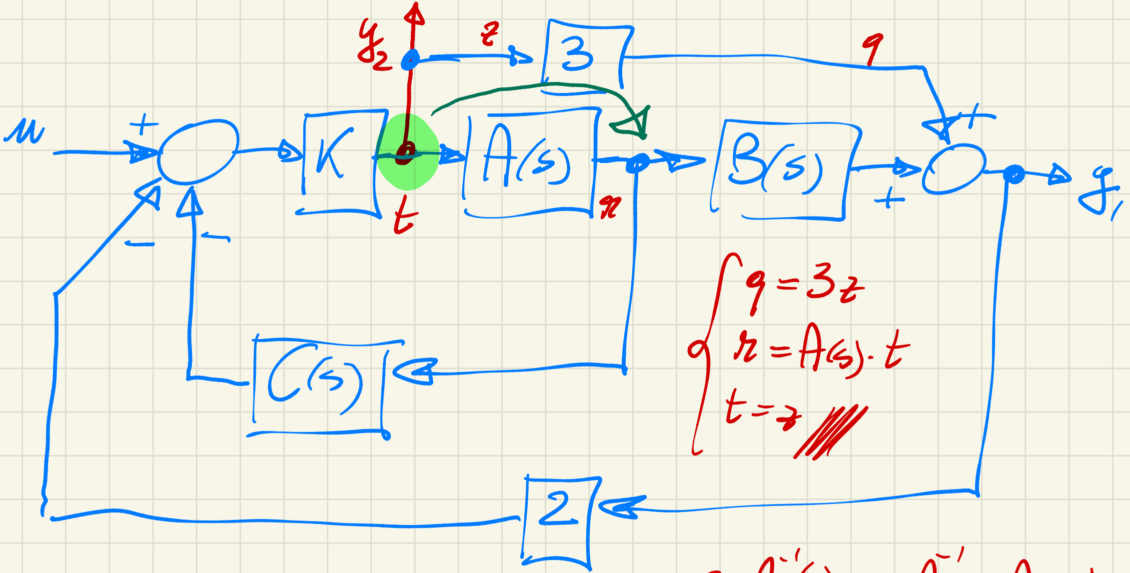
Fondamenti di Automatica

esercizi

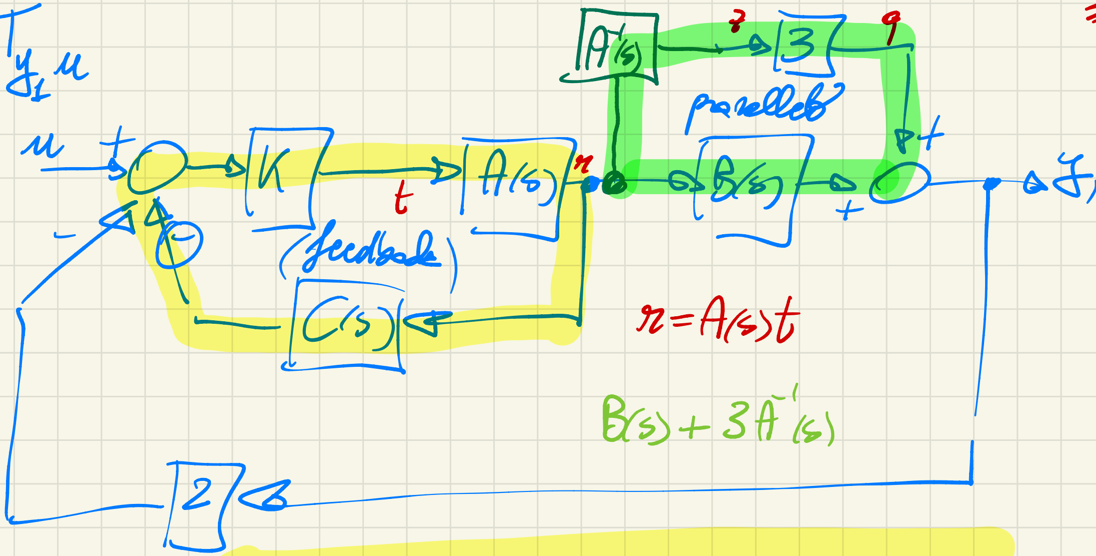
15/04/2021



$$\begin{cases} q = 3z \\ r = A(s) \cdot t \\ t = z \end{cases}$$

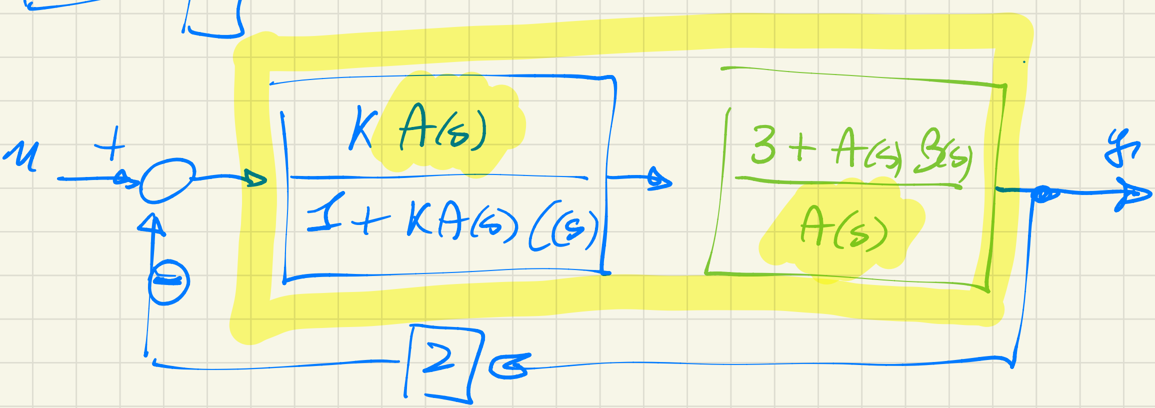
$$z = A^{-1}(s)\, r = A^{-1}(s) A(s)\, t = t$$

$T_{y_1 u}$



$r = A(s) t$

$B(s) + 3A^{-1}(s)$



$$T_{yu} = \frac{\dfrac{K\cancel{A(s)}}{1+KA(s)C(s)} \cdot \dfrac{3+A(s)B(s)}{\cancel{A(s)}}}{1 + 2 \cdot \dfrac{K\cancel{A(s)}}{1+KA(s)C(s)} \cdot \dfrac{3+A(s)B(s)}{\cancel{A(s)}}}$$

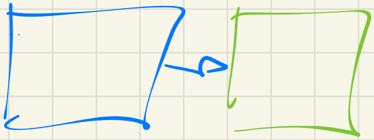
$$A(s) = \frac{2s}{s+1} \qquad B(s) = \frac{1}{s-1} \qquad C(s) = \frac{3}{s}$$

$$\frac{K\,A(s)}{1+KA(s)C(s)} = \frac{\cancel{2Ks/s+1}}{1+\cancel{\dfrac{6K\cancel{s}}{\cancel{s}(s+1)}}} = \frac{2Ks/s+1}{\dfrac{(s+1)+6K}{(s+1)}}$$

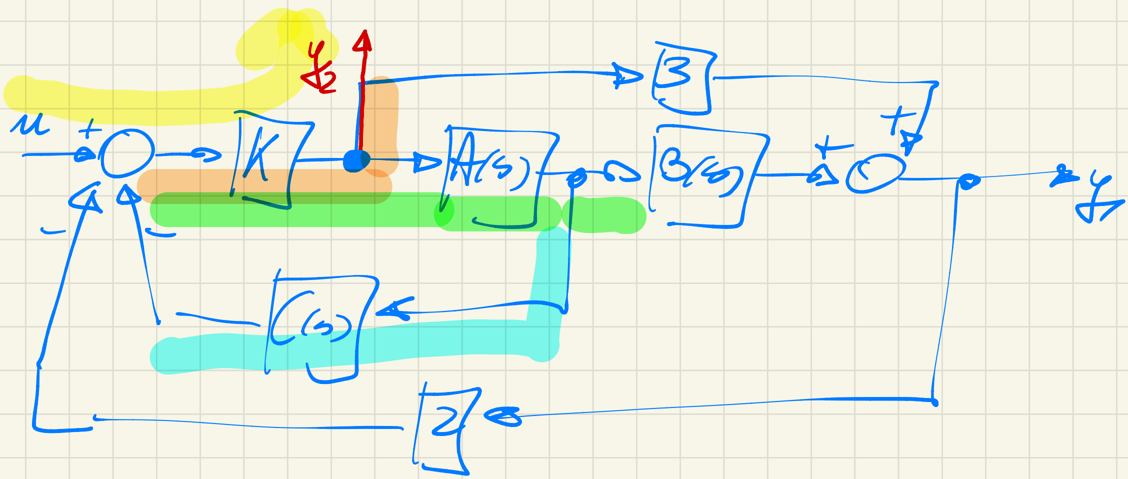
$$= \frac{2Ks}{(s+1)+6K}$$

$$\frac{3+A(s)\cdot B(s)}{A(s)} = \frac{3+\dfrac{2s}{s+1}\cdot\dfrac{1}{s-1}}{\dfrac{2s}{s+1}}$$

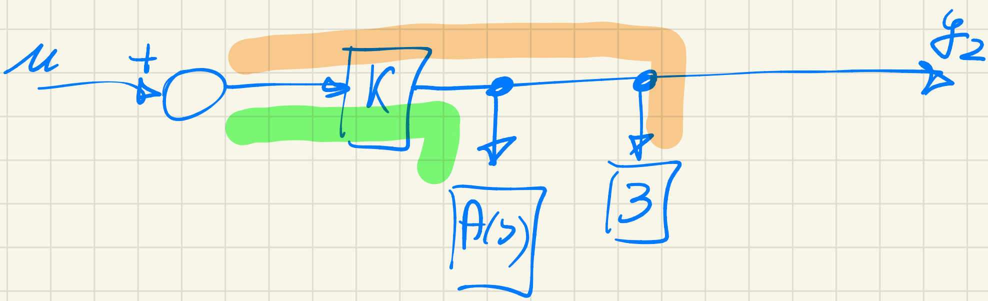
$$= \frac{\left[3(s+1)(s-1)+2s\right]}{\cancel{(s+1)}(s-1)} \cdot \frac{\cancel{s+1}}{2s}$$

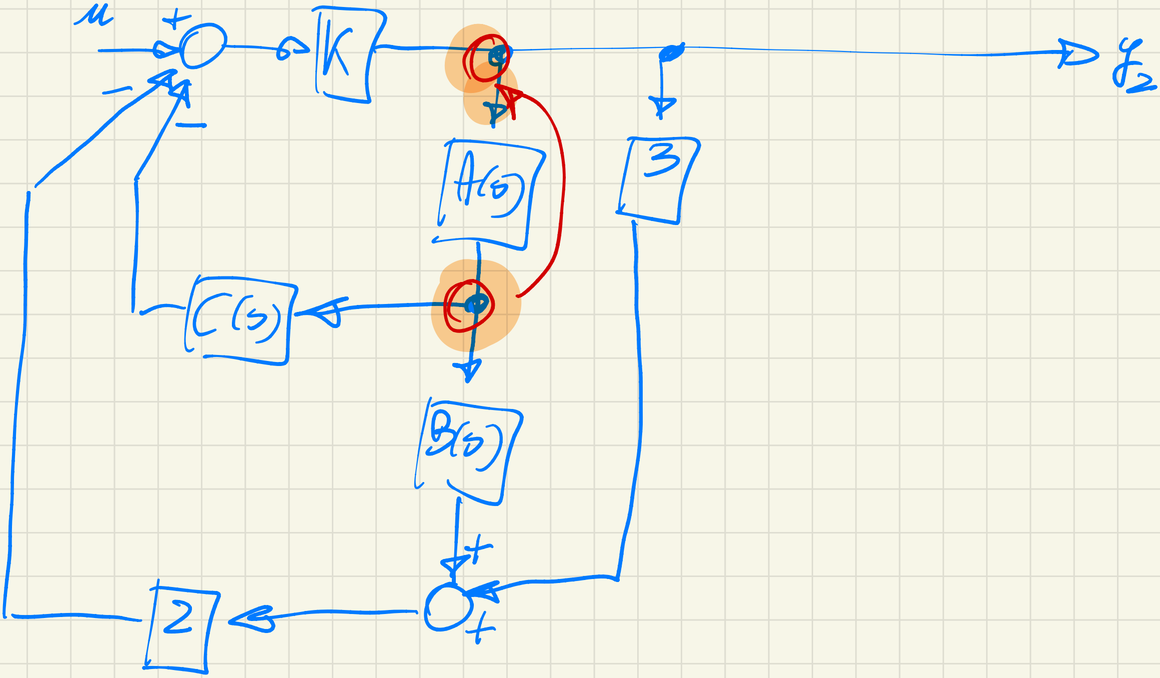


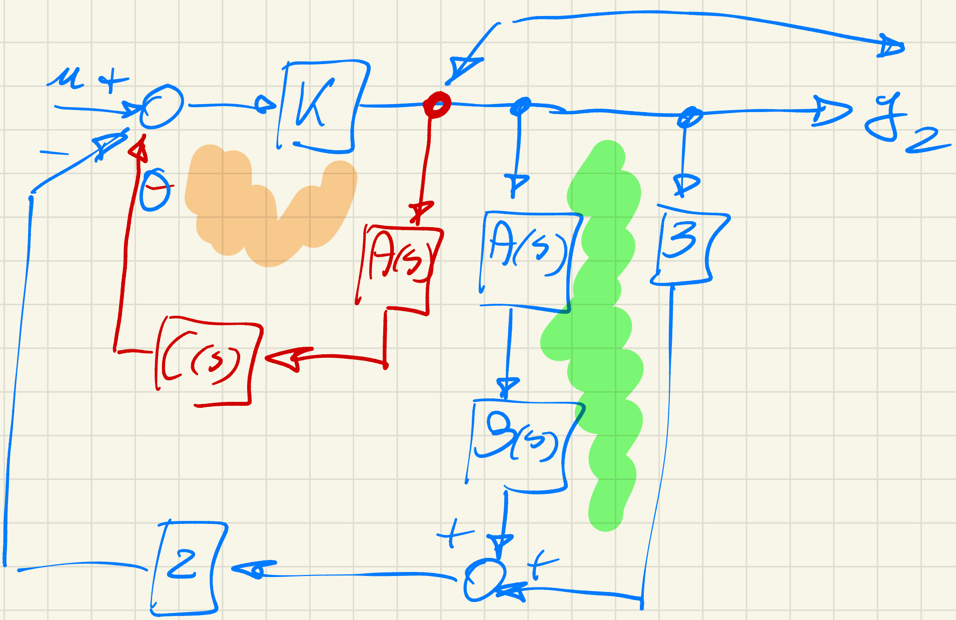
$$\frac{2K\cancel{s}}{(s+1)+6K}\cdot\frac{[3s^2-3+2s]}{2\cancel{s}(s-1)}$$



$T_{y_2 u}$




u
+
−
−
K
A(s)
3
C(s)
B(s)
+
+
2
y_2


u
+
−
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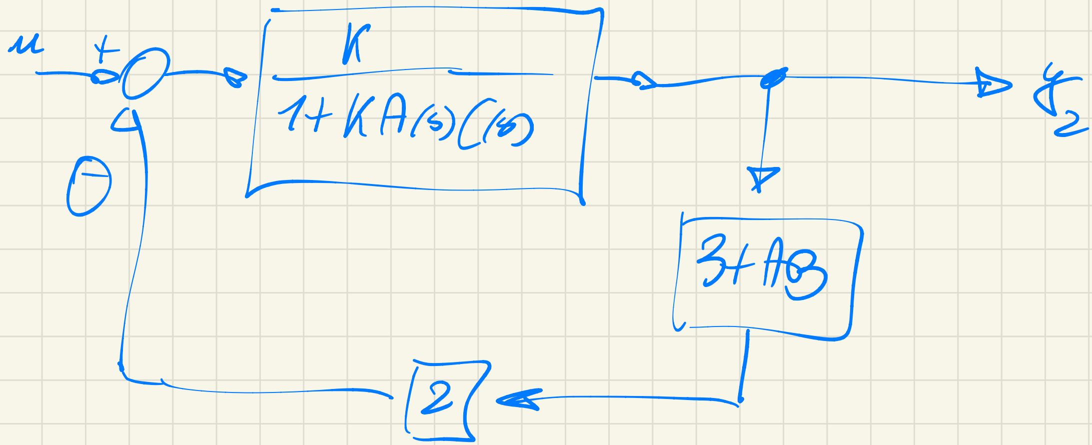
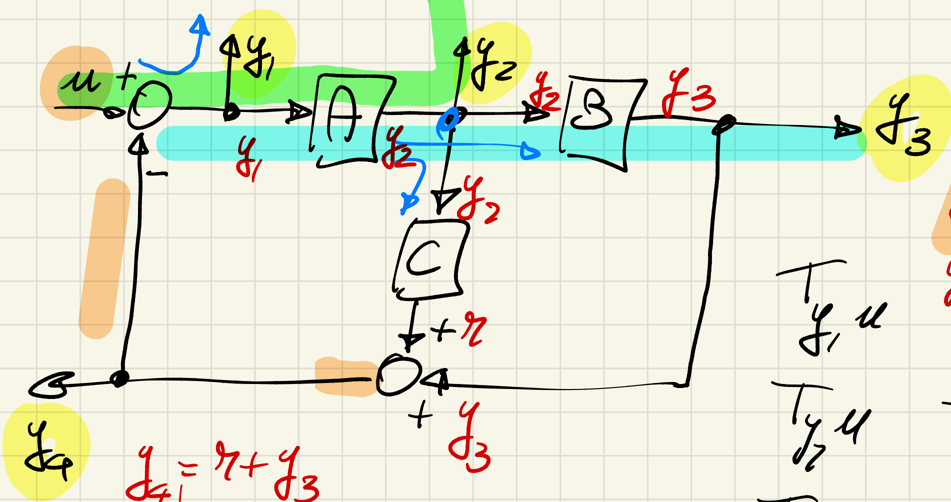
K
A(s)
A(s)
3
C(s)
B(s)
+
+
2
y_2



$$T_{y_2 u} = \frac{\dfrac{K}{1+KA(s)C(s)}}{1+2(3+AB)\dfrac{K}{1+KA(s)C(s)}} = \qquad K$$

$$= \frac{K}{1+KA(s)C(s)+2(3+A(s)B(s))K}$$



$y_2 = A y_1$

$y_3 = B y_2$

$T_{y_1 u}$

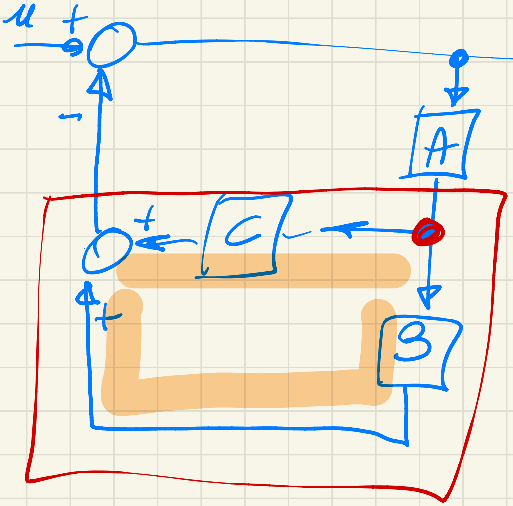
$T_{y_2 u}$ $\qquad$ $T_{y_4 u}$

$T_{y_3 u}$

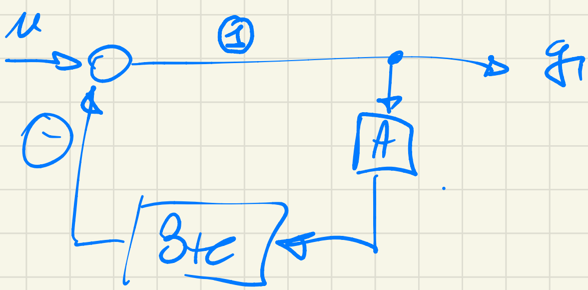
$y_4 = r + y_3$

$= C y_2 + y_3$

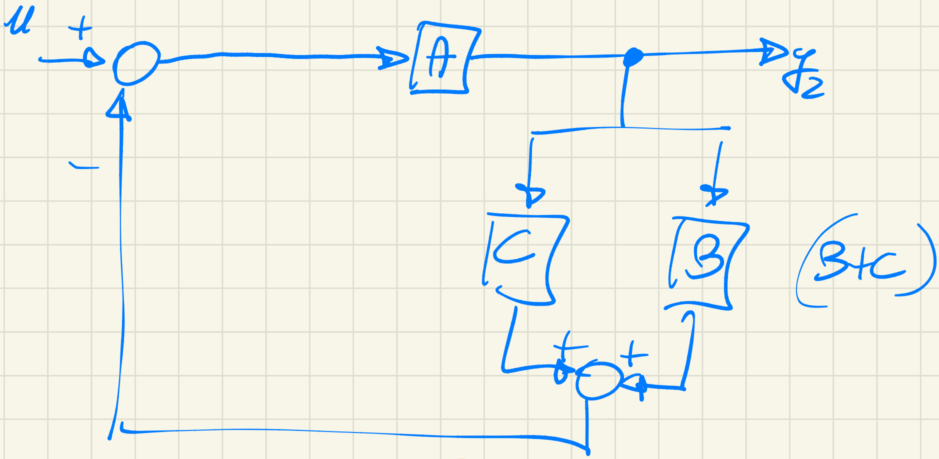
$T_{y u}$



$B + C$



$$T_{y_1 u} = \frac{1}{1 + 1 \cdot A(B+C)}$$



$$T_{y_2 u} = \frac{A}{1 + A(B+C)}$$

$$y_2 = A y_1$$

$$= A \cdot T_{y_1 u}$$

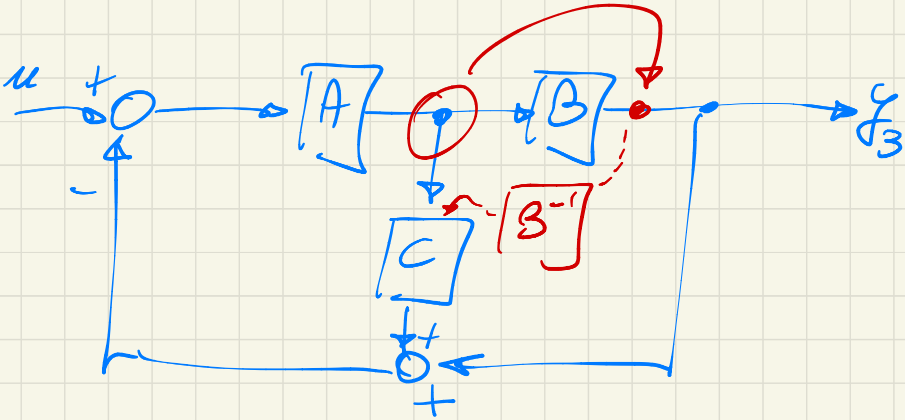
$$T_{y_1 u} = \dots$$

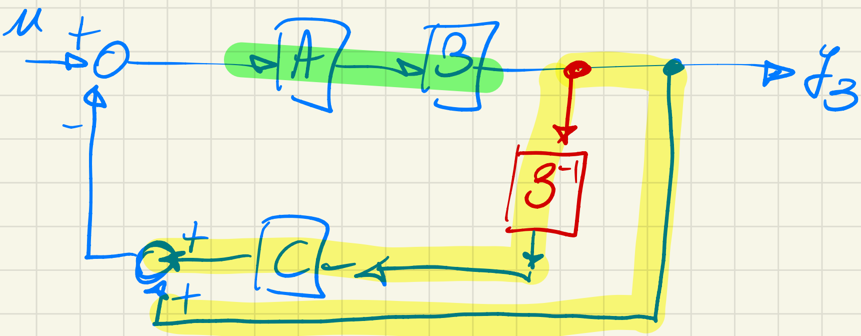
$$T_{y_3 u} = B \cdot A \cdot T_{y_1 u}$$

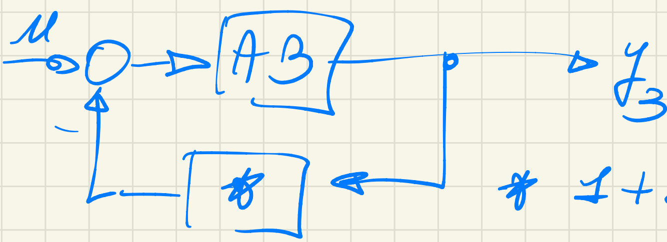
$$y_3 = B y_2 = B A y_1$$

$$y_4 = C y_2 + y_3 = C A y_1 + B A y_1 = (C+B) A y_1$$

$T_{y_3 u}$







$$* \quad 1 + B^{-1}C = \frac{B+C}{B}$$

$$T_{y_3 u} = \frac{AB}{1 + A\cancel{B} \cdot \frac{B+C}{\cancel{B}}} = \frac{AB}{1 + A(B+C)}$$

$$= AB \underbrace{\left[\frac{1}{1 + A(B+C)} \right]}_{T_{y_1 u}}$$

$T_{y_4 u} = ?$

$$T_{y_4 u} = \frac{A(B+C)}{1 + A(B+C)}$$

Realizzazione in equazioni di stato per FdT di ordine 1

approccio valido

SOLO

per sistemi LTI SISO

di ordine $n = 1$

LTI SISO n=1 tempo continuo

$$\begin{cases} \dot{x}(t) = a\,x(t) + b\,u(t) \\ y(t) = c\,x(t) + d\,u(t) \end{cases}$$

$a, b, c, d \in \mathbb{R}$

$d = 0$ sistema strettamente proprio

FdT $F(s) = c(sI - a)^{-1} b = \dfrac{cb}{s-a}$

$d \neq 0$ sistema non strettamente proprio

FdT $F(s) = c(sI - a)^{-1} \cdot b + d = \dfrac{cb}{s-a} + d = \dfrac{ds + (cb - da)}{s-a}$

FdT $\boxed{n=1}$ $F(s) = \dfrac{N(s)}{D(s)}$ m grado di $N(s)$

n grado di $D(s)$

$$\begin{cases} n = 1 \\ m = 0 \end{cases} \qquad F(s) = \frac{f}{ps - q}$$

$f, p, q \in \mathbb{R}$

$p \neq 0$

e)

$$F(s) = \frac{3}{4s-17}$$

$$F(s) = \frac{f}{ps-q} \overset{?}{\longleftrightarrow} \frac{cb}{s-a}$$

$$= \frac{f/p}{s-q/p} \overset{?}{=} \frac{cb}{s-a}$$

$$\begin{cases} cb = f/p \\ -a = -q/p \end{cases}$$

$$\begin{cases} a = q/p \\ cb = f/p \end{cases}$$

?

$c=1 \rightarrow b = f/p$

$b=1 \rightarrow c = f/p$

$$F(s) = \boxed{\frac{3}{4s-17}} = \frac{3/4}{s-17/4} \qquad \frac{cb}{s-a}$$

$$\begin{cases} a = 17/4 \\ cb = 3/4 \end{cases}$$

$a = 17/4 \quad b = 3/4 \quad c = 1$

$b = 1 \quad c = 3/4$

$$u \rightarrow \boxed{F(s)} \rightarrow y$$

$$\begin{cases} \dot{x} = \frac{17}{4}x + \frac{3}{4}u \\ y = x \end{cases} \qquad \begin{cases} \dot{x} = \frac{17}{4}x + u \\ y = \frac{3}{4}x \end{cases}$$

FdT $n=1$
$m=1$

$$F(s) = \frac{rs - w}{ps - q}$$

r, w
p, q $\in \mathbb{R}$

$r \neq 0$
$p \neq 0$

$$F_2(s) = \frac{2s+3}{4s+5}$$

$$F(s) = \frac{cb}{s-a} + d$$

$$\frac{rs - w}{ps - q} = t + \frac{h}{ps - q}$$

$$\begin{array}{l|l} rs - w & ps - q \\ \hline -rs + q\frac{r}{p} & \left(\frac{r}{p}\right) \text{ quoziente } t \\ \hline \left(\frac{qr}{p} - w\right) \text{ resto } h & \end{array}$$

$$\frac{rs - w}{ps - q} = \left(\frac{r}{p}\right) + \frac{\left(\frac{qr}{p} - w\right)}{ps - q}$$

$$d + \frac{cb}{s-a}$$

$\to d = \dfrac{r}{p}$

$\dfrac{qr}{p} - w \triangleq h \qquad \dfrac{h}{ps-q} = \dfrac{h/p}{s-q/p} \longleftrightarrow \dfrac{cb}{s-a}$

$\begin{cases} a = q/p \\ cb = h/p \end{cases} \qquad \begin{cases} a = q/p \\ cb = \left(\dfrac{qr}{p} - w\right)/p \end{cases}$

$F(s) = \dfrac{2s+3}{4s+5} = \dfrac{1}{2} + \dfrac{1/2}{4s+5} = \dfrac{1}{2} + \dfrac{1/8}{s+5/4}$

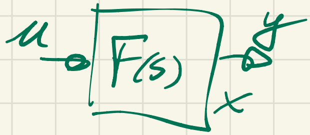
$$\begin{array}{r|l} 2s+3 & 4s+5 \\ \hline -2s-\frac{5}{2} & \frac{1}{2} \\ \hline / \quad 3-\frac{5}{2} = \frac{1}{2} & \end{array}$$

$= \dfrac{1}{2} + \dfrac{1/8}{s+\frac{5}{4}}$

$d + \dfrac{cb}{s-a} \qquad \begin{cases} d = \frac{1}{2} \\ a = -\frac{5}{4} \end{cases} \qquad cb = \frac{1}{8}$

$\begin{cases} a = -\frac{5}{4} \\ b = 1 \\ c = \frac{1}{8} \quad d = \frac{1}{2} \end{cases} \qquad \begin{cases} a = -\frac{5}{4} \\ c = 1 \quad b = \frac{1}{8} \\ d = \frac{1}{2} \end{cases}$

$$F(s) = \frac{2s+3}{4s+5}$$



$$\begin{cases} \dot{x} = -\frac{5}{4}x + u \\ y = \frac{1}{8}x + \frac{1}{2}u \end{cases} \qquad \begin{cases} \dot{x} = -\frac{5}{4}x + \frac{1}{8}u \\ y = x + \frac{1}{2}u \end{cases}$$

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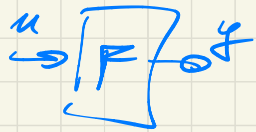
(1) $F(s) = \frac{2}{s+1} \longleftrightarrow \frac{cb}{s-a}$



$$\begin{cases} a = -1 \\ cb = 2 \end{cases} \qquad \begin{cases} \dot{x}_1 = -x_1 + 2u \\ y = x_1 \end{cases}$$

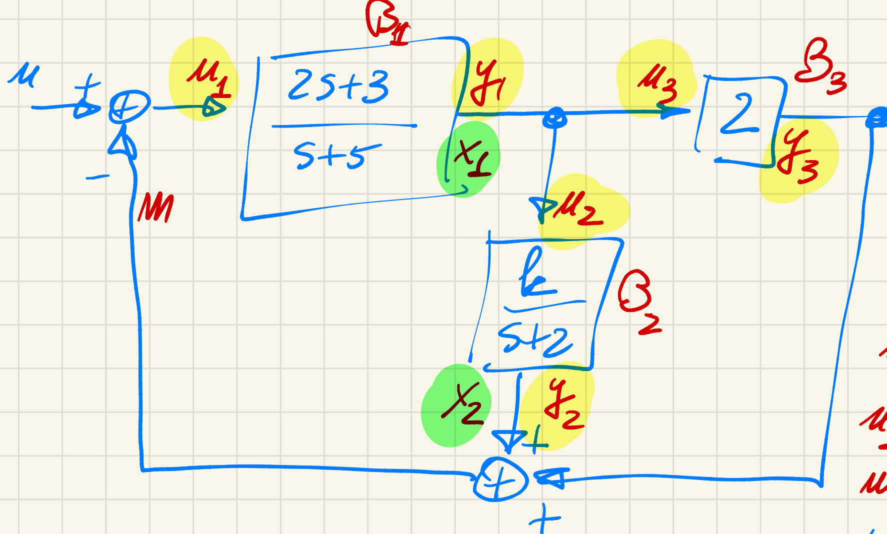
(2) $T(s) = \frac{2s}{3s-1}$

$$= \frac{2}{3} + \frac{2/3}{3s-1}$$

$$d \quad \frac{2}{3} + \frac{2/9}{s - \frac{1}{3}}$$

$$\begin{array}{l|l} 2s & 3s-1 \\ \hline -2s + 2/3 & 2/3 \\ \hline +\frac{2}{3} & \end{array}$$

$$\frac{cb}{s-a} \qquad cb = 2/9 \qquad a = \frac{1}{3}$$



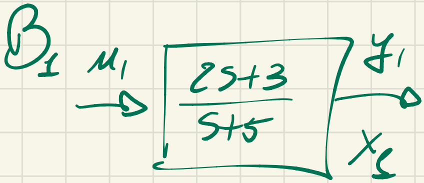
$y_3 = y$

$u_3 = y_1$

$u_2 = y_1$

$u_1 = u - m$

$m = y_2 + y_3$

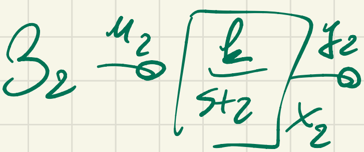


$$\frac{2s+3}{s+5} = \frac{\overset{cb}{-7}}{s\underset{-a}{+5}} + \underset{d}{2}$$

$$\begin{array}{r|l} 2s+3 & s+5 \\ \hline -2s-10 & 2 \\ \hline -7 & \end{array}$$

$$\begin{cases} \dot{x}_1 = -5x_1 - 7u_1 \\ y_1 = x_1 + 2u_1 \end{cases}$$

$$\begin{cases} \dot{x}_1 - 5x_1 + u_1 \\ y_1 = -7x_1 + 2u_1 \end{cases}$$



$\frac{cb}{s-a}$

$$\begin{cases} \dot{x}_2 = -2x_2 + ku_2 \\ y_2 = x_2 \end{cases}$$

$$\begin{cases} \dot{x}_2 = -2x_2 + u_2 \\ y_2 = kx_2 \end{cases}$$

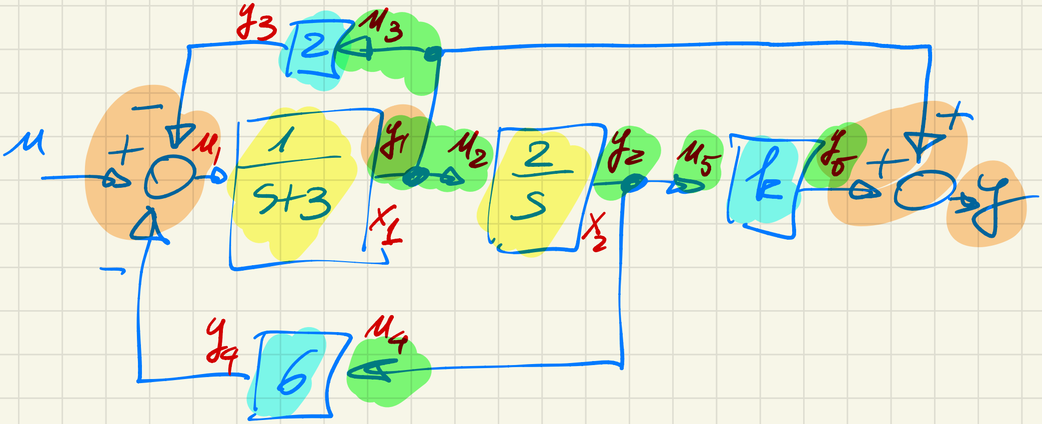
S_3 $\quad u_3 \to \boxed{2} \to y_3$ $\qquad y_3 = 2u_3$

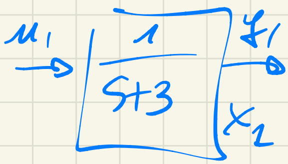
$$\dot{x}_1 = -5x_1 + u_1$$
$$\dot{x}_2 = -2x_2 + 6u_2$$

$$y_1 = -7x_1 + 2u_1$$
$$y_2 = x_2$$
$$y_3 = 2u_3$$

$$\dot{x}_1 = -5x_1 \ldots$$

$$u_3 = u_2 = y_1$$
$$y_3 = y$$
$$u_1 = u - (y_2 + y_3)$$
$$= u - x_2 - 2[-7x_1 + 2u_1]$$

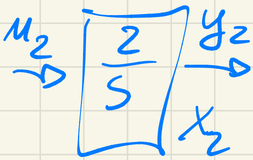




$$\frac{cb}{s-a}$$

$$\begin{cases} \dot{x}_1 = -3x_1 + u_1 \\ y_1 = x_1 \end{cases}$$

$c = 1$



$$\frac{cb}{s-a}$$

$$\begin{cases} \dot{x}_2 = 2u_2 \\ y_2 = x_2 \end{cases}$$

$c = 1$

$$\begin{cases} \dot{x}_1 = -3x_1 + u_1 \\ \dot{x}_2 = 2u_2 \\ y_1 = x_1 \\ y_2 = x_2 \\ u_2 = u_3 = y_1 \quad y_3 = 2u_3 \quad u_5 = y_2 \quad y = y_5 + y_1 \\ u_4 = y_2 \quad y_4 = 6u_4 \quad y_5 = k\,u_5 \quad u_1 = u - y_3 - y_4 \end{cases}$$

$$\begin{cases} \dot{x}_1 = -5x_1 - 6x_2 + u \\ \dot{x}_2 = 2x_1 \\ y = x_1 + k x_2 \end{cases}$$