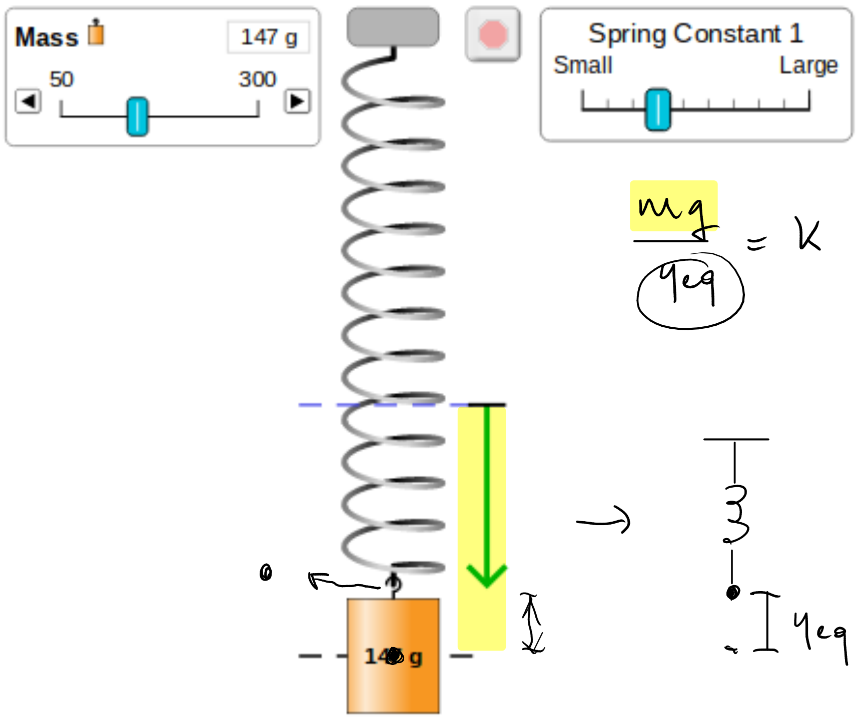
Misura del periodo su PHET

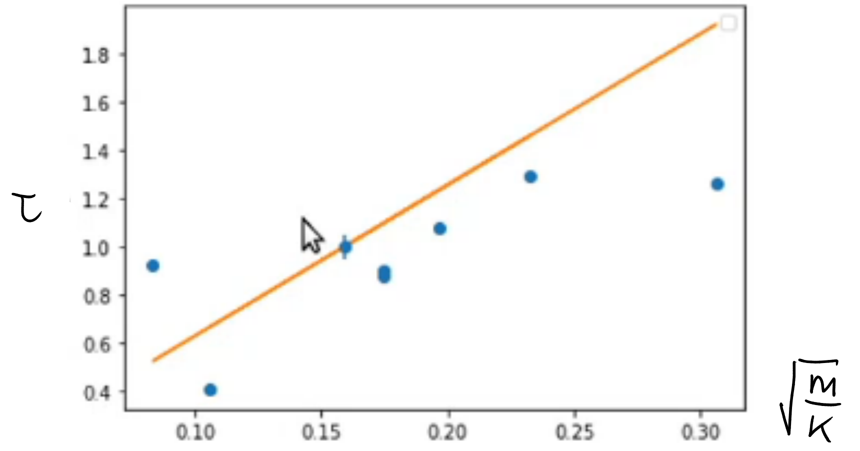
K

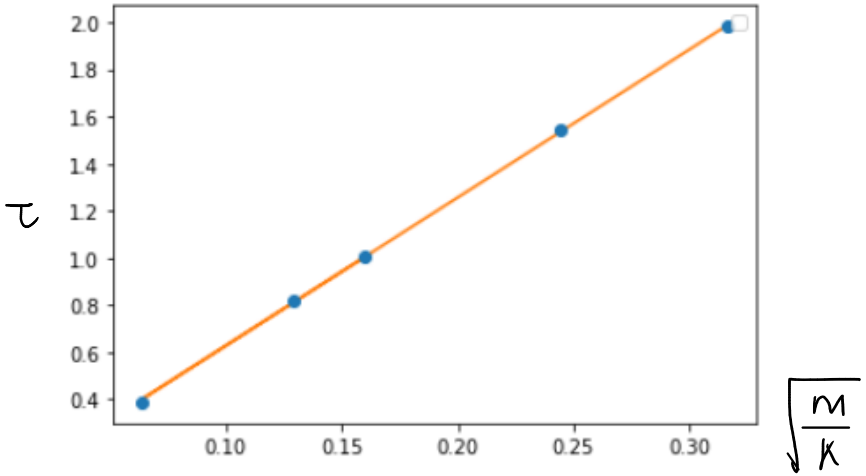
$\uparrow$



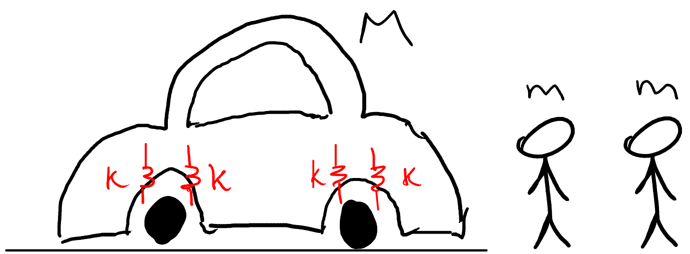
$$\frac{mg}{y_{eq}} = K$$

$\rightarrow$ errore sistematico !





Esempio: oscillazioni di un automobile



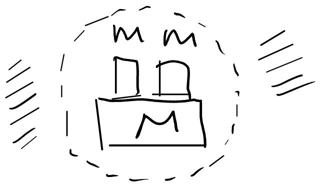
$M = 1300$ kg

$m = 80$ kg

$k = 20000$ N/m

f = frequenza delle oscillazioni verticali = ?

Sistema: auto + persone



→ diagramma corpo libero

III Newton: $\sum \vec{F}_{int} = \vec{0}$

$\rightarrow \sum \vec{F}_{est}$ forze esterne

$\vec{F}_{el} \uparrow\uparrow\uparrow\uparrow$

$m\vec{g} \downarrow \quad M\vec{g} \downarrow \quad m\vec{g} \downarrow$

$\uparrow \vec{e}_y$

$$\vec{F}_{el}^{\,tot} = \sum_{i=1}^{4} \vec{F}_{el} = 4\vec{F}_{el}$$

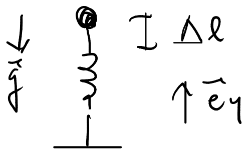
$$\boxed{\begin{array}{l} f = \frac{1}{2\pi}\sqrt{\frac{4k}{M}} \\ f \sim \frac{1}{\sqrt{m_{tot}}} \quad m_{tot} \downarrow \ f \uparrow \\ f = 1.25 \text{ Hz} \end{array}}$$

$\Uparrow$

$\Delta l = ?$

Equilibrio statico: $\sum \vec{F} = \vec{0} \Rightarrow 4\vec{F}_{el} + (M+2m)\vec{g} = \vec{0}$

$4k\Delta l = (M+2m)g \Rightarrow \Delta l = \frac{M+2m}{4k} g$



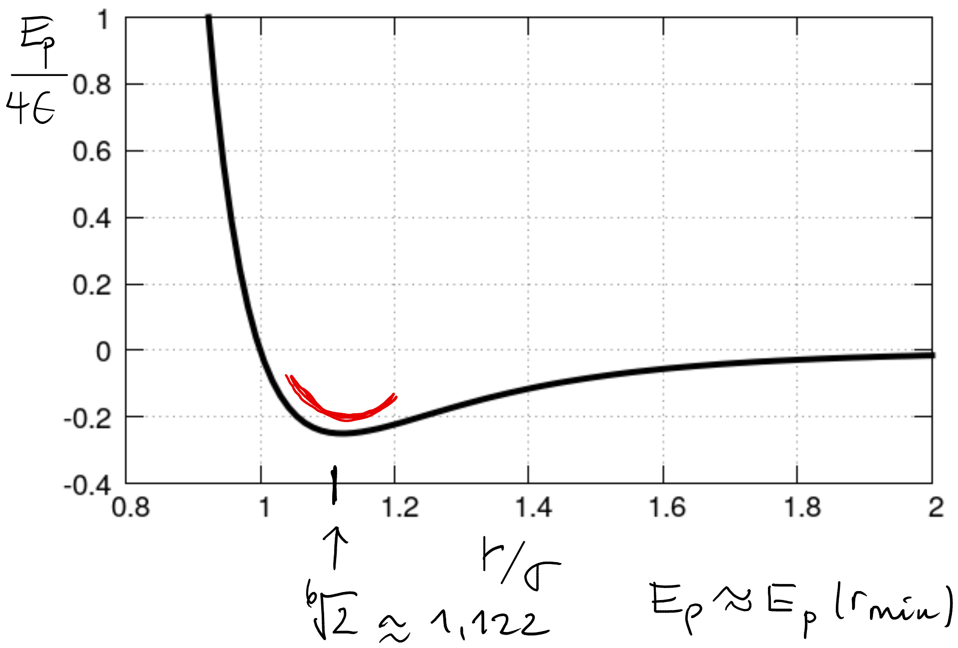
$$(M+2m)\frac{d^2y}{dt^2} = -(M+2m)g + 4ky \rightarrow (M+2m)\frac{d^2y'}{dt^2} = -4ky'$$

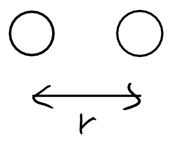
$$\frac{d^2y'}{dt^2} = -\underbrace{\frac{4k}{M+2m}}_{\omega^2} y' \Rightarrow \omega = \sqrt{\frac{4k}{M+2m}} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4k}{M+2m}} = 1.18 \text{ Hz}$$

freq.
SI: Hz $\equiv s^{-1}$

Oscillatore armonico come modello per diversi fenomeni fisici

Interazione tra atomi neutri : → gas rari Ar, Ne, Xe   potenziale Lennard-Jones





$$E_p(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

$$\frac{dE_p}{dr}(r_{min}) = 0$$

$$\frac{dE_p}{dr} = 4\epsilon\left[-12\frac{\sigma^{12}}{r^{13}} + 6\frac{\sigma^6}{r^7}\right]$$

$$\frac{dE_p}{dr} = 24\epsilon\left[-2\frac{\sigma^{12}}{r^{13}} + \frac{\sigma^6}{r^7}\right] = 0$$

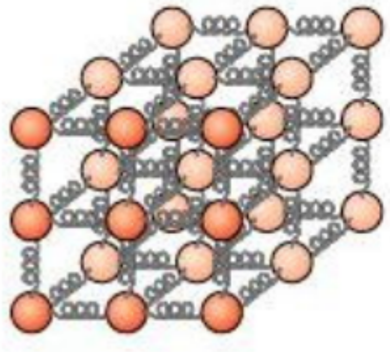
$$2\frac{\sigma^{\cancel{12}\,6}}{r^{\cancel{13}\,6}} = \frac{\cancel{\sigma^6}\,1}{\cancel{r^7}\,1} \Rightarrow r_{min}^6 = 2\sigma^6 \Rightarrow r_{min} = \sqrt[6]{2}\,\sigma$$

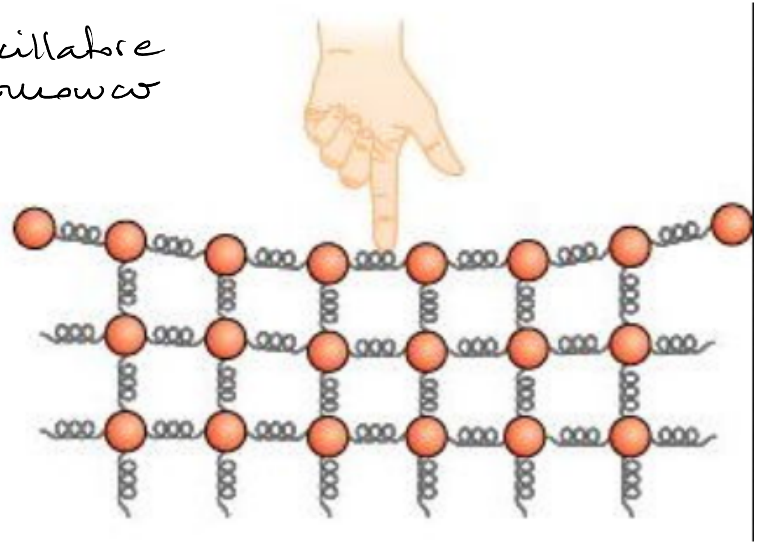
$$E_p \approx E_p(r_{min}) + \frac{1}{2}\frac{d^2E_p}{dr^2}\Big|_{r_{min}}(r-r_{min})^2 \rightarrow \text{oscillatore armonico}$$

Oscillazioni e deformazioni nei solidi

→ oscillatori armonici accoppiati

→ oscillatori armonici indipendenti

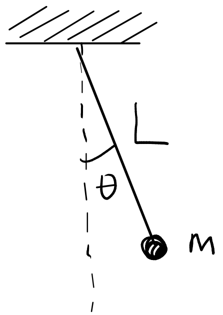




Source : compadre.org

Pendolo semplice

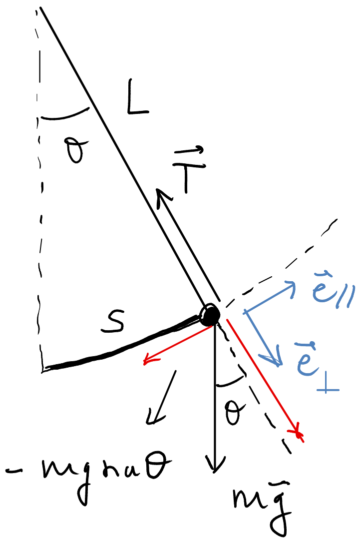
Con attrito → oscillatore armonico SMORZATO



assenza resistenza dell'aria

semplice: massa puntiforme, filo ideale

fisico: massa estesa → rotazione corpo rigido



sistema: massa

forze: peso, tensione

$$\left(\frac{d^2x}{dt^2} = -\omega^2 x \right)^*$$

II Newton: $\sum \vec{F} = m\vec{a}$

$$\sum F_{/\!/} = m a_{/\!/}$$

$$-\cancel{m} g \sin\theta = \cancel{m} \frac{d^2 s}{dt^2}$$

$$\theta = \frac{s}{L} \rightarrow s = \theta L$$

$$L \frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

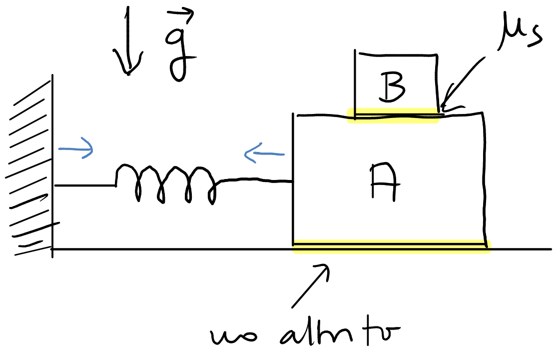
eq. moto pendolo semplice

→ oscillazioni non armoniche

Piccole oscillazioni: $\theta \ll 1 \rightarrow \sin\theta \simeq \theta$ (Taylor I ordine) ok $\theta \lesssim 20° - 30°$

oscillatore armonico $\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}^* \Rightarrow \omega = \sqrt{\frac{g}{L}} \rightarrow \boxed{\tau = 2\pi\sqrt{\frac{L}{g}}}$ $L\uparrow \tau\uparrow$

Problema di ricapitolazione (SJ 12.47)

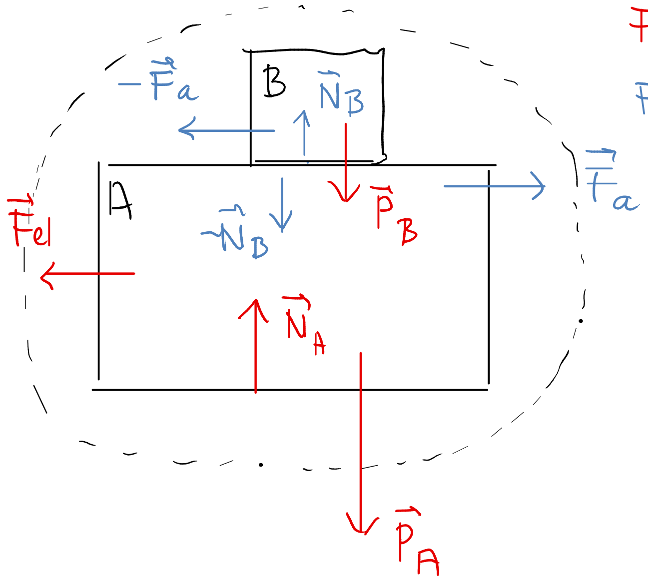


Altrito tra A e B ($\mu_s = 0.6$), non tra A e il suolo

Oscillazioni armoniche di A+B : $f = 1.5\,\text{Hz} \rightsquigarrow K$

Massima ampiezza Δx delle oscillazioni senza che B scivoli su A?

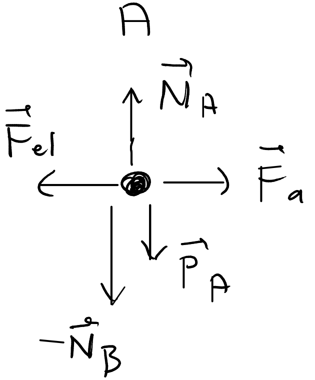
Sistema : A + B   III Newton : coppie forze uguali e opposte in segno

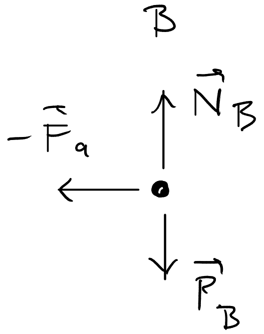


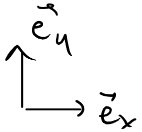
Forze esterne

Forze interne

$\longrightarrow$ corpo libero







$\vec{N}_A = N_A \vec{e}_y \quad , \quad \vec{N}_B = N_B \vec{e}_y \quad , \quad \vec{P}_A = -m_A g \vec{e}_y \quad , \quad \vec{P}_B = -m_B g \vec{e}_y$

$\vec{F}_{el} = F_{el} \vec{e}_x \quad , \quad \vec{F}_a = F_a \vec{e}_x$

Equilibrio verticale:

$A: \quad \vec{N}_A - \vec{N}_B + \vec{P}_A = \vec{0} \qquad \vec{N}_A + \vec{P}_A + \vec{P}_B = \vec{0}$

$B: \quad \vec{N}_B + \vec{P}_B = \vec{0} \qquad N_A = (m_A + m_B) g$

moto solidale A e B: $\vec{a}_A = \vec{a}_B \rightarrow a_{Ax} = a_{Bx}$

$$\begin{cases} m_A a_{Ax} = \sum_A F_x \\ m_B a_{Bx} = \sum_B F_x \end{cases} \Rightarrow \frac{1}{m_A} \sum_A F_x = \frac{1}{m_B} \sum_B F_x$$

$$\frac{1}{m_A} (F_{el} + F_a) = -\frac{1}{m_B} F_a \Rightarrow m_B F_{el} + m_B F_a = -m_A F_a$$

$$F_a = \frac{m_B}{m_A + m_B} F_{el} = \frac{\cancel{m_B}}{m_A + m_B} k \Delta l \leq \mu_s |\vec{N}_B| = \mu_s \cancel{m_B} g \qquad f = \frac{1}{2\pi} \sqrt{\frac{K}{m_A + m_B}}$$

$$\Rightarrow \Delta l \leq \frac{\mu_s (m_A + m_B) g}{K} = \frac{\mu_s g}{4\pi^2 f^2} = \underline{6{,}63 \text{ cm}} \quad \square \qquad 4\pi^2 f^2 = \frac{k}{m_A + m_B}$$