal)

- Laplace transform is equivalent to finding the complex representation $e^{st}$ of a signal for each moment t :

$$u(t) = e^{\sigma t} cos(\omega t)$$



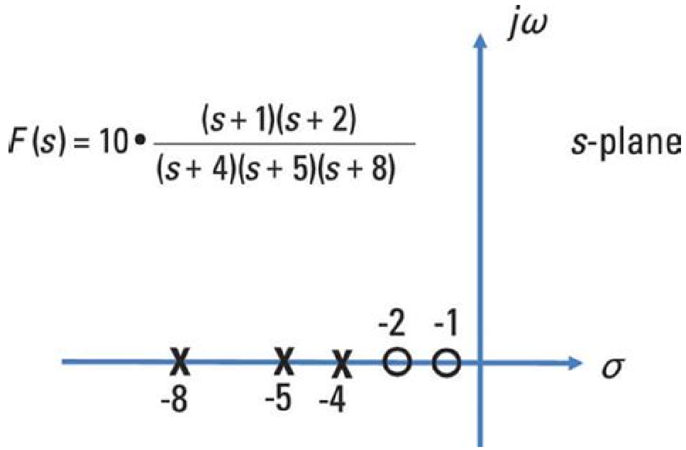
# LTI Systems in the Frequency Domain

- Intuition behind the Laplace Transform of a <u>system</u>

- H(s) is the L-transform “impulse response” of a system (response to ideal input, Dirac or Kronecker delta )
- Output response to input u(t) is the convolution with impulse response h(t)
- H(s) represents the natural “modes” of system S = {A,B,C,D}

$$H(s) = C(sI - A)^{-1}B + D = \frac{num(s)}{den(s)}$$

- Denominator is $den(s) = det(sI - A)$
- H(s) is represented with zeros/poles on the complex plane

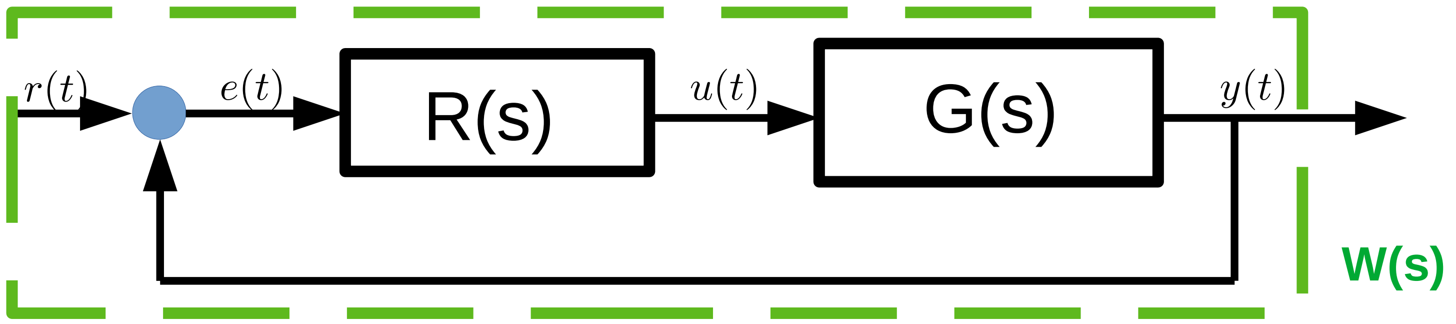


# LTI Systems in the Frequency Domain

Frequency-domain controller design

- G(s) poles: $p_0 = +1,\ p_{1,2} = -1 \pm j$

$$G(s) = \frac{s(s-2)}{(s-1)(s^2+2s+2)}$$




$$y(t) = R(s)G(s)e(t)$$

$$e(t) = r(t) - R(s)G(s)e(t)$$

$$e(t) = \frac{1}{1+R(s)G(s)} r(t)$$

$$y(t) = \frac{R(s)G(s)}{1+R(s)G(s)} r(t)$$

$$W(s) = \frac{R(s)G(s)}{1+R(s)G(s)}$$

Poles allocation of
FeedBack system
- Root Locus

FeedForward Path
- Bode Plot
- Nyquist Plot

# From Transfer Function to State-Space

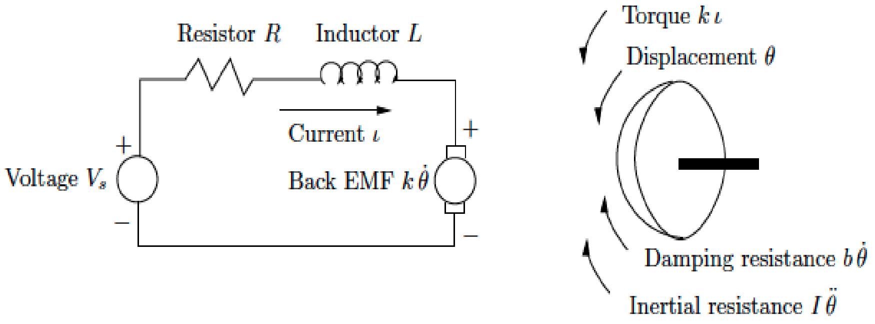
- Controllable canonical form

$$G(s) = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad \Rightarrow \quad C(sI - A)^{-1}B$$

$$C = \begin{bmatrix} b_0 & b_1 & \cdots & b_{n-2} & b_{n-1} \end{bmatrix}$$

# DC Motor Example (ss → tf)




$$A = \begin{bmatrix} -b/I & k \\ -k/L & -R \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

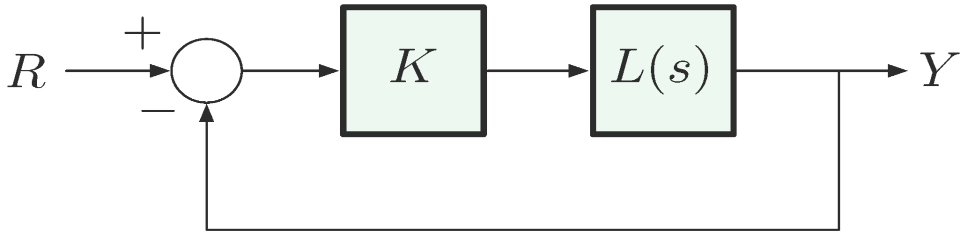
$$\mathrm{G(s)} = \mathrm{C(sI\text{-}A)}^{-1} B = [1\ 0] \begin{bmatrix} s + b/I & -k \\ k/L & s + R \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

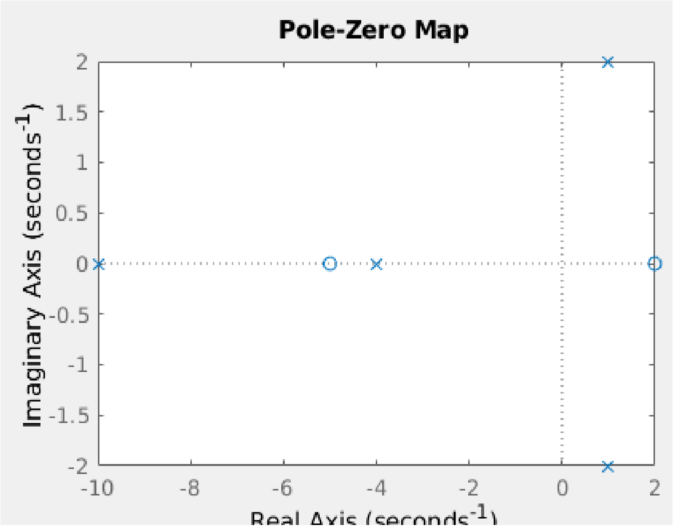
$$= \frac{[1\ 0] \begin{bmatrix} s + R & k \\ -k/L & s + b/I \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{(s + b/I)(s + R) + k^2/L} = \frac{k}{(s + b/I)(s + R) + k^2/L}$$

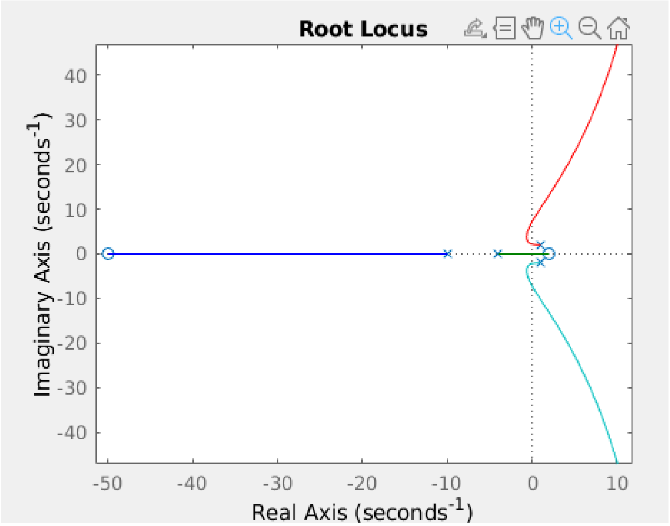
# Classic Control System Design: Root Locus

- how do close-loop dynamics change with K?

$$L(s) = \frac{-20(s-2)(s+5)}{(s+4)(s+10)(s^2+2s+5)}$$




- Additional material
https://www.youtube.com/watch?v=eTVddYCeiKI

# Bode Plot

- Stability of the feedback system is a hard constraint, but not the only one
- Control problem is always a trade-off: fast vs. smooth response
- Bode Plot offers a frequency view of an open loop system
- Complex conjugate system can yield “resonance effect” and influence oscillatory behaviour of a system