MECCANICA RAZIONALE

Ing Civile & Ambientale

Navale

20 Aprile 2021

Dinamica $\longrightarrow$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} = Q_i$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0$$

con $\mathcal{L} = K - V$

$$\begin{cases} \underline{R}^e = \dfrac{d}{dt}\underline{P} \\ \underline{M}^e(O) = \dfrac{d}{dt}\underline{L}(O) + \underline{v}_O \wedge \underline{P} \end{cases}$$

$$\underline{P} = \sum_{B \in S}' m_B \underline{v}_B = M \underline{v}_G$$

$$\underline{L}(O) = \sum_{B \in S}' (\underline{x}_B - \underline{x}_O) \wedge m_B \underline{v}_B$$

Abbiamo visto cosa sono K e $\underline{L}(O)$

per un corpo rigido

K per un rigido

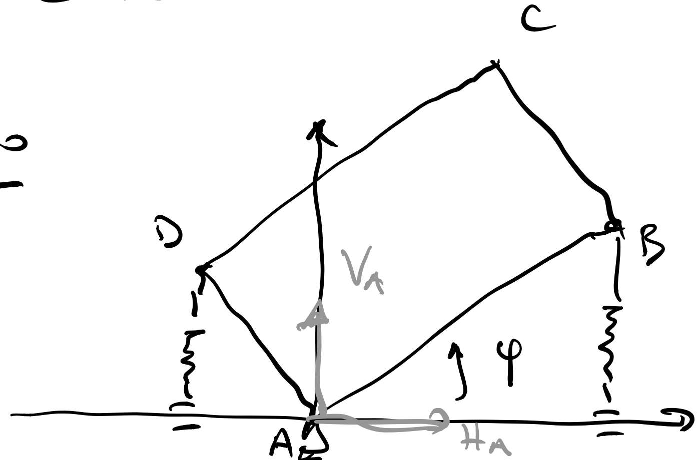
- $K = \frac{1}{2} M v_G^2 + \frac{1}{2}\underline{\omega}\cdot I_G(\underline{\omega})$ generico
- $K = \frac{1}{2}\underline{\omega}\cdot I_O(\underline{\omega})$ O fisso
- $K = \frac{1}{2} I_z \omega^2$ z fisso
- $K = \frac{1}{2} M v_G^2$ Traslazione

$\underline{L}(O)$ per un rigido

- $O \in R$ $\quad \underline{L}(O) = M(\underline{x}_G - \underline{x}_O)\wedge \underline{v}_O + I_O(\underline{\omega})$
- $O \notin R$ $\quad \underline{L}(O) = \underline{L}(A) + (\underline{x}_A - \underline{x}_O)\wedge M\underline{v}_G$

  $A \in R$

<u>Esercizio</u>



$\overline{AB} = 2\ell$

$\overline{BC} = \ell$

lamina rettangolare omogenea

Eq. del moto

$$K = \frac{1}{2} I_{A,3} \dot\varphi^2 = \frac{1}{2}\left[\underbrace{\frac{m}{3}(\ell^2 + 4\ell^2)}_{\frac{m}{3}(a^2+b^2)} \dot\varphi^2\right]$$

$$= \frac{1}{2}\left(\frac{5}{3} m \ell^2 \dot\varphi^2\right)$$

$$V = \frac{c}{2} y_B^2 + \frac{c}{2} y_D^2 = \frac{c}{2}(2\ell)^2 \sin^2\varphi + \frac{c}{2}\ell^2\cos^2\varphi$$

$$= \frac{c}{2}\ell^2(3\sin^2\varphi + 1)$$

$$\mathcal{L} = \underbrace{\frac{1}{2}\left(\frac{5}{3} m \ell^2 \dot\varphi\right)}_{K} \underbrace{-}_{-} \underbrace{\frac{c}{2}\ell^2(3\sin^2\varphi + 1)}_{V}$$

$$\boxed{\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\varphi} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0}$$

$$\frac{5}{3} m \ell^2 \ddot\varphi + 3 c \ell^2 \sin\varphi\cos\varphi = 0$$

condizioni iniziali
$$\begin{cases} \varphi(0) = \varphi_0 = \frac{\pi}{4} \\ \dot\varphi(0) = \omega_0 \end{cases}$$

Energia : $E = K + V$

$$\boxed{\begin{aligned} &= \frac{1}{2}\left(\frac{5}{3} m \ell^2 \dot\varphi^2\right) + \frac{c}{2}\ell^2(3\sin^2\varphi + 1) \\ &= \frac{1}{2}\left(\frac{5}{3} m \ell^2 \omega_0^2\right) + \frac{c\ell^2}{2}\left(\frac{3}{2} + 1\right) \end{aligned}}$$

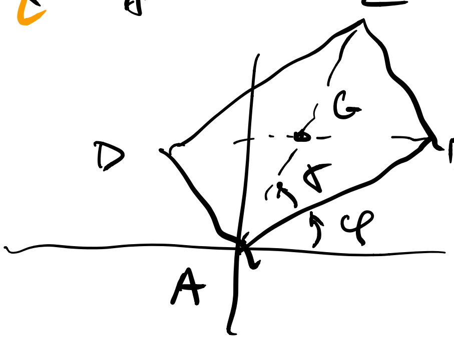
Reazioni vincolari in A

Abbiamo H_A, V_A $\to$ ECD per la risultante

$$\underline{R}^e = H_A\,\underline{e}_1 + V_A\,\underline{e}_2 - c\,y_B\,\underline{e}_2 - c\,y_D\,\underline{e}_2$$

$$= \dot{\underline{P}} \qquad (m\,\dot{\underline{v}}_G) \to \begin{matrix} m\,\ddot{x}_G\,\underline{e}_1 \\ m\,\ddot{y}_G\,\underline{e}_2 \end{matrix}$$

$$\begin{cases} H_A = m\,\ddot{x}_G \\ V_A = c\,2\ell\sin\varphi + c\,\ell\cos\varphi + m\,\ddot{y}_G \end{cases}$$



$$\begin{matrix} x_G \\ y_G \end{matrix} \to \begin{matrix} \ddot{x}_G \\ \ddot{y}_G \end{matrix}$$

$$\begin{cases} x_G = \overline{AG}\cos(\gamma+\varphi) \\ y_G = \overline{AG}\sin(\gamma+\varphi) \end{cases}$$

$$\overline{AG} = \frac{1}{2}\sqrt{\ell^2 + 4\ell^2} = \frac{\ell}{2}\sqrt{5} \qquad \overline{AC} = \sqrt{\ell^2+4\ell^2} = \ell\sqrt{5}$$

$$\gamma: \quad \sin\gamma = \frac{\overline{BC}}{\overline{AC}} = \frac{1}{\sqrt{5}}, \quad \cos\gamma = \frac{\overline{AB}}{\overline{AC}} = \frac{2}{\sqrt{5}}$$

$$\begin{cases} \dot{x}_G = -\overline{AG}\sin(\gamma+\varphi(t))\,\dot{\varphi}(t) \\ \dot{y}_G = \overline{AG}\cos(\gamma+\varphi(t))\,\dot{\varphi}(t) \end{cases}$$

$$\begin{cases} \ddot{x}_G = -\overline{AG}\,\sin(\gamma+\varphi(t))\,\ddot{\varphi}(t) - \overline{AG}\,\cos(\gamma+\varphi(t))\,\dot{\varphi}^2(t) \\ \ddot{y}_G = \overline{AG}\,\cos(\gamma+\varphi(t))\,\ddot{\varphi}(t) - \overline{AG}\,\sin(\gamma+\varphi(t))\,\dot{\varphi}^2(t) \end{cases}$$

Usiamo eq di moto & conservazione dell'energia $\longrightarrow$ reazioni in funzione di φ

$$\frac{1}{2}\left(\frac{5}{3} m l^2 \dot{\varphi}^2\right) + \frac{c}{2} l^2 (3\sin^2\varphi + 1) = E$$

$$\rightarrow \boxed{\dot{\varphi}^2 = \frac{6}{5}\frac{1}{m l^2}\left[E - c\,\frac{l^2}{2}(3\sin^2\varphi+1)\right] =: f(\varphi)}$$

Eq. moto $\quad \frac{5}{3} m l^2 \ddot{\varphi} + 3\frac{c}{2} l^2\, 2\sin\varphi\cos\varphi = 0$

$$\boxed{\ddot{\varphi} = -\frac{9}{5}\frac{c}{m}\sin\varphi\cos\varphi =: g(\varphi)}$$

$$H_A = -m\,\overline{AG}\,\sin(\gamma+\varphi)\,g(\varphi) - \overline{AG}\,m\cos(\gamma+\varphi)\,f(\varphi)$$

$$V_A = 2cl\sin\varphi + cl\cos\varphi + m\,\overline{AG}\left[\cos(\gamma+\varphi)\,g(\varphi) - \sin(\gamma+\varphi)\,f(\varphi)\right]$$

$\rightarrow$ abbiamo H_A, V_A in funzione di φ

Usiamo queste espressioni per trovare un limite per H_A, V_A.

Possiamo trovare max, sin e cos $\in [-1, 1]$

ad esempio $|\ddot{\varphi}| \leq \frac{9}{5} \frac{c}{m}$ - - -

$|\sin\varphi \cos\varphi|$

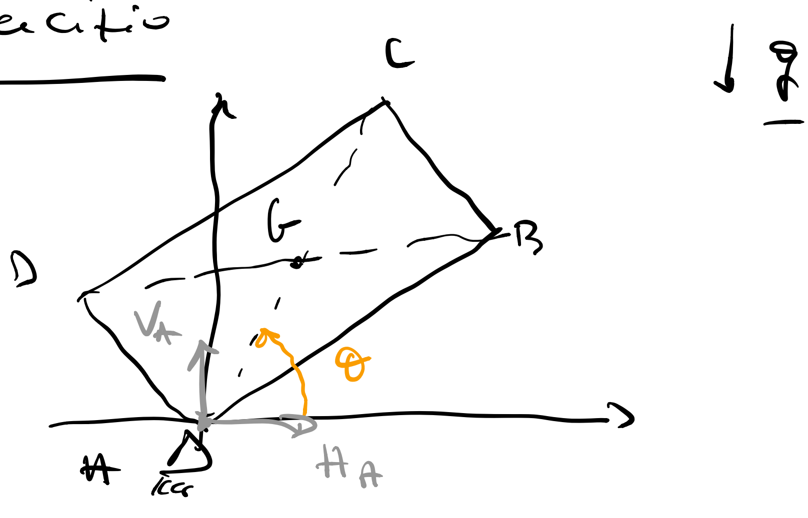
Allo stesso modo $\dot{\varphi}^2 \leq$ valore max

Troveremo

$|H_A| \leq$ espressioni che non dipendono più da φ

$|V_A| \leq$

Esercizio



lamina rettangolare piano verticale

1. eq. di moto
2. reazioni in A per $\boxed{\theta = -\frac{\pi}{2}}$

$\theta(0) = 0$, $\dot{\theta}(0) = 0$

$\left[\theta = \varphi + \ell\right]$

es precedente

Eq. di moto $\mathcal{L} = K - V$

$$K = \frac{1}{2} I_{A,3}\, \dot\theta^2 = \frac{1}{2}\left(\frac{5}{3} m \ell^2\right)\dot\theta^2$$

$$\frac{1}{3} m \,(a^2 + b^2)$$

$$V = mg\, AG \sin\theta \qquad \overline{AG} = \frac{1}{2}\sqrt{4\ell^2 + \ell^2} = \frac{\sqrt{5}\,\ell}{2}$$

Allora $\mathcal{L} = K - V$

$$= \underbrace{\frac{1}{2}\left(\frac{5}{3} m \ell^2\right)\dot\theta^2} - \underbrace{mg \frac{\sqrt{5}}{2}\ell \sin\theta}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = \frac{5}{3} m \ell^2 \ddot\theta + mg\frac{\sqrt{5}}{2}\ell\cos\theta = 0$$

2) Reazioni vincolari in A: E.C.D.

$$\underline{R}^e = H_A\, \underline{e}_1 + V_A\, \underline{e}_2 - mg\, \underline{e}_2 = \underline{\dot P}$$

$$\begin{cases} H_A = m\ddot x_G \\ V_A = m\ddot y_G + mg \end{cases} \qquad \begin{aligned} x_G &= \overline{AG}\cos\theta(t) \\ y_G &= \overline{AG}\sin\theta(t) \end{aligned}$$

$$\ddot x_G = \frac{d}{dt}\left(-\overline{AG}\sin\theta(t)\,\dot\theta(t)\right) =$$

$$= -\overline{AG}\cos\theta(t)\,\dot\theta^2(t) - \overline{AG}\sin\theta(t)\,\ddot\theta(t)$$

$$\ddot{y}_G = \frac{d}{d\tau}\left(\overline{AG} \cos\theta(\tau)\, \dot{\theta}(\tau) \right) =$$

$$= -\overline{AG} \sin\theta(\tau)\, \dot{\theta}(\tau)^2 + \overline{AG} \cos\theta(\tau)\, \ddot{\theta}(\tau)$$

usiamo la conservazione dell' energia

$$E = K + V = \frac{1}{2}\left(\frac{5}{3} m l^2 \dot{\theta}^2\right) + mg\frac{l\sqrt{5}}{2}\sin\theta$$

$$= E\Big|_{\tau=0} = 0 + 0 = 0$$

condizioni iniziali $\theta(0) = 0$, $\dot{\theta}(0) = 0$

$$\hookrightarrow \quad \dot{\theta}^2 = -\frac{g}{l}\frac{3}{\sqrt{5}}\sin\theta$$

Dall' eq. di moto

$$\ddot{\theta} = -\frac{3\sqrt{5}}{10}\frac{g}{l}\cos\theta$$

Andiamo a vedere H_A e V_A per

$$\theta = -\frac{\pi}{2}$$

$$\dot{\theta}^2\Big|_{\theta=-\frac{\pi}{2}} = +\frac{3}{\sqrt{5}}\frac{g}{l}$$

$$\ddot{\theta}\Big|_{\theta=-\frac{\pi}{2}} = 0$$

Per $\theta = -\frac{\pi}{2}$

$$\ddot{x}_G = 0$$

$$\ddot{y}_G = -\left(\frac{\ell}{2}\sqrt{3}\right)(-1)\left(\frac{3}{\sqrt{3}}\frac{g}{\ell}\right)$$

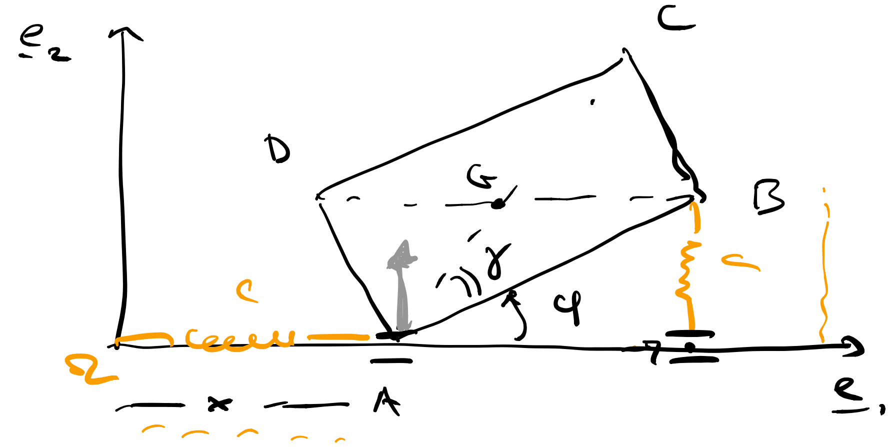
$$= \frac{3}{2}g$$

Per finire

$$\begin{cases} H_A = 0 \\ V_A = \underbrace{mg}_{\text{statica}} + \underbrace{m\frac{3}{2}g}_{\text{dalla dinamica } (\ddot{y}_G)} = \frac{5}{2}mg \end{cases}$$

Seconda parte

Esercizio



lamina rettangolare

molle : c

coordinate libere

(x, φ)

Eq. di moto

$$K = \frac{1}{2} m v_G^2 + \frac{1}{2} I_{G,3}\, \omega^2$$

$$\underline{\omega} = \dot{\varphi}\, \underline{e}_3 \qquad I_{G,3} = \frac{M}{12}\left(\ell^2 + 4\ell^2\right) = \frac{M5}{12}\ell^2$$

$$\frac{M}{12}\left(a^2 + b^2\right)$$

$$\underline{v}_G = \frac{d}{dt}\underline{x}_G$$

$$\underline{x}_G = \left[x(t) + \overline{AG}\cos(\gamma + \varphi)\right]\underline{e}_1 + \overline{AG}\sin(\gamma + \varphi)\,\underline{e}_2$$

dove $\overline{AG} = \ell\frac{\sqrt{5}}{2}$, $\sin\gamma = \frac{1}{\sqrt{5}}$, $\cos\gamma = \frac{2}{\sqrt{5}}$

$$\underline{v}_G = \frac{d}{dt}\underline{x}_G = \left[\dot{x}(t) - \overline{AG}\sin(\gamma + \varphi)\dot{\varphi}\right]\underline{e}_1 + \overline{AG}\cos(\gamma + \varphi)\dot{\varphi}\,\underline{e}_2$$

$$v_G^2 = \underline{v}_G \cdot \underline{v}_G = \left(\dot{x}(t) - \overline{AG}\sin(\gamma + \varphi)\dot{\varphi}\right)^2 + \left(\overline{AG}\cos(\gamma + \varphi)\dot{\varphi}\right)^2$$

$$= \dot{x}(t)^2 + \overline{AG}^2\dot{\varphi}^2\left(\sin^2(\gamma + \varphi) + \cos^2(\gamma + \varphi)\right)$$

$$- \; 2 \; \dot{x}(t) \; \overline{AG} \; \dot{\varphi}(t) \; \sin(\gamma + \varphi(t))$$

Mettiamo tutto insieme

$$K = \frac{1}{2} m v_G^2 + \frac{1}{2} I_{G,3} \omega^2 =$$

$$= \frac{1}{2} m \left(\dot{x}^2 + \underline{\dot{\varphi}^2 \frac{5}{4} \ell^2} - \ell \sqrt{5} \, \dot{x} \dot{\varphi} \cdot \sin(\gamma + \varphi) \right.$$
$$\left. + \underline{\dot{\varphi}^2 \frac{5}{12} \ell^2} \right)$$

$$\frac{5}{4} + \frac{5}{12} = \frac{5}{3}$$

$$V = \frac{c}{2} x^2 + \frac{c}{2} (2\ell \sin\varphi)^2 = \frac{c}{2} x^2 + 2c\ell^2 \sin^2\varphi$$

$$\mathcal{L} = K - V = \frac{1}{2} m \left(\dot{x}^2 + \frac{5}{3} \ell^2 \dot{\varphi}^2 + \right.$$
$$\left. - \ell \sqrt{5} \, \dot{x} \dot{\varphi} \sin(\gamma + \varphi) \right) - \left(\frac{c}{2} x^2 + 2c\ell^2 \sin^2\varphi \right)$$

eq. di moto

$$x) \quad \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} =$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}} \left(\frac{1}{2} m \dot{x}^2 \right) = m\dot{x} \quad \xrightarrow{\frac{d}{dt}} \quad m\ddot{x}$$

$$= \frac{d}{dt} \left[m\dot{x} - \frac{1}{2} m \ell \sqrt{5} \, \dot{\varphi} \sin(\gamma + \varphi) \right] + cx$$

$$= \boxed{m\ddot{x} - \frac{1}{2} m \ell \sqrt{5} \, \ddot{\varphi} \sin(\gamma + \varphi) - \frac{1}{2} m \ell \sqrt{5} \, \dot{\varphi}^2 \cos(\gamma + \varphi) + cx = 0}$$

4) $$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot\varphi} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

$$= \frac{d}{d\tau}\left[\frac{5}{3}\dot\varphi\, m\ell^2 - \frac{1}{2} m\ell\sqrt{5}\,\dot x \sin(\gamma+\varphi)\right]$$

$$- \left[-\frac{1}{2} m\ell\sqrt{5}\,\dot x\dot\varphi\cos(\gamma+\varphi) - 4c\ell^2\cos\varphi\sin\varphi\right]$$

$$= \frac{5}{3}\ddot\varphi\, m\ell^2 - \frac{1}{2} m\ell\sqrt{5}\,\ddot x \sin(\gamma+\varphi) +$$

$$- \cancel{\frac{1}{2} m\ell\sqrt{5}\,\dot x\dot\varphi\cos(\gamma+\varphi)} +$$

$$+ \cancel{\frac{1}{2} m\ell\sqrt{5}\,\dot x\dot\varphi\cos(\gamma+\varphi)} + 4c\ell^2\cos\varphi\sin\varphi$$

$$= \frac{5}{3} m\ell^2\ddot\varphi - \frac{1}{2} m\ell\sqrt{5}\,\ddot x \sin(\gamma+\varphi) + 4c\ell^2\cos\varphi\sin\varphi = 0$$

Calcolare reazioni in A : V_A

eq. cardinali lungo $\underline{e}_2$

$$V_A - c\,y_B = V_A - c\,2\ell\sin\varphi = m\ddot y_G$$

$$\left(\underline{R}^c = \frac{d}{d\tau}\underline{P} \text{ lungo } \underline{e}_2\right)$$

$$V_A = 2\ell c\sin\varphi + m\left[\overline{AG}\cos(\gamma+\varphi)\ddot\varphi - \overline{AG}\sin(\gamma+\varphi)\dot\varphi^2\right)$$

$\ddot{\varphi}$, $\dot{\varphi}^2$ → coordinate libere.

Conservazione energia → non ci trova perché è un'eq sola

$\ddot{\varphi}$, $\ddot{x}$, $\dot{\varphi}$, $\dot{x}$

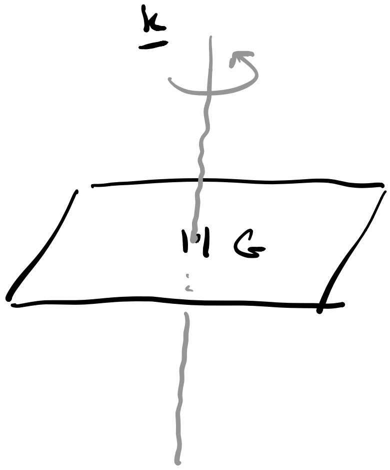
Per trovare v_A in funzione di (x, φ) dobbiamo risolvere le eq. del moto.

Terza parte

Rigido con un asse fisso

Esempio

Lamina rettangolare



$\downarrow \underline{g}$ omogenea

Cerniera cilindrica in G

Vincoli lisci

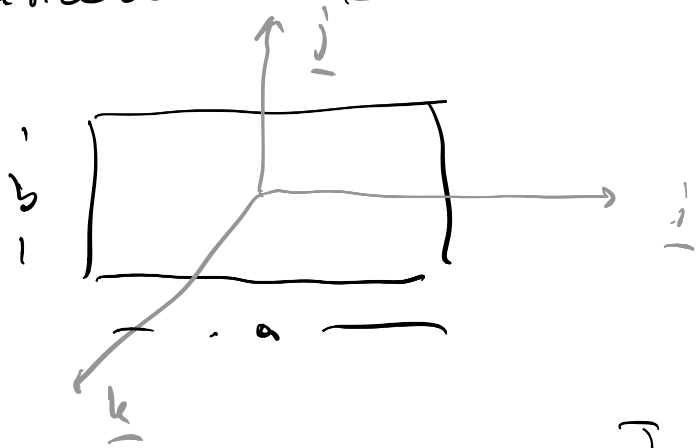
ECD : $\underline{R}^e = M\underline{g} + \underline{F}_0^{\ell} = \dot{\underline{P}} = \underline{0}$

perché G fisso

$$\underline{M}^e(G) = \underline{\mu}_G^e = \frac{d}{dt}\underline{L}(G)$$

La velocità angolare $\underline{\omega} = \omega\,\underline{k}$

Introduciamo la terna



$$\underline{L}(G) = I_G(\underline{\omega}) = \omega J_3\,\underline{k}$$

$$J_3 = \frac{M}{12}(a^2+b^2)$$

$$\frac{d}{dt}\underline{L}(G) = \dot{\omega} J_3\,\underline{k}$$

Ritorniamo E.C.D.

- momenti lungo $\underline{k}$: $J_3\dot{\omega} = 0$

  $\to \omega(t) =$ costante $= \omega_0$ rotazione uniforme

- lungo $\underline{i}, \underline{j}$: eq. dei momenti ci dà $\underline{\mu}_G^e = \underline{0}$

- risultanti: $\underline{F}_0^e = -M\underline{g}$

Nota: se c'è attrito, ad esempio $-\nu \underline{\omega} = -\nu \omega \underline{k}$

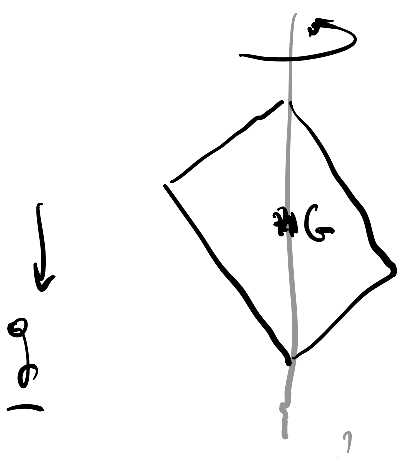
eq. momenti lungo $\underline{k}$

$$J_3 \dot{\omega} = -\nu \omega$$

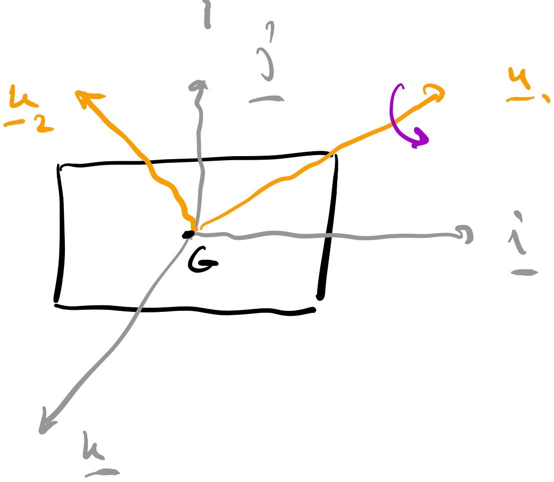
$$\Rightarrow \quad \omega(t) = \omega_0 \exp\left(-\frac{\nu}{J_3} t\right)$$

condizione iniziale

Esempio



l'asse di rotazione è la diagonale



Terna solidale

$$S(G; \underline{u}_1, \underline{u}_2, \underline{k})$$

$$I_G = \begin{pmatrix} I_{11} & I_{12} & 0 \\ I_{12} & I_{22} & 0 \\ 0 & 0 & I_{33} \end{pmatrix}$$

$$\underline{\omega} = \omega \underline{u}_1 \qquad \underline{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{L}(G) = I_G(\underline{\omega}) = \omega\, I_G(\underline{u}_1)$$

$$= \omega\left(I_{11}\, \underline{u}_1 + I_{12}\, \underline{u}_2 \right)$$

Abbiamo visto

$$\frac{d}{dt}\, \underline{L}(G) = I_G(\dot{\underline{\omega}}) + \underline{\omega} \wedge I_G(\underline{\omega})$$

Poisson

$$= \dot{\omega}\, I_G(\underline{u}_1) + \omega^2\, \underline{u}_1 \wedge I_G(\underline{u}_1)$$

$$= \dot{\omega}\, I_{11}\, \underline{u}_1 + \dot{\omega}\, I_{12}\, \underline{u}_2 + \omega^2 I_{12}\, \underline{k}$$

ECD : $\underline{M}^e(G) = \underline{\mu}_G = \dfrac{d}{dt}\, \underline{L}(G)$

$$\begin{cases} I_{11}\, \dot{\omega} = 0 & \text{eq. di moto} \\ \underline{\mu}_G^1 \cdot \underline{u}_2 = I_{12}\, \dot{\omega} & \text{momenti di reazione} \\ \underline{\mu}_G^2 \cdot \underline{k} = I_{12}\, \omega^2 & \end{cases}$$

$I_{12} \neq 0$

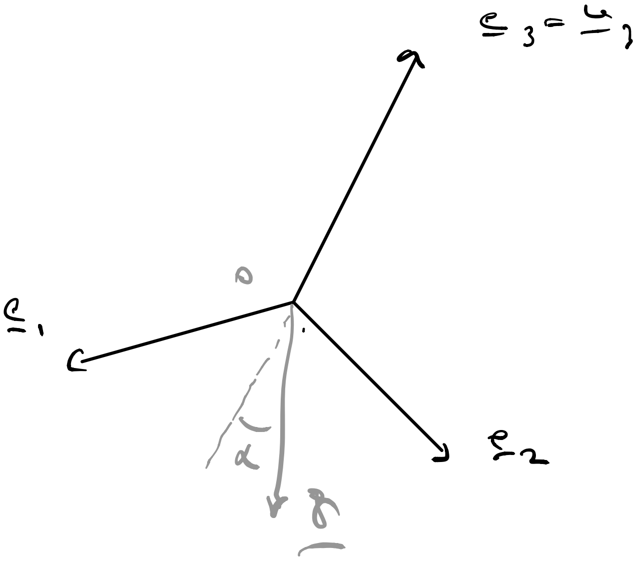
Esempio

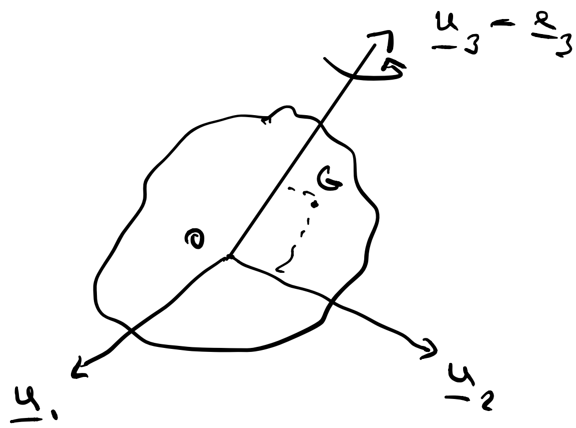
Terna fissa

$\Sigma(O, \underline{e}_1, \underline{e}_2, \underline{e}_3)$

Terna solidale

$S(O; \underline{u}_1, \underline{u}_2, \underline{u}_3)$





$$(\underline{x}_G - \underline{x}_O) = y_G \underline{u}_2 + z_G \underline{u}_3$$

$$\underline{g} = g \sin\alpha \, \underline{e}_2 - g \cos\alpha \, \underline{e}_3$$

ECD

$$\begin{cases} M\underline{g} + \underline{F}^{e} = \dot{\underline{P}} \\ \underbrace{(\underline{x}_G - \underline{x}_O)}_{\underline{u}} \wedge \underbrace{M\underline{g}}_{\underline{e}} + \underline{M}_O^{e} = \dfrac{d}{dt} \underline{L}(O) \end{cases}$$

$$(\underline{g})_S = R^T(\varphi)\, (\underline{g})_\Sigma = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \sin\alpha\, g \\ -\cos\alpha\, g \end{pmatrix}$$

$$= g\left(\sin\alpha \sin\varphi \, \underline{u}_1 + \sin\alpha \cos\varphi \, \underline{u}_2 - \cos\alpha \, \underline{u}_3\right)$$

Allora momento del peso

$$(\underline{x}_G - \underline{x}_0) \wedge M\underline{g} = (y_G \underline{u}_2 + z_G \underline{u}_3) \wedge$$

$$\wedge \left(-\sin\alpha \sin\varphi\, \underline{u}_1 + \sin\alpha \cos\varphi\, \underline{u}_2 - \cos\alpha\, \underline{u}_3\right)$$

$$= Mg\Big(-y_G \sin\alpha \sin\varphi\, \underline{u}_3 - y_G \cos\alpha\, \underline{u}_1$$

$$+ z_G \sin\alpha \sin\varphi\, \underline{u}_2 - z_G \sin\alpha \cos\varphi\, \underline{u}_1\Big)$$

$$\underline{L}(O) = I_O(\omega\, \underline{u}_3) \qquad \underline{\omega} = \omega\, \underline{u}_3$$

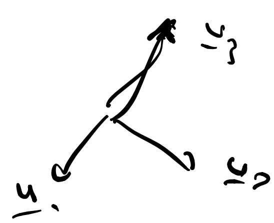
$$\frac{d}{dt}\underline{L}(O) = \dot{\omega}\, I_O(\underline{u}_3) + \omega^2\, \underline{u}_3 \wedge I_O(\underline{u}_3)$$

Prima

$$I_O(\underline{u}_3) = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} I_{13} \\ I_{23} \\ I_{33} \end{pmatrix}$$

$$\underline{u}_3 \wedge I_O(\underline{u}_3) = \underline{u}_3 \wedge \left(I_{13}\underline{u}_1 + I_{23}\underline{u}_2 + I_{33}\underline{u}_3\right)$$

$$= -I_{23}\, \underline{u}_1 + I_{13}\, \underline{u}_2$$



$$\frac{d}{dt}\underline{L}(O) = \underline{u}_1\left(I_{13}\,\dot{\omega} - I_{23}\,\omega^2\right) + \underline{u}_2\left(I_{23}\,\dot{\omega} + I_{13}\,\omega^2\right) + \dot{\omega}\, I_{33}\,\underline{u}_3$$

Mettiamo tutto insieme $\omega = \dot{\varphi}$

$$(\underline{x}_G - \underline{x}_O)\wedge Mg + \mu_O^2 = \frac{d}{dt}\underline{L}(O)$$

$\underline{u}_1$ $\quad \mu_O^2 \cdot \underline{u}_1 = Mg\left(y_G\cos\alpha + z_G\sin\alpha\cos\varphi\right) + I_{13}\,\ddot{\varphi} - I_{23}\,\dot{\varphi}^2$

$\underline{u}_2$: $\quad \mu_O^2 \cdot \underline{u}_2 = -Mg\,z_G\sin\alpha\sin\varphi + I_{23}\,\ddot{\varphi} + I_{13}\,\dot{\varphi}^2$

$\underline{u}_3$: $\quad I_{33}\,\ddot{\varphi} = -\left(Mg\,y_G\sin\alpha\right)\sin\varphi$

risolvere φ dall' eq di moto