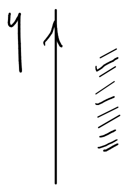
Modello atmosfera isoterma

$$\frac{dP}{dy} = -\rho g = -\frac{P M_A}{RT} g = -\frac{M_A g}{RT} P \Rightarrow$$



$\rho = \rho(y)$

$PV = nRT$

separazione delle variabili

$$\frac{dP}{P} = -\frac{M_A g}{RT} dz$$

$$\int_{P_i}^{P_f} \frac{dP}{P} = -\frac{M_A g}{RT} \int_{z_i}^{z_f} dz \rightarrow \int_{P_0}^{P} \frac{dP'}{P'} = ---$$

$$\ln \frac{P_f}{P_i} = -\frac{M_A g}{RT}(z_f - z_i)$$

$$\frac{P_f}{P_i} = \exp\left[-\frac{M_A g}{RT}(z_f - z_i)\right]$$

$$P_f = P_i \exp\left[-\frac{z_f - z_i}{L}\right]$$

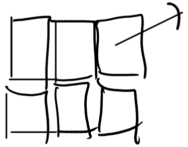
$$P = P_0 \exp\left[-\frac{z - z_0}{L}\right]$$

DINAMICA DEI FLUIDI

→ fluidodinamica: fluidi comprimibili HARD

→ idrodinamica: liquidi ad alta velocità

a bassa velocità ⇝ OK



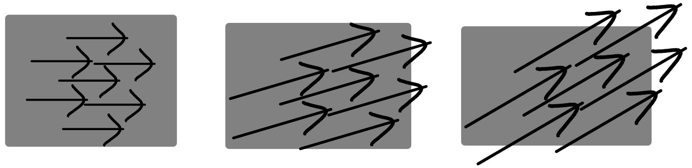
elemento di fluido

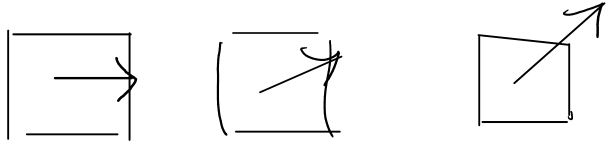
3. + 4. $\boxed{V}$

1. liquido incomprimibile: $\rho = \text{cost}$ → si deforma ma volume cost

2. liquido non viscoso: trascuro attrito tra elementi di fluido vicini

3. 4. $\boxed{X}$





$\vec{v}$

$\vec{v}(\vec{r}, t)$

1. + 2. liquido ideale

corrente = flusso

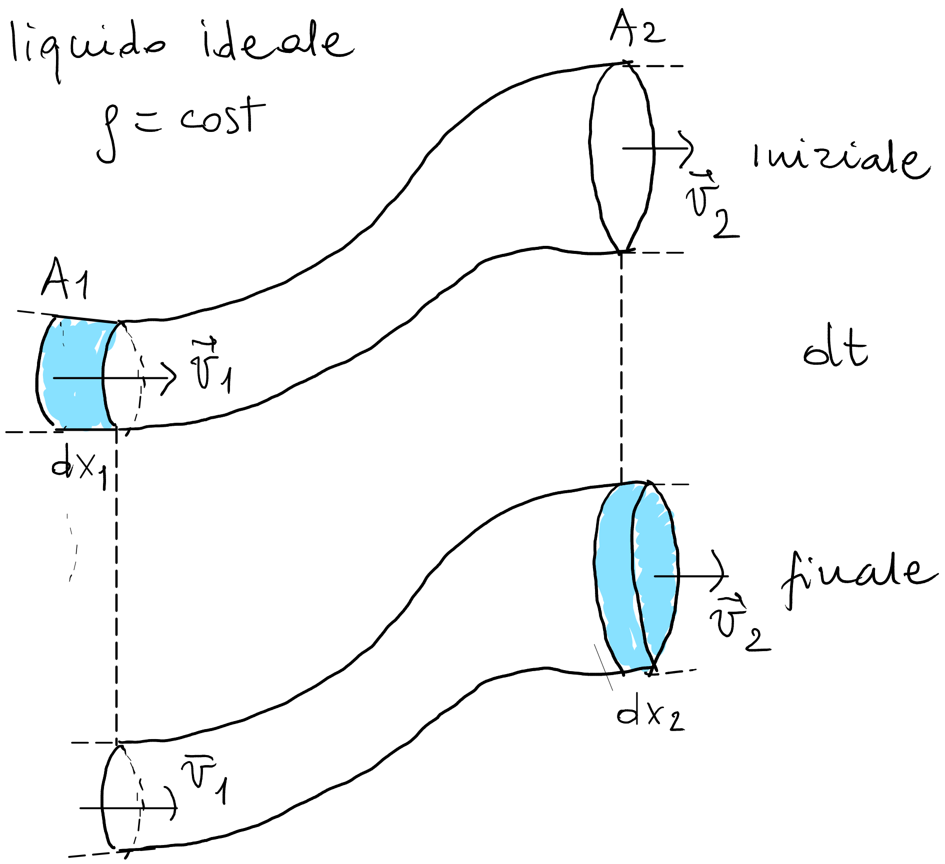
3. corrente stazionaria $\vec{v}(\vec{r})$ indip. da t

4. corrente irrotazionale → no rotazione

Equazione di continuità per fluidi incomprimibili

liquido ideale

$\rho = \text{cost}$



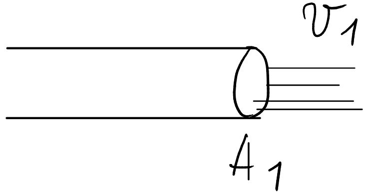
$$\rho A_1 dx_1 = \rho A_2 dx_2$$

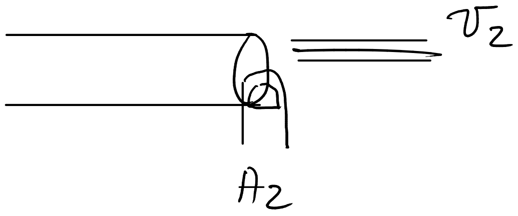
$$\cancel{\rho} A_1 |\vec{v}_1| d\cancel{t} = \cancel{\rho} A_2 |\vec{v}_2| d\cancel{t}$$

$$\Rightarrow A_1 |\vec{v}_1| = A_2 |\vec{v}_2|$$

$$\boxed{A_1 v_1 = A_2 v_2}$$ eq. continuità

$[A v] = \frac{L^3}{T}$ SI: $\frac{m^3}{s}$ **portata** o **corrente di volume**



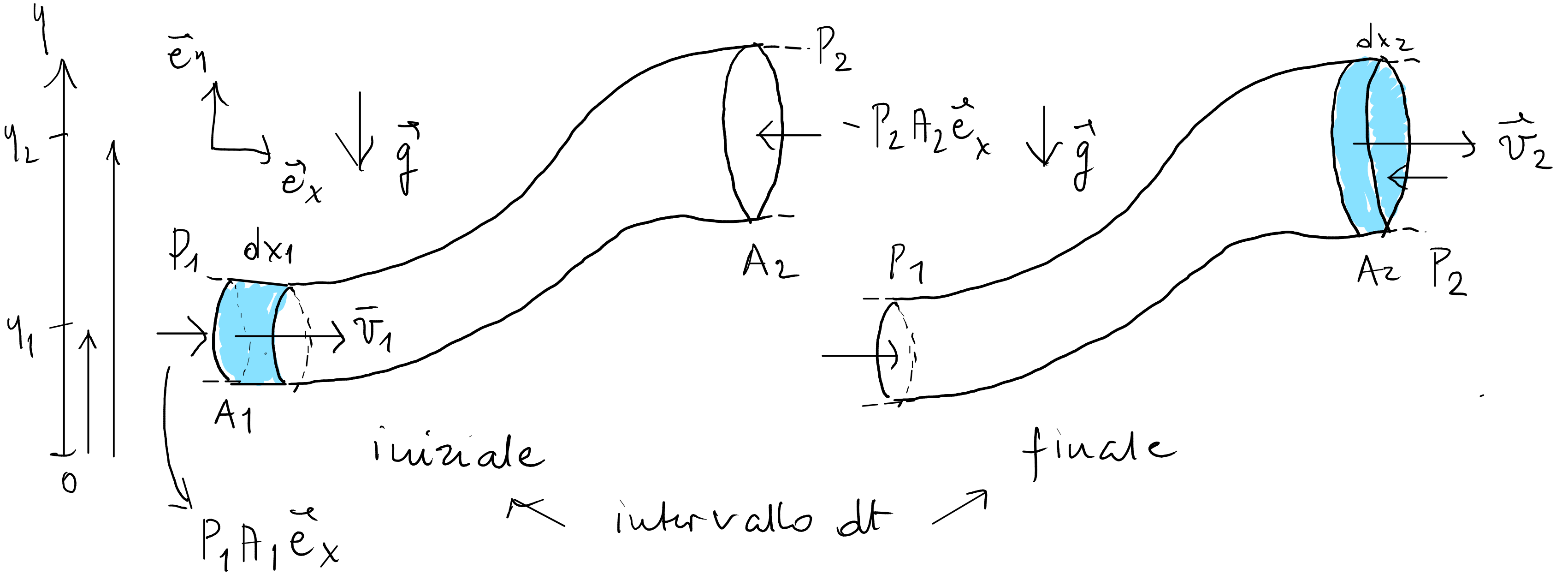


$$v_2 = \frac{A_1}{A_2} v_1$$

$A_2 = \frac{1}{2} A_1 \Rightarrow v_2 = 2 v_1$

Teorema di Bernoulli:

Liquido ideale → forze esterne: pressione liquido, gravità



Bernoulli 1738

Teor. energia cinetica: $\Delta E_c = W(\Sigma \vec{F}) = W(\Sigma \vec{F}_{est})$

③ ① p $\vec{g}$ ②

① $W_1 = P_1 A_1 \vec{e}_x \cdot \vec{v}_1 dt = P_1 A_1 dx_1 = P_1 V_1$

$W_2 = -P_2 A_2 \vec{e}_x \cdot \vec{v}_2 dt = -P_2 A_2 dx_2 = -P_2 V_2$ $\Rightarrow$ $W_1 + W_2 = (P_1 - P_2) V$

② $W[\vec{g}] = -\Delta E_p$

$$\Delta E_p = (mgy_2 + E_p^0) - (mgy_1 + E_p^0) = mg(y_2 - y_1)$$

③ $\Delta E_c = \left(\frac{1}{2}mv_2^2 + E_c^0\right) - \left(\frac{1}{2}mv_1^2 + E_c^0\right) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$$\Delta E_c + \Delta E_p = W_1 + W_2$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1 = P_1V - P_2V \qquad /V \qquad \frac{m}{V} = \rho$$

$$\frac{1}{2}\rho v_2^2 + \rho g y_2 + P_2 = \frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 \Rightarrow \boxed{\frac{1}{2}\rho v^2 + \rho g y + P = \text{cost}}$$

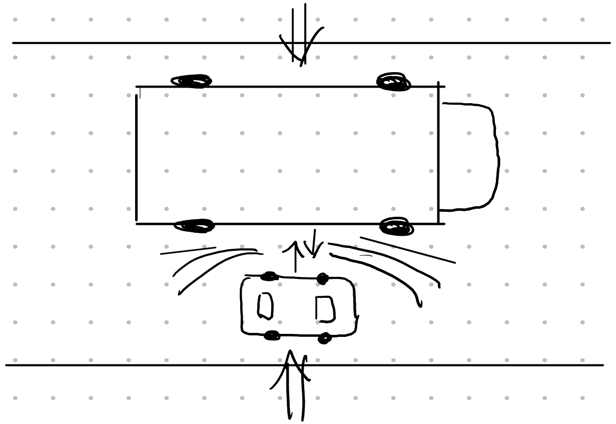
<u>Casi particolari:</u>

1) $v_1 = v_2 = 0 \Rightarrow P_2 + \rho g y_2 = P_1 + \rho g y_1 \Rightarrow P_2 = P_1 + \rho g \Delta y$

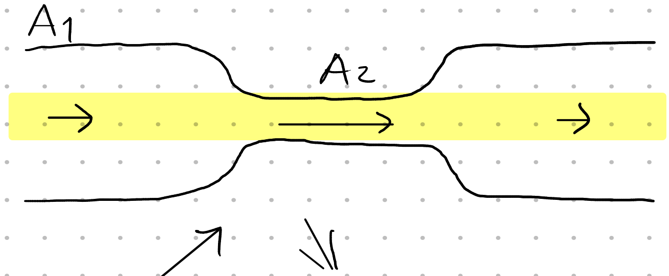
liquido statico

$$P = P_0 + \rho g \Delta y \quad \text{Stevino}$$

2) $y_1 = y_2 = \text{cost}$



effetto Venturi

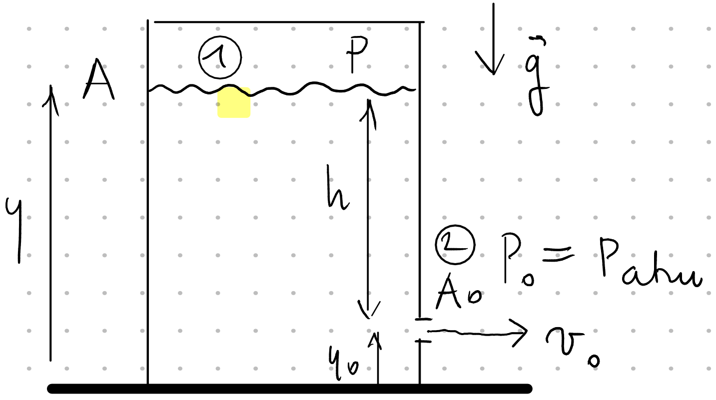


$$\frac{1}{2}\rho v^2 + P = cost$$

eq. continuità: $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1}{A_2} v_1$$

Esempio: $\rho = cost$ $\quad h = y - y_0 \quad \Rightarrow \quad v_0 = ?$



$\sim$ estintore

Bernoulli:

$$\frac{1}{2}\rho v^2 + \rho g y + P = \frac{1}{2}\rho v_0^2 + \rho g y_0 + P_0$$

1 $\qquad\qquad$ 2

$A \gg A_0 \Rightarrow v \approx 0$

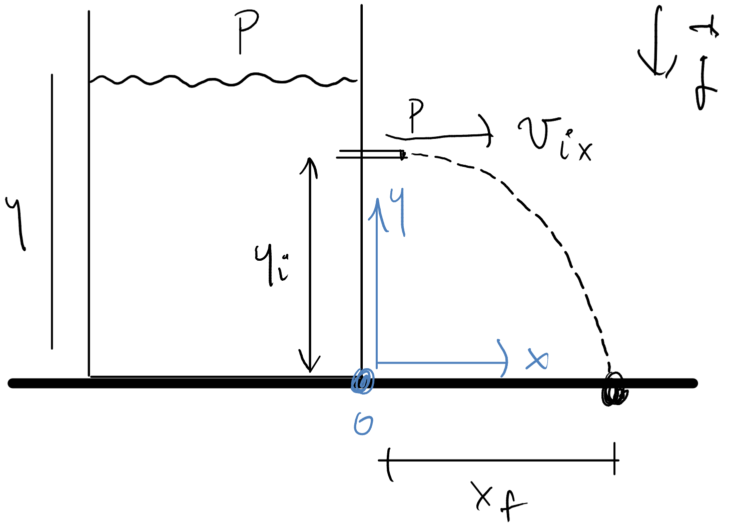
$$\rho g y + P = \frac{1}{2}\rho v_0^2 + \rho g y_0 + P_0$$

$$\frac{1}{2}\rho v_0^2 = (P - P_0) + \rho g (y - y_0) = (P - P_0) + \rho g h$$

$$v_0^2 = 2\,\frac{P-P_0}{\rho} + 2gh \Rightarrow \boxed{v_0 = \sqrt{2\,\frac{P-P_0}{\rho} + 2gh}}$$

- $P \gg P_0 \Rightarrow v_0 \approx \sqrt{2\,\frac{P-P_0}{2}}$
- contenitore aperto $\Rightarrow P = P_0 \Rightarrow v_0 = \sqrt{2gh} \rightarrow$ caduta libera! legge di Torricelli

Variante : contenitore aperto, $\rho = \text{cost} \Rightarrow y_i$ tale che gittata x_f è massima?



$$v_{ix} = \sqrt{2gy_i}$$

$$\begin{cases} y = y_i + v_{iy}t - \frac{1}{2}gt^2 = y_i - \frac{1}{2}gt^2 \\ x = x_i + v_{ix}t = \sqrt{2gh}\cdot t = \sqrt{2g(y-y_i)} \end{cases}$$

$$0 = y_i - \frac{1}{2}gt_f^2 \Rightarrow t_f = \sqrt{2gy_i}$$

$$x_f = \sqrt{2g(y-y_i)}\sqrt{2gy_i} = 2g\sqrt{(y-y_i)y_i}$$

$$= 2g\sqrt{yy_i - y_i^2} \rightarrow \text{gittata}$$