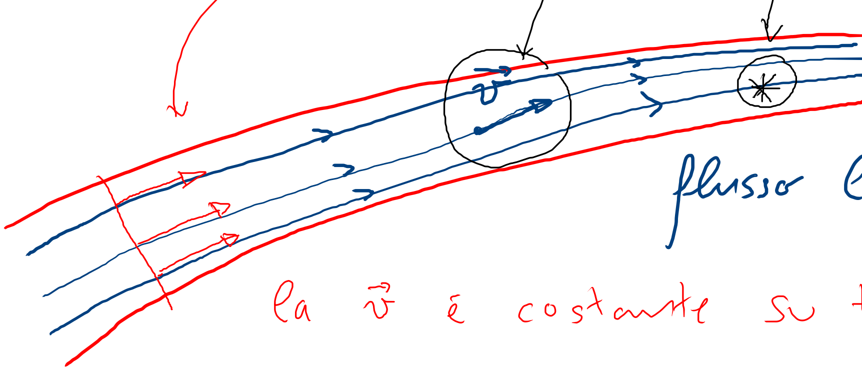
FLUIDODINAMICA

FLUIDO IDEALE $\begin{cases} \rho \text{ costante (incomprimibile)} \\ \eta = 0 \quad \text{(no attrito)} \end{cases}$

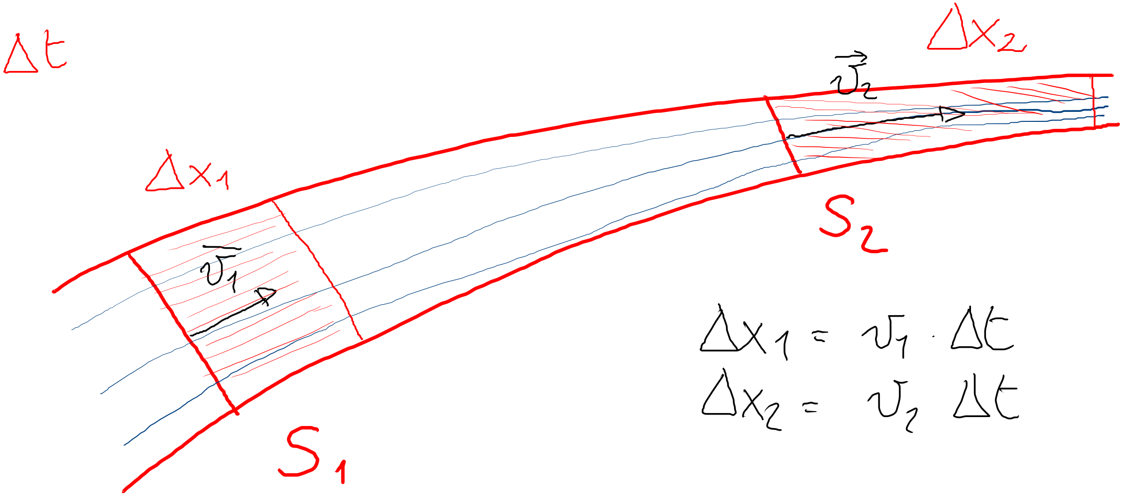
FLUSSO $\begin{cases} \text{stazionario} & \vec{v} \text{ costante nel tempo} \\ \text{irrotazionale} & \text{i mulinelli non girano} \end{cases}$



flusso laminare

la $\vec{v}$ è costante su tutta la sezione

EQUAZIONE DI CONTINUITA'



$\Delta x_1 = v_1 \cdot \Delta t$

$\Delta x_2 = v_2 \, \Delta t$

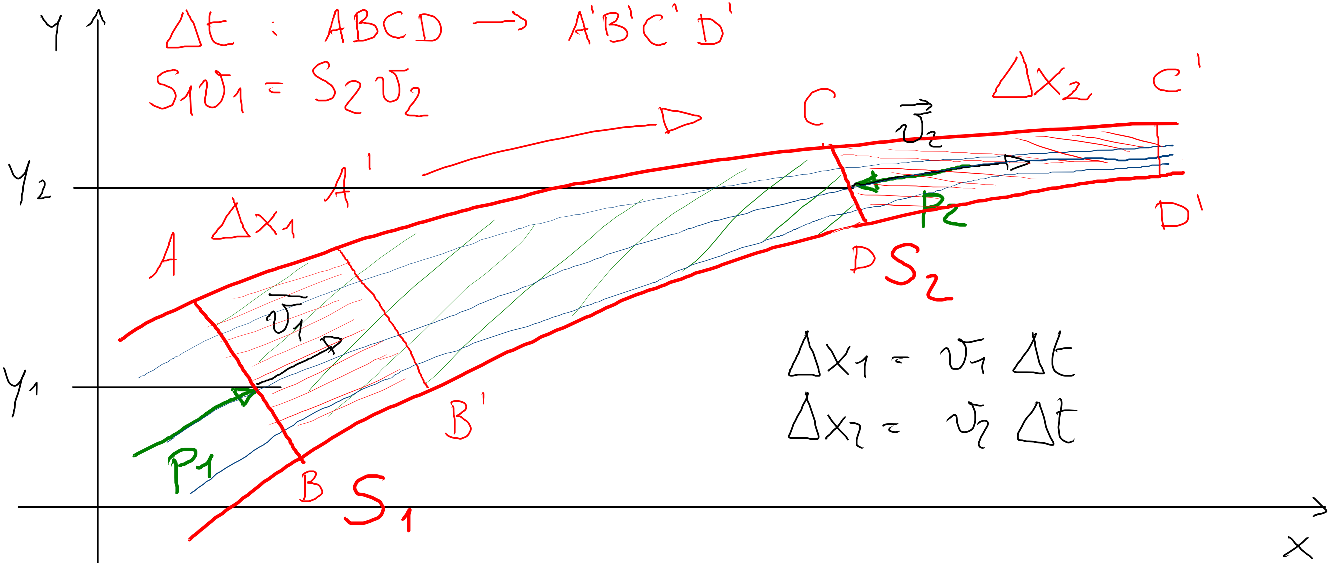
Q portata $\frac{m^3}{s}$ $\qquad Q = S v$

$$Q_1 = \frac{S_1 \Delta x_1}{\Delta t} = \frac{S_1 v_1 \cdot \cancel{\Delta t}}{\cancel{\Delta t}} = S_1 v_1$$

$$Q_2 = \frac{S_2 \, \Delta x_2}{\Delta t} = \frac{S_2 v_2 \, \cancel{\Delta t}}{\cancel{\Delta t}} = S_2 v_2$$

$$Q_1 = Q_2 \Rightarrow \boxed{S_1 v_1 = S_2 v_2}$$

TEOREMA DI BERNOULLI



$$\mathcal{L} = \Delta K$$

$$\mathcal{L}_p + \mathcal{L}_g = \Delta K \qquad\qquad \mathcal{L}_g = -\Delta U_g$$

$$\mathcal{L}_p = \Delta K + \Delta U_g$$

(vedi slide successiva)

$$(p_1 - p_2)\cancel{V} = \frac{1}{2}\rho \cancel{V}(v_2^2 - v_1^2) + \rho \cancel{V} g (y_2 - y_1)$$

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

- $\mathcal{L}_p = F_1 \cdot \Delta x_1 - F_2 \cdot \Delta x_2 = p_1 S_1 \Delta x_1 - p_2 S_2 \Delta x_2 = p_1 S_1 v_1 \Delta t - p_2 S_2 v_2 \Delta t = (p_1 - p_2) V$

$F_1 = p_1 S_1$
$F_2 = p_2 S_2$

$V = S_1 v_1 \Delta t = S_2 v_2 \Delta t$

- $\Delta U_g = U_{g\,A'B'C'D'} - U_{g\,ABCD} = U_{g\,CDD'C'} - U_{g\,ABB'A'} = \rho V g y_2 - \rho V g y_1$

- $\Delta K = K_{A'B'C'D'} - K_{ABCD} = K_{CDD'C'} - K_{ABB'A'} = \frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2$

Ricordo che in meccanica
$U = mgh$
$K = \frac{1}{2} m v^2$

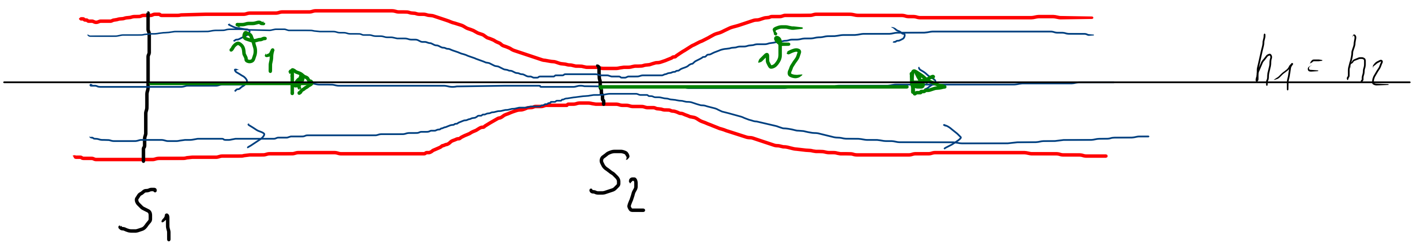
$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{cost}$$

$$\rho = \frac{m}{V}$$

$$\frac{J}{m^3} \qquad \frac{J}{m^3} \qquad \frac{J}{m^3}$$

$$[p] = \frac{N}{m^2} \cdot \frac{m}{m} = \frac{J}{m^3}$$

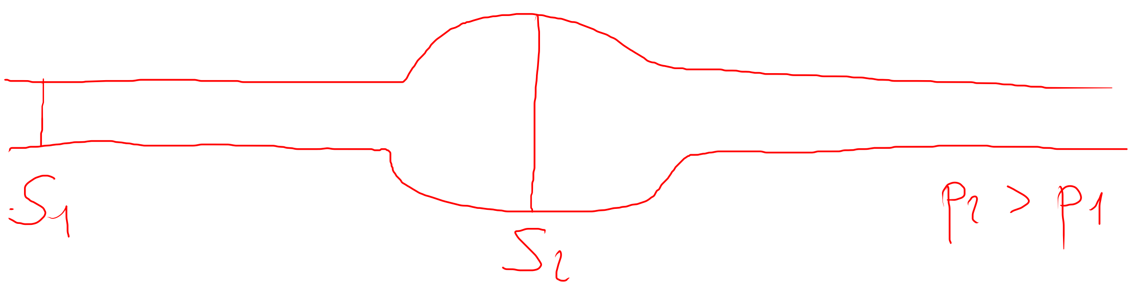
EFFETTO VENTURI



$$S_1 v_1 = S_2 v_2 \qquad v_2 = v_1 \frac{S_1}{S_2} \qquad v_2 > v_1$$

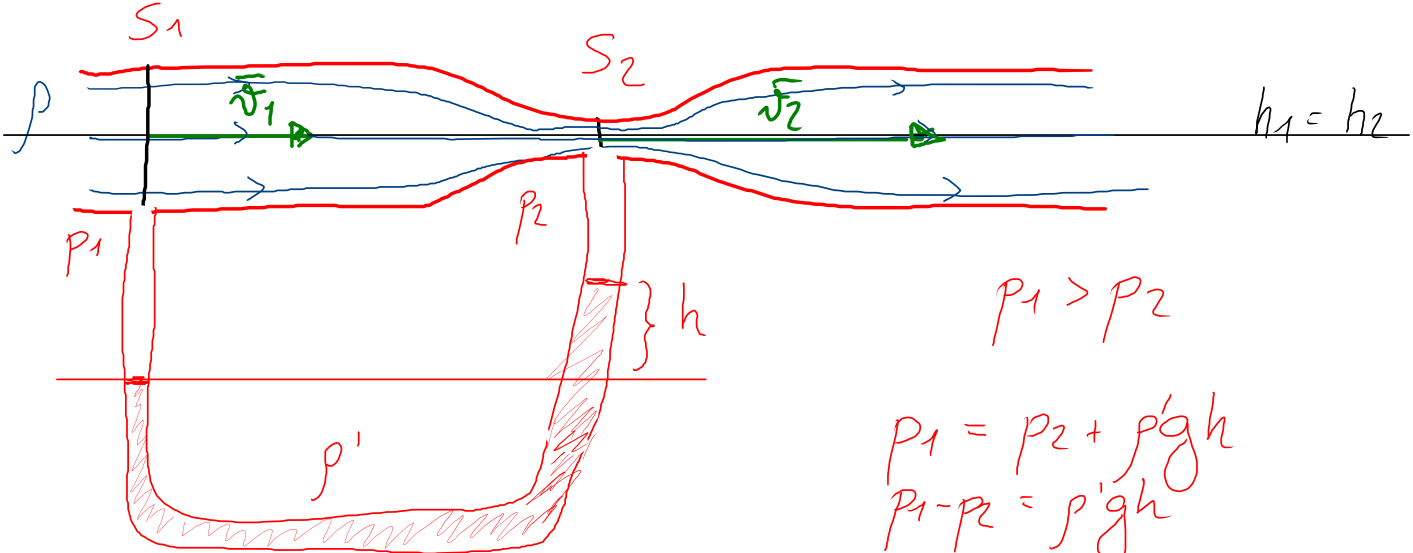
$$p_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = p_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad \Rightarrow \quad p_2 < p_1$$



$p_2 > p_1$

VENTURIMETRO



$p_1 > p_2$

$p_1 = p_2 + \rho' g h$

$p_1 - p_2 = \rho' g h$

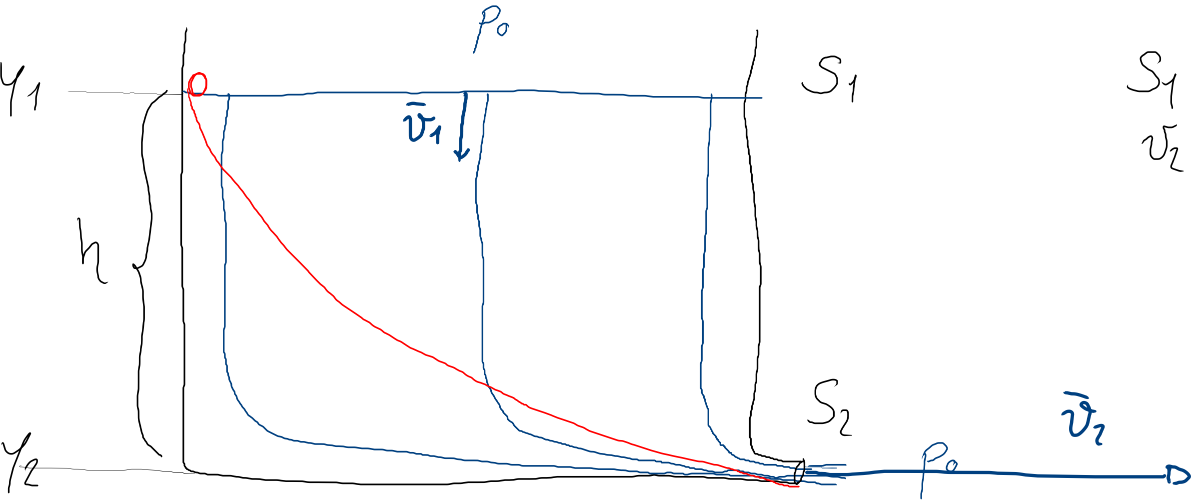
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \qquad v_2 = v_1 \frac{S_1}{S_2} > v_1$$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\stackrel{!}{=} \frac{1}{2}\rho\left(v_1^2 \frac{S_1^2}{S_2^2} - v_1^2\right) = \frac{1}{2}\rho v_1^2\left(\frac{S_1^2}{S_2^2} - 1\right)$$

$$\rho' g h = \frac{1}{2}\rho v_1^2\left(\frac{S_1^2}{S_2^2} - 1\right) \qquad v_1^2 = \underbrace{\frac{\rho' g}{\frac{1}{2}\rho\left(\frac{S_1^2}{S_2^2} - 1\right)}}_{K} h$$

TORRICELLI

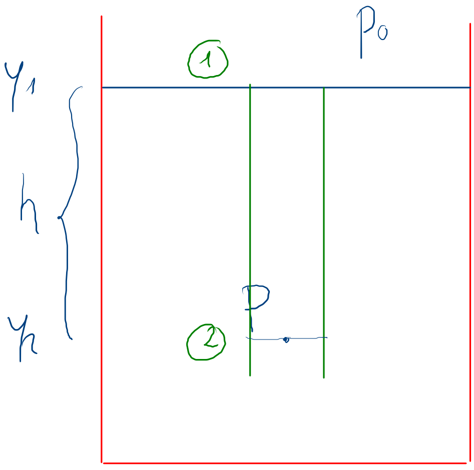


$$S_1 \gg S_2$$
$$v_2 \gg v_1$$

$$\cancel{p_0} + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \cancel{p_0} + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\frac{1}{2}\cancel{\rho} v_2^2 \cong \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \rho g (y_1 - y_2) = \cancel{\rho} g h$$

$$v_2^2 = 2gh \qquad \boxed{v_2 = \sqrt{2gh}}$$



$$p_1 + \cancel{\frac{1}{2}\rho v_1^2} + \rho g y_1 = p_2 + \cancel{\frac{1}{2}\rho v_2^2} + \rho g y_2$$

$$p_2 - p_1 = \rho g (y_1 - y_2)$$

$$p_2 = \underset{\substack{\downarrow \\ p_0}}{p_1} + \rho g \underbrace{(y_1 - y_2)}_{h}$$

$$p_2 = p_0 + \rho g h$$