Cyber-Physical Systems

Laura Nenzi

Università degli Studi di Trieste Il Semestre 2020

Lecture 15: STL applications

[Many Slides due to J. Deshmukh, S. Silvetti]

Falsification re-framed

Given:

- Set of all such input signals : U
- ▶ Input signal $\mathbf{u} : \mathbb{T} \to D_1 \times \cdots \times D_m$, where $\mathbb{T} \subseteq [0, T], D_i \subset \mathbb{R}$ compact set
- Model *M* s.t. $M(\mathbf{u}) = \mathbf{y}, \quad \mathbf{y}: \mathbb{T} \to \mathbb{R}^n$ *M* maps **u** to some signal **y** with the same domain as **u**, and co-domain some subset of \mathbb{R}^n
- Property φ that can be evaluated to true/false over given **u** and **y**

Check: $\exists \mathbf{u} \in U : (\mathbf{y} = M(\mathbf{u})) \vDash \neg \varphi(\mathbf{u}, \mathbf{y})$

Input/Output Properties for Closed-loop Models

- Properties/Specifications/Requirements are rarely monolithic formulas $\varphi(\mathbf{u}, \mathbf{y})$
- Function Typically specified as a pair: a pre-condition φ_I on the inputs, and a post-condition φ_O on the outputs
- Verification problem then stated as:

Prove that: $\forall \mathbf{u} \in U$: $(\mathbf{u} \models \varphi_I) \land (\mathbf{y} = M(\mathbf{u})) \Rightarrow (\mathbf{y} \models \varphi_O)$

Testing problem stated as:

Find *u* such that $(\mathbf{u} \models \varphi_I) \land (\mathbf{y} = M(\mathbf{u})) \land (\mathbf{y} \not\models \varphi_O)$

Input Properties/Pre-conditions

- Common practice in control theory to excite closed-loop models with input signals of certain special shapes
- Motivation comes from theory of linear systems, where a step-response or impulse-response are enough to characterize all behaviors of the system
- Such special shapes do not provide comprehensive information for nonlinear closed-loop systems, yet, it is still common to excite these systems with a few common patterns
- Frequently, input signal patterns come from engineering insights or application-specific domain expertise

Common input patterns used for testing



Testing in practice

- Each time-point in a signal is an independent dimension, i.e. the signal can change arbitrarily at each time-point in the signal
- Number of independent domains is infinite (e.g. consider a signal defined over rational time-points)
- Fypical testing approach is to find a *test-suite*: This is a **finite** number of test input signals (satisfying φ_I) and then obtain output behaviors using these signals as test inputs.
- If each corresponding output signal satisfies the output property φ₀, then testing concludes, indicating that the model is correct for the given test-suite (i.e. no output in the test-suite satisfies φ₀).

Signal Generation

- Find a signal generator for the property φ_I
 - Function that uses random-ness to generate an input signal that satisfies φ_I (hopefully, an input signal different from previously generated ones!)
- Signal generation usually relies on defining a *finite parameterization* for the input signal
 - ▶ For the chosen class of signals, find parameters that define the shape
 - Define acceptable ranges for the parameters
 - Define a generation function that takes the *parameter values* as inputs and generates an input signal

Finite Parameterization



Finite parameterization using control points



Finite parameterization using control points



Finite parameterization using linear interpolation



Finite parameterization using interpolation



Piecewise cubic interpolation

 $\lambda = [20, 40, 10, 40, 10]$ t = [0, 5, 10, 15, 20]

Finite parameterization variable control point times



Finite parameterization variable control point times



Signal Generator



- Signal Generation controlled by the testing algorithm
 - Parameter space could be sampled all at once
 - Parameter space could be sampled in a sequential fashion, e.g. using a method such as Markov Chain Monte Carlo
 - Sampling scheme could be application-specific: uniform random, quasi-random (more evenly spread out), truncated normal, grid-based sampling (points from a fixed grid), etc.

Black-box Optimization



- Given:
 - Function $M: U \rightarrow Y$ with unknown symbolic representation
 - Ability to query the value of M at any given u; query will return some y
 - Cost function $C: X \times Y \to \mathbb{R}$
- Objective of black-box optimizer
 - Let $x^* = \min_{x \in \mathbf{X}} C(x, f(x))$
 - Find \hat{x} such that $\|\hat{x} x^*\|$ is small
- Let $\hat{x_i}$ be the best answer found by optimizer in its i^{th} iteration

Falsification using Optimization



Step-by-step of how falsification works

- Given: a finite parameterization for input signals, a model that can be simulated and an STL property
- While the number of allowed iterations is not exhausted do:
 - pick values for the signal parameters
 - generate an input signal
 - run simulation with generated input signal to get output signal
 - compute robustness value of given property w.r.t. the input/output signals
 - if robustness value is negative, HALT
 - pick a new set of values for the signal parameters based on certain heuristics

Picking new parameter values to explore

- Pick random sampling as a (not very good) strategy!
- Basic method: locally approximate the gradient of the function ρ locally, and chose the direction of steepest descent (greedy heuristic to take you quickly close to a local optimum)
- Challenge 1: cost surface may not be convex, thus you could have many local optima
- Challenge 2: cost surface may be highly nonlinear and even discontinuous, using just gradient-based methods may not work well

Heuristics rely on:

- combining gradient-based methods with perturbing the search strategy (e.g. simulated annealing, stochastic local search with random restarts)
- evolutionary strategies: Covariance Matrix Adaptation Evolution Strategy (CMA-ES), genetic algorithms etc.
- probabilistic techniques: Ant Colony Optimization, Cross-Entropy optimization, Bayesian optimization

Model



Model



Black Box Assumption



- Less information
- A more general Approach (interesting for industries)

Falsification of CPS



Goal:

Find the inputs (1) which falsify the requirements (4)

Problems:

- Falsify with a low number of simulations
- Functional Input Space



Active Learning Adaptive Parameterization

Gaussian Processes

Definition

$$f \sim GP(m,k) \iff (f(t_1), f(t_2), \dots, f(t_n)) \sim N(m,K)$$

where $m = (m(t_1), m(t_2), \dots, m(t_n))$ is the vector mean

 $K \in \mathbb{R}^{n \times n}$ is the covariance matrix, such that $K_{ij} = k(f(t_i), f(t_j))$



Gaussian Processes



Domain Estimation Problem

Finding the trajectories which falsify the requirements, finding $u \in B$

 $\mathsf{B}=\{\boldsymbol{u}\in\mathsf{U}\mid\rho(\phi,\boldsymbol{u},0)<0\}\subseteq U$

> Training Set: $K = \{u_i, \rho(\phi, u_i, 0)\}_{i \le n}$ (the partial knowledge after n iterations)

→ Gaussian Process: $\rho_K(u) \sim GP(m_K(u), \sigma_K(u))$ (the partial model)

$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$

Idea: implementing an iterative sample strategy in order to increase the probability to sample a point in B, as the number of iterations increases.

Algorithm 1

1: procedure [B, d] = DOMAINESTIMATION (maxIter, ce, m, f, I) $i \leftarrow 0, \ B \leftarrow \emptyset, \ d \leftarrow +\infty$ 2: 3: INITIALIZE(K(f))4: while $(|B| \leq ce \text{ and } i \leq maxIter)$ do 5: $f_{K(f)} \sim \text{TRAINGAUSSIANPROCESS}(K(f))$ 6: $D_{arid} \leftarrow \text{LHS}(m)$ 7: $x_{new} \leftarrow \text{SAMPLE}\{(x, P(x \in \mathcal{B})), x \in D_{arid}\}$ 8: $f_{new} \leftarrow f(x_{new})$ $d \leftarrow \min(d, \text{DISTANCE}(f_{new}, I))$ 9: $K(f) \leftarrow K(f) \cup \{(x_{new}, f_{new})\}$ 10: if $f_{new} \in I$ then 11: $B = B \cup \{x_{new}\}$ 12:13:end if 14: $i \leftarrow i+1$ 15:end while 16: end procedure











$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$





$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$



$$u_2$$

$$P(\rho_K(\boldsymbol{u}) < 0) = CDF(\frac{0 - m_K(\boldsymbol{u})}{\sigma_K(\boldsymbol{u})})$$

Domain Estimation Problem

Finding the trajectories which falsify the requirements, finding $u \in B$

$$\mathsf{B} = \{ \boldsymbol{u} \in \mathsf{U} \mid \rho(\phi, \boldsymbol{u}, 0) < 0 \} \subseteq U$$

We call B the counterexample set and its elements counterexamples

If B is empty then $\rho(\phi, \boldsymbol{u}, 0) \geq 0$

Solving the domain estimation problem could be extremely difficult because of the infinite dimensionality of the input space, which is a space of functions

Finite Parameterization



Domain Estimation Problem

Finding the trajectories which falsify the requirements, finding $\hat{c} \in \hat{B}$

$$\hat{B} = \{ \hat{c} \in U_{n_1} \times \cdots \times U_{n_{|U|}} \mid \rho(\phi, P_n(\hat{c}), 0)) < 0 \}$$

Where
$$c_k = \{(t_1^k, u_{n_k}^k), \dots, (t_{n_k}^k, u_{k_n})\}$$
 and $P_n = (P_{n_1}, \dots, P_{n|U|})$

Piecewise linear or polynomial functions are known to be dense in the space of continuous functions!

Then, B has at least one element $\Leftarrow \exists n \in \omega^{|U|}$, \hat{B} has at least one element.



Adaptive Parameterization



46

Adaptive Parameterization

Algorithm 2



Tests Case & Results

- $\phi_1(\bar{v},\bar{\omega}) = \mathbf{G}_{[0,30]}(v \leq \bar{v} \wedge \omega \leq \bar{\omega})$ (in the next 30 seconds the engine and vehicle speed never reach $\bar{\omega}$ rpm and \bar{v} km/h, respectively)
- φ₂(v̄, ω̄) = G_[0,30](ω ≤ ω̄) → G_[0,10](v ≤ v̄) (if the engine speed is always less than ω̄ rpm, then the vehicle speed can not exceed v̄ km/h in less than 10 sec)
- φ₃(v̄, ω̄) = F_[0,10](v ≥ v̄) → G_[0,30](ω ≤ ω̄) (the vehicle speed is above v̄ km/h than from that point on the engine speed is always less than ω̄ rpm)



	Adaptive DEA		Adaptive GP-UCB		S-TaLiRo		
Req	nval	times	nval	times	nval	times	Alg
ϕ_1	4.42 ± 0.53	2.16 ± 0.61	4.16 ± 2.40	0.55 ± 0.30	5.16 ± 4.32	0.57 ± 0.48	UR
ϕ_1	6.90 ± 2.22	5.78 ± 3.88	8.7 ± 1.78	1.52 ± 0.40	39.64 ± 44.49	4.46 ± 4.99	SA
ϕ_2	3.24 ± 1.98	1.57 ± 1.91	7.94 ± 3.90	1.55 ± 1.23	12.78 ± 11.27	1.46 ± 1.28	CE
ϕ_2	10.14 ± 2.95	12.39 ± 6.96	23.9 ± 7.39	9.86 ± 4.54	59 ± 42	6.83 ± 4.93	SA
ϕ_2	8.52 ± 2.90	9.13 ± 5.90	13.6 ± 3.48	4.12 ± 1.67	43.1 ± 39.23	4.89 ± 4.43	SA
ϕ_3	5.02 ± 0.97	2.91 ± 1.20	5.44 ± 3.14	0.91 ± 0.67	10.04 ± 7.30	1.15 ± 0.84	CE
ϕ_3	7.70 ± 2.36	7.07 ± 3.87	10.52 ± 1.76	2.43 ± 0.92	11 ± 9.10	1.25 ± 1.03	UR

```
(atomicExpression)
             ! Formula
2
             Formula & Formula
3
             Formula | Formula
4
             Formula => Formula
5
             Formula until [a b] Formula
6
            Formula since [a b] Formula
7
            eventually [a b] Formula
8
             globally [a b] Formula
9
             once [a b] Formula
10
             historically [a b] Formula
11
             escape(distanceExpression)[a b] Formula
12
             Formula reach (distanceExpression)[a b] Formula
13
             somewhere(distanceExpression) [a b] Formula
14
             everywhere (distanceExpression) [a b] Formula
15
             {Formula}
16
```

Model



https://it.mathworks.com/help/simulink/slref/modeling-an-automatic-transmission-controller.html

	.
Specification	Natural Language
Safety $(\Box_{[0,\theta]}\phi)$	ϕ should always hold from time 0 to θ .
Liveness $(\diamond_{[0,\theta]}\phi)$	ϕ should hold at some point from 0 to θ (or now).
	ϕ_1 through ϕ_n should hold at some point in the future (or now), not necessarily in order or at the same time.
Stabilization $(\Diamond \Box \phi)$	At some point in the future (or now), ϕ should always hold.
Recurrence $(\Box \diamondsuit \phi)$	At every point in time, ϕ should hold at some point in the future (or now).
Reactive Response $(\Box(\phi \rightarrow \psi))$	At every point in time, if ϕ holds then ψ should hold.

Automatic Transmission						
	Natural Language	MTL				
ϕ_1^{AT}	The engine speed never reaches $\bar{\omega}$.	$\Box(\omega < \bar{\omega})$				
ϕ_2^{AT}	The engine and the vehicle speed never reach $\bar{\omega}$ and \bar{v} , resp.	$\Box((\omega < \bar{\omega}) \land (v < \bar{v}))$				
ϕ_3^{AT}	There should be no transition from gear two to gear one and back to gear two in less than 2.5 sec.	$\Box((g_2 \wedge Xg_1) \to \Box_{(0,2.5]} \neg g_2)$				
ϕ_4^{AT}	After shifting into gear one, there should be no shift from gear one to any other gear within 2.5 sec.	$\Box((\neg g_1 \wedge Xg_1) \to \Box_{(0,2.5]}g_1)$				
ϕ_5^{AT}	When shifting into any gear, there should be no shift from that gear to any other gear within 2.5sec.	$\wedge_{i=1}^{4} \Box((\neg g_i \land Xg_i) \to \Box_{(0,2.5]}g_i)$				
ϕ_6^{AT}	If engine speed is always less than $\bar{\omega}$, then vehicle speed can not exceed \bar{v} in less than T sec.	$\neg(\diamondsuit_{[0,T]}(v > \bar{v}) \land \Box(\omega < \bar{\omega}))$				
ϕ_7^{AT}	Within T sec the vehicle speed is above \bar{v} and from that point on the engine speed is always less than $\bar{\omega}$.	$\diamondsuit_{[0,T]}((v \ge \bar{v}) \land \Box(\omega < \bar{\omega}))$				
ϕ_8^{AT}	A gear increase from first to fourth in under 10secs, ending in an RPM above $\bar{\omega}$ within 2 seconds of that, should result in a vehicle speed above \bar{v} .	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				

Bibliography

Falsification:

- Silvetti S., Policriti A., Bortolussi L. (2017) An Active Learning Approach to the Falsification of Black Box Cyber-Physical Systems. IFM 2017. LNCS, vol 10510. Springer, Cham.
- Several excellent papers on the first development of falsification technology can be found on the web-site of S-TaLiRo : <u>https://sites.google.com/a/asu.edu/s-taliro/references</u>
- Jyotirmoy Deshmukh, Marko Horvat, Xiaoqing Jin, Rupak Majumdar, and Vinayak S. Prabhu. 2017. Testing Cyber-Physical Systems through Bayesian Optimization. ACM Trans. Embed. Comput. Syst. 16, 5s, Article 170 (September 2017)
- Deshmukh, Jyotirmoy, Xiaoqing Jin, James Kapinski, and Oded Maler. Stochastic Local Search for Falsification of Hybrid Systems. In International Symposium on Automated Technology for Verification and Analysis, pp. 500-517.