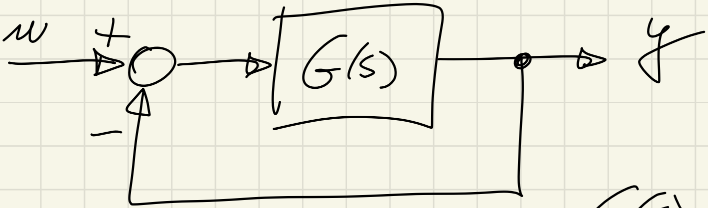
Fondamenti di Automatica

Esercizi

23 aprile 2021



$$G(s) = k \, \frac{s+4}{s(s+3)}$$

$$k \in \mathbb{R}, \quad k > 0$$

- k: sistema as. stabile
- k: (as. stabile) sistema a ciclo chiuso deve avere risposta a $u(t) = 1(t)$ con oscillazioni smorzate

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$\boxed{\text{as. stab}}$

$$F(s) = \frac{G(s)}{1+G(s)} = \frac{N(s)}{N(s)+D(s)}$$

$$D_k(s) = s^2 + (3+k)s + 4k$$

$$\begin{array}{c|cc} 2 & +1 & +4k \\ 1 & (3+k) & \\ 0 & +4k & \end{array}$$

$$\begin{cases} 3+k > 0 \\ k > 0 \end{cases} \quad \begin{cases} k > -3 \\ k > 0 \end{cases}$$

$k > 0$

$k > 0 \longrightarrow$ ris. stabile

$$F(s) = \frac{k(s+4)}{s^2+(3+k)s+4k} = \frac{\cancel{4k}(1+s/4)}{\cancel{4k}\left[1+\frac{3+k}{4k}s+\frac{s^2}{4k}\right]}$$

FA Posto? #72

$$\frac{\mu(1+sT)}{\underbrace{1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}}}$$

$F(0)=1$ $\quad z_1 \rightarrow T_1 = \frac{1}{4}$

abbia radici complesse

$k: \Delta < 0 \longrightarrow \bar{K}$

$k \in \bar{K}$ poli complessi coniugati

$$\omega_n = \sqrt{4k} = 2\sqrt{k} \qquad \xi = \frac{\sqrt{k}}{4}\left(1+\frac{3}{k}\right)$$

$$\exists\, k: \quad 0 < \xi < 1$$

$k>0$ $\quad 0 < \frac{\sqrt{k}}{4}\left(1+\frac{3}{k}\right) < 1$

$$\sqrt{k}\,(k+3) < 4k \qquad /^{2}$$

$$k^2 - 10k + 9 < 0$$

$$1 < k < 9 \Longleftrightarrow \bar{K}$$

$k=1$? $\quad k=9$?

$0<k<1$? $\quad k>9$

$1<k<9$ ris. stab. oscill. smorzata

$\exists k$; $\max \Delta_s$?

$$\max_{k} \Delta_s = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}$$

$$\Downarrow$$

$$\min_{k} \left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right) \longleftarrow \xi = \frac{\sqrt{k}}{4}\left(1+\frac{3}{k}\right)$$

$$\min_{k} \pi^2 \left(\frac{k^2+6k+9}{-k^2+10k-9} \right)$$

$$\frac{d(\,)}{dk} = 0 \qquad 8k - 24 = 0$$

$$k = 3$$

$k > 0$ $\quad \boxed{1 < k < 9}$

$\exists\, k:$ $\quad t_a \simeq 1s$

poli c.c. $\quad t_{a_{1\%}} \simeq \dfrac{5}{\xi \omega_n} = \dfrac{5}{|\sigma|}$

$$D_F(s) \Rightarrow s^2 + (3+k)s + 4k = 0 \qquad p_{1,2} = -\sigma \pm j\omega$$

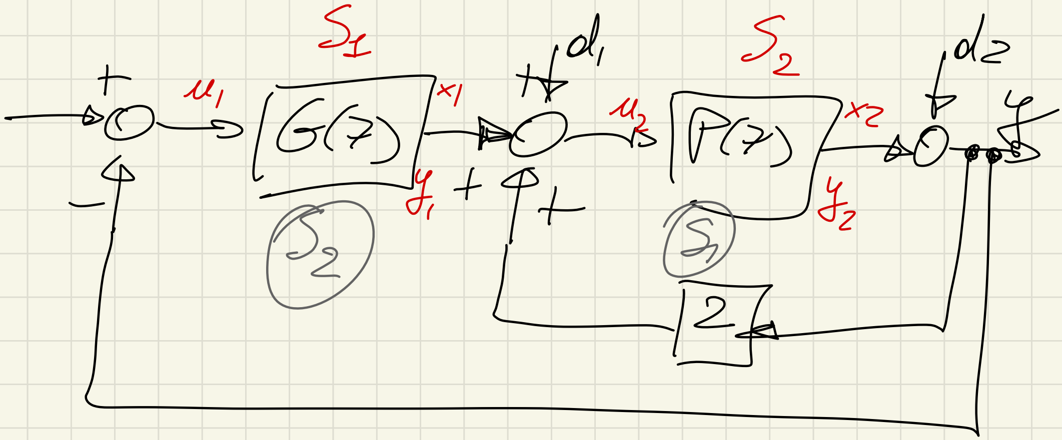
$$(3+k) = -2(-\sigma) = 2\sigma$$

$$t_a \simeq 1 = \frac{5}{5}$$

$$3+k = 10$$

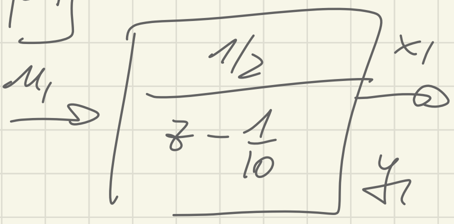
$$k = 7$$

$k = 7 \rightarrow p_{1,2} = -5 \pm j\sqrt{3}$



$$G(z) = \frac{1/2}{z - 1/10} \qquad P(z) = \frac{z + 2/5}{z + 1/2}$$

$\boxed{S_1}$



$$\frac{cb}{z - a} \quad \begin{cases} x(k+1) = a\,x(k) + b\,u(k) \\ y(k) = c\,x(k) \end{cases}$$

$a = \frac{1}{10}$

b

$cb = \frac{1}{2} \quad \begin{cases} b = 1 \\ c = \frac{1}{2} \end{cases} \quad \begin{cases} b = 1/2 \\ c = 1 \end{cases}$

$$\begin{cases} x_1(k+1) = \frac{1}{10} x_1(k) + \frac{1}{2} u_1(k) \\ y_1(k) = x_1(k) \end{cases}$$

S_2: $u_2 \longrightarrow \boxed{\dfrac{z + 2/5}{z + 1/2}} \longrightarrow y_2$ (x_2)

$$T(z) = d + \frac{cb}{z - a} = 1 - \frac{1/10}{z + \frac{1}{2}}$$

$$\begin{array}{l|l} z + \frac{2}{5} & z + \frac{1}{2} \\ \hline -z - \frac{1}{2} & 1 \\ \hline -\frac{1}{10} & \end{array}$$

$$\boxed{\begin{array}{ll} d = 1 & \\ a = -\frac{1}{2} & cb = -\frac{1}{10} \end{array}}$$

$$\begin{cases} x_2(k+1) = -\frac{1}{2} x_2(k) + \frac{1}{10} u_2(k) \\ y_2(k) = x_2(k) + u_2(k) \end{cases}$$

$$x(k+1) = A\,x(k) + B\,[u(k)\ \ d_1(k)\ \ d_2(k)]^T$$

$$y(k) = C\,x(k) + D\,[u(k)\ \ d_1(k)\ \ d_2(k)]^T$$

$$A = \begin{bmatrix} \frac{3}{5} & -\frac{1}{2} \\ -\frac{1}{10} & -\frac{3}{10} \end{bmatrix} \qquad B = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 0 & -\frac{1}{10} & -\frac{1}{5} \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & +1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & -1 & -1 \end{bmatrix}$$

$$\boxed{S_1 \leftrightarrow S_2}$$

$$\hat{x}(k+1) = \hat{A}\,x(k) + \hat{B}\begin{bmatrix} u \\ d_1 \\ d_2 \end{bmatrix}$$

$$y(k) = \hat{C}\,\hat{x}(k) + \hat{D}\begin{bmatrix} u \\ d_1 \\ d_2 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} -\frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{2} & \frac{3}{5} \end{bmatrix}$$

$$\mathrm{eig}(A) = \mathrm{eig}(\hat{A})$$

$$\exists T \quad \hat{A} = TAT^{-1}$$