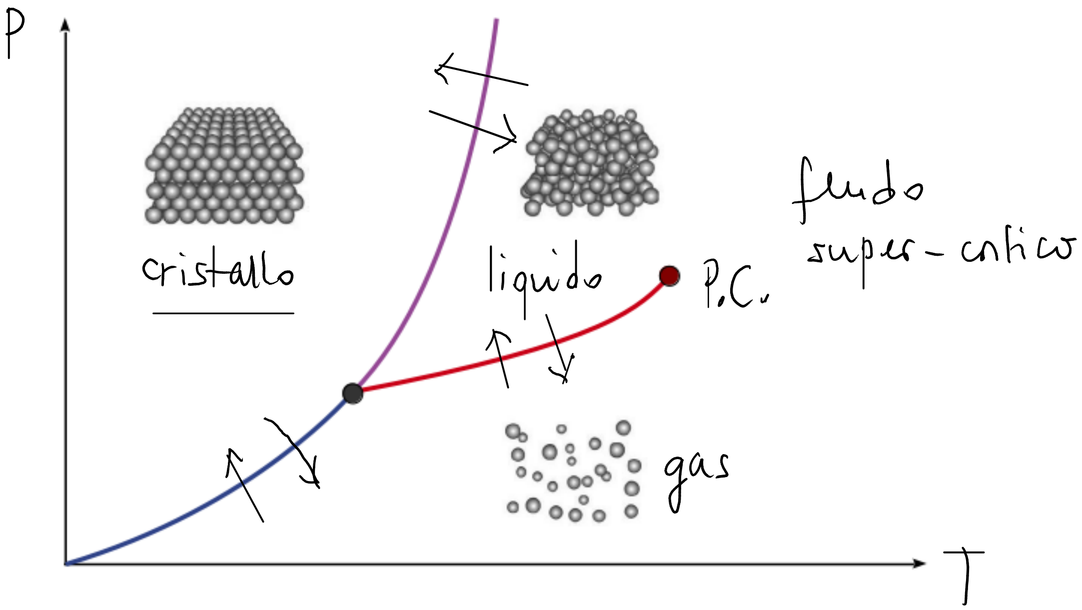
MECCANICA DEI FLUIDI



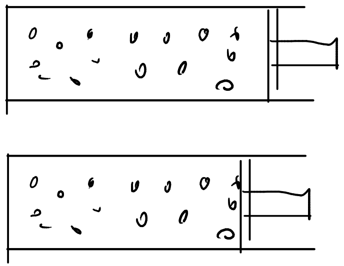
Fluido → gas o liquido

Differenza tra gas e liquido:

- gas poco denso, molto comprimibile
- liquido molto comprimibile, poco comprimibile

↑ densità ρ ↑ comprimibilità

V_i, P_i



P_{est}

V_f, P_f $P'_{est} > P_{est}$

$\Delta P = P_f - P_i$

$\Downarrow$

$\Delta V = V_f - V_i$

$\Delta V \sim \Delta P$ $\Delta V = K \, \Delta P$

$K < 0$

Coefficiente di comprimibilità isoterma $(T = \text{cost})$ $V = V(P, T)$

SI: Pa^{-1}

$\chi_T = 10^{-2} \, Pa^{-1}$

$$\frac{\Delta V}{V} = -\chi_T \, \Delta P \longrightarrow \chi_T \equiv -\frac{1}{V}\frac{dV}{dP} \rightarrow \boxed{\chi_T \equiv -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T}$$

χ_T [Pa^{-1}]

liquidi $10^{-11} - 10^{-10}$

gas 10^{-5}

gas perfetto : $PV = nRT$

$$V = \frac{nRT}{p}$$

$$\rightarrow \chi_T = -\frac{1}{V}\, nRT\, \frac{\partial}{\partial p}\left(\frac{1}{p}\right)$$

$$= -\underbrace{\boxed{\frac{nRT}{V}}}_{p}\left(-\frac{1}{p^2}\right) = \frac{1}{p}$$

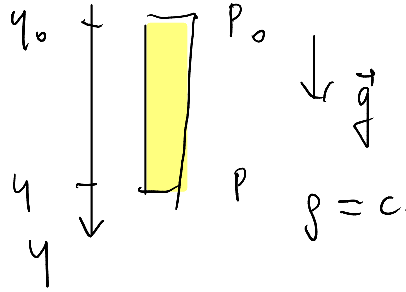
- traslazione $\boxed{V}$
- rotazione $\boxed{X}$
- deformazione $\boxed{V}$

statica → gas, liquido } fluidostatica $\boxed{V}$

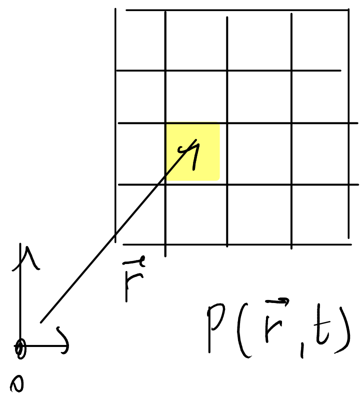
dinamica → gas, liquido → idrodinamica $\boxed{V}$

FLUIDOSTATICA

Legge di Stevino: $P = P_0 + \rho g \underbrace{\Delta y}_{(y-y_0)} \rightarrow \Delta P = \rho g \Delta y$



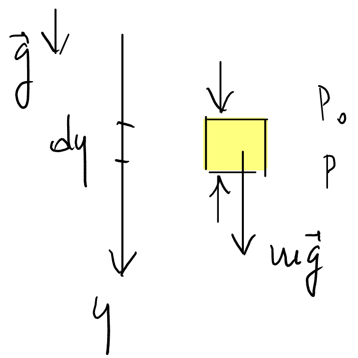
$\rho = cost$



$P(\vec{r}, t) \rightarrow$ campo scalare

$\rho(\vec{r}, t)$

Principio fondamentale della fluidostatica



$\rho(y)$ ben definita nell'elemento di volume $dx\,dy$

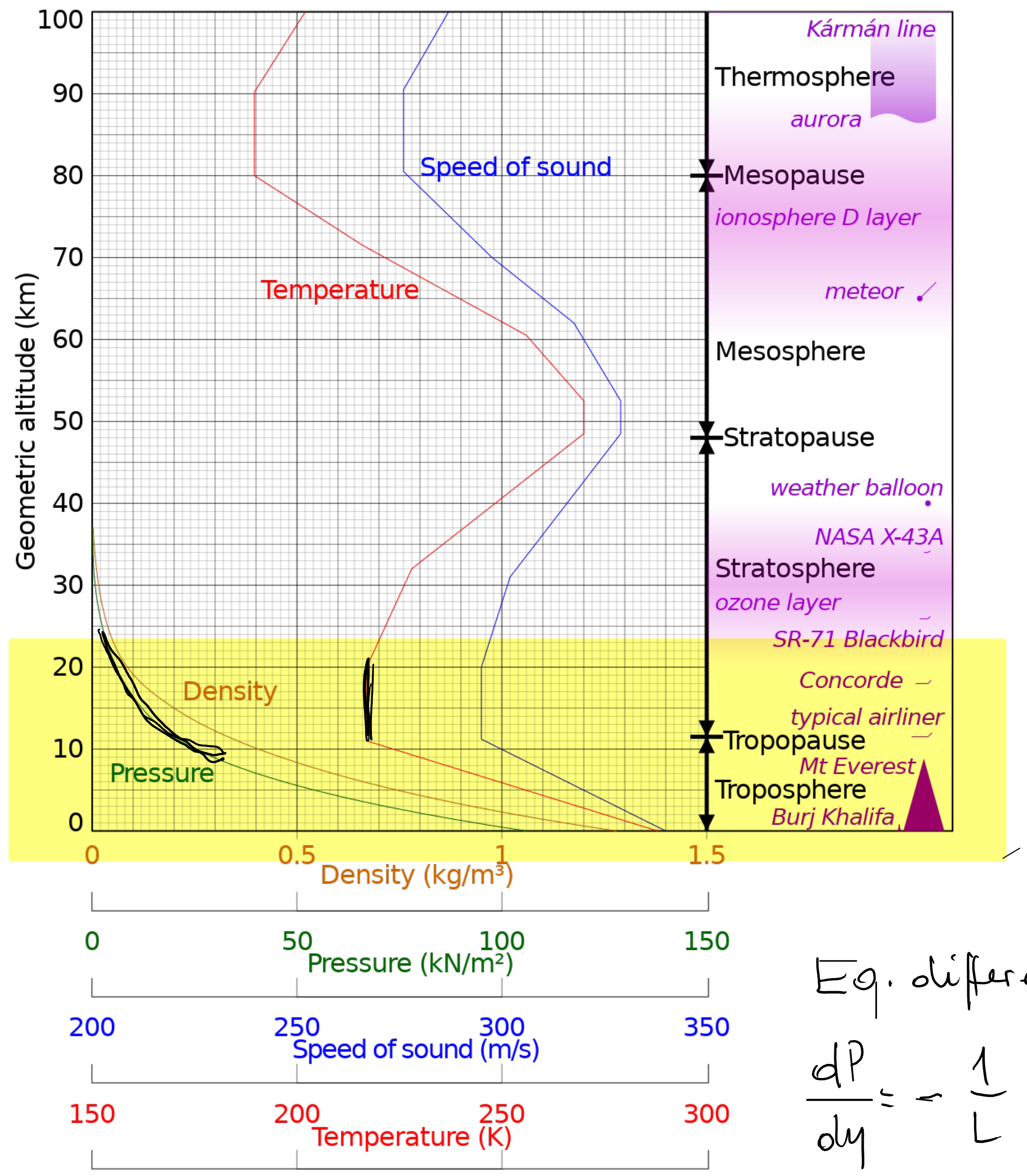
$dP = \rho g \, dy$ ⚠ $\rho = \rho(y)$

$\frac{dP}{dy} = \rho g$ principio fond. fluidostatica

Se oriento asse y verso l'alto

$dP = -\rho g \, dy$

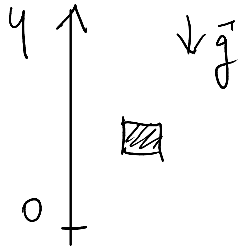
$\vec{g} \Rightarrow \frac{dP}{dx} = \rho g_x ; \frac{dP}{dy} = \rho g_y \rightarrow \vec{\nabla} P = \frac{\partial P}{\partial x} \vec{e}_x + \frac{\partial P}{\partial y} \vec{e}_y = \rho \vec{g}$



Modellizzazione dell' atmosfera terrestre

→ pressione, densità variano vs. altezza

- $T = \text{cost}$ atmosfera isoterma
- gas perfetto = aria, $M_A = 28\ \frac{g}{mol}$
- → fluidostatica | $L = \frac{RT}{M_A g} \approx \frac{\cancel{8}\times 2\cancel{00}}{\cancel{28}\times 10^{-3}\times 1\cancel{0}} \approx 10^4\ m$ $= 10\ km$



$$dP = -\rho g\, dy \qquad \rho = \rho(y)$$

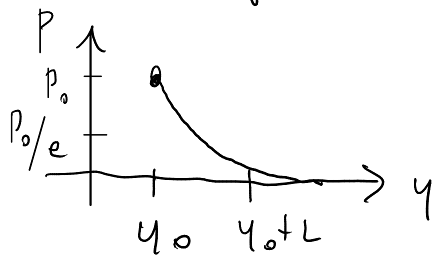
$$\frac{dP}{dy} = -g\boxed{\rho} = -\boxed{\frac{M_A g}{RT}} P = -\frac{1}{L} P$$

$$\rho = \frac{M}{V} \underset{\uparrow\, M_A}{=} \frac{n M_A}{V} \underset{\uparrow\, g.p.}{=} \frac{\cancel{n} M_A}{\cancel{n} RT} \cdot P = \frac{M_A P}{RT}$$

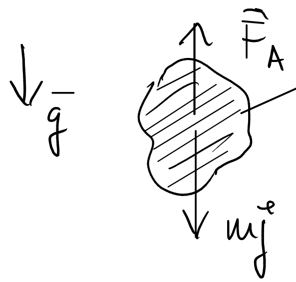
$$L = \frac{RT}{M_A g}$$

Eq. differenziale:

$$\frac{dP}{dy} = -\frac{1}{L} P \Rightarrow P = P_0 \exp\left(-\frac{y-y_0}{L}\right)$$



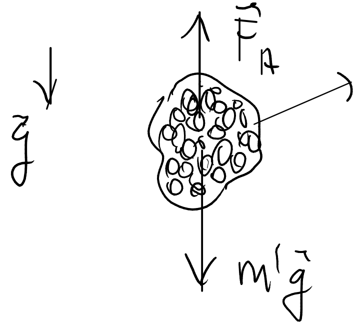
Principio di Archimede



→ elemento fluido : densità fluido ρ

$$\sum \vec{F} = m\vec{g} + \vec{F}_A = \vec{0}$$

↑ forze pressione fluido circostante



→ corpo densità ρ'

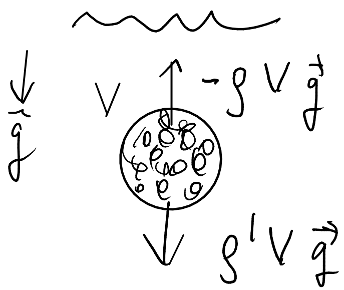
$$\boxed{\vec{F}_A = -\underbrace{\rho V}_{m}\,\vec{g}}$$

Principio Archimede: corpo immerso in un fluido subisce una forza verso l'alto pari in modulo al peso del volume di fluido spostato

→ forza uguale in modulo e direzione al peso del volume del fluido spostato e opposto in verso

Casi particolari:

1) corpo completamente immerso : ρ' densità corpo, ρ del fluido



$$\sum \vec{F} = (\rho' - \rho) V \vec{g}$$

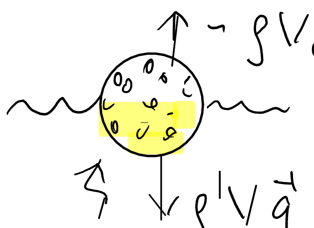
$\rho' > \rho$: $\sum \vec{F} \sim \vec{g}$ verso basso

$\rho' < \rho$: $\sum \vec{F} \sim -\vec{g}$ verso alto

$\rho' = \rho$: $\sum \vec{F} = \vec{0}$ equilibrio

↑ indifferente

2) corpo parzialmente immerso : $\rho' < \rho \Rightarrow V_0 \equiv$ volume immerso



$$\sum \vec{F} = (\rho' V - \rho V_0)\vec{g} = \vec{0} \Rightarrow \rho' V = \rho V_0 \Rightarrow \frac{V_0}{V} = \frac{\rho'}{\rho} < 1$$

galleggio $\Rightarrow$ equilibrio

frazione volume immerso