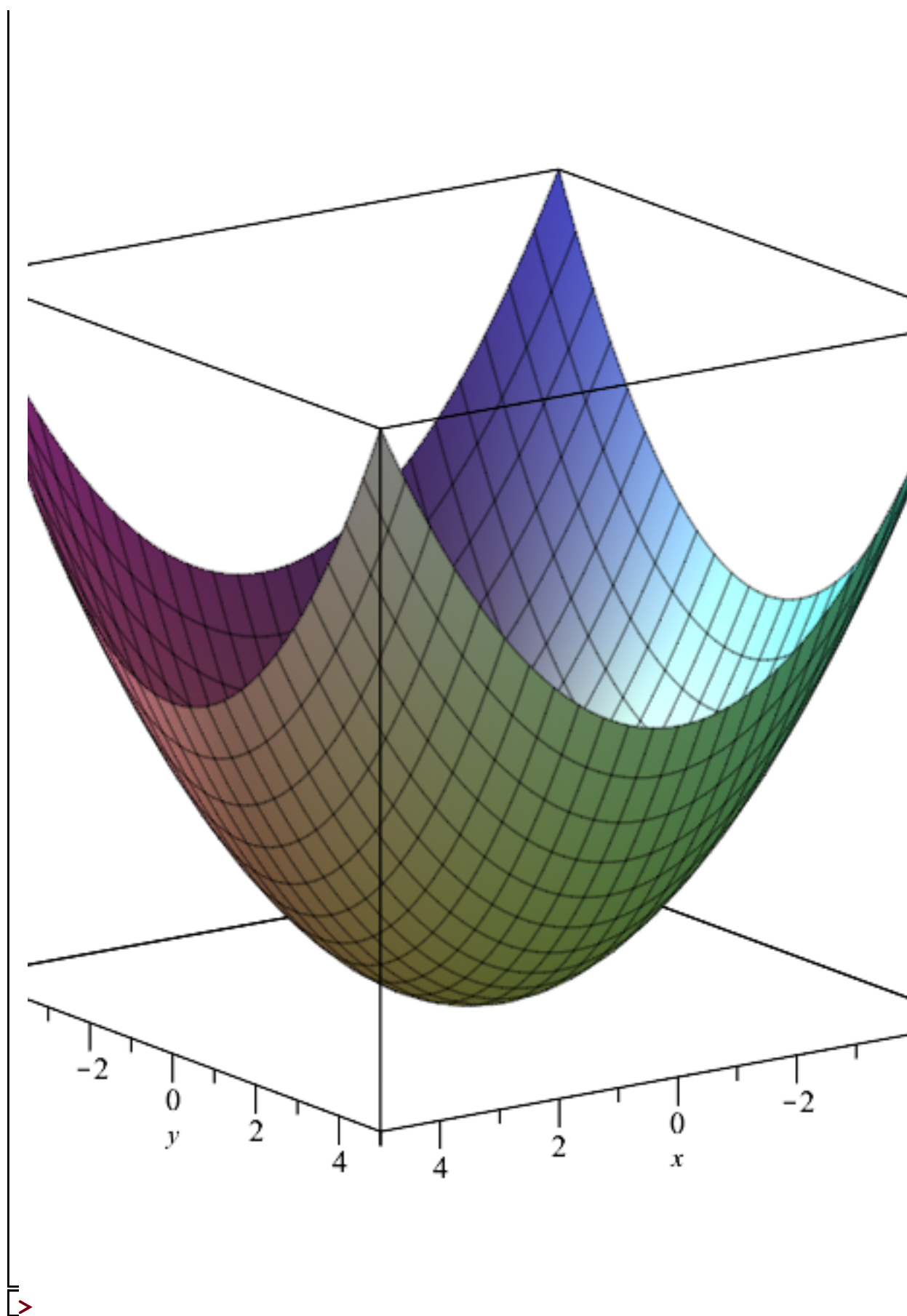


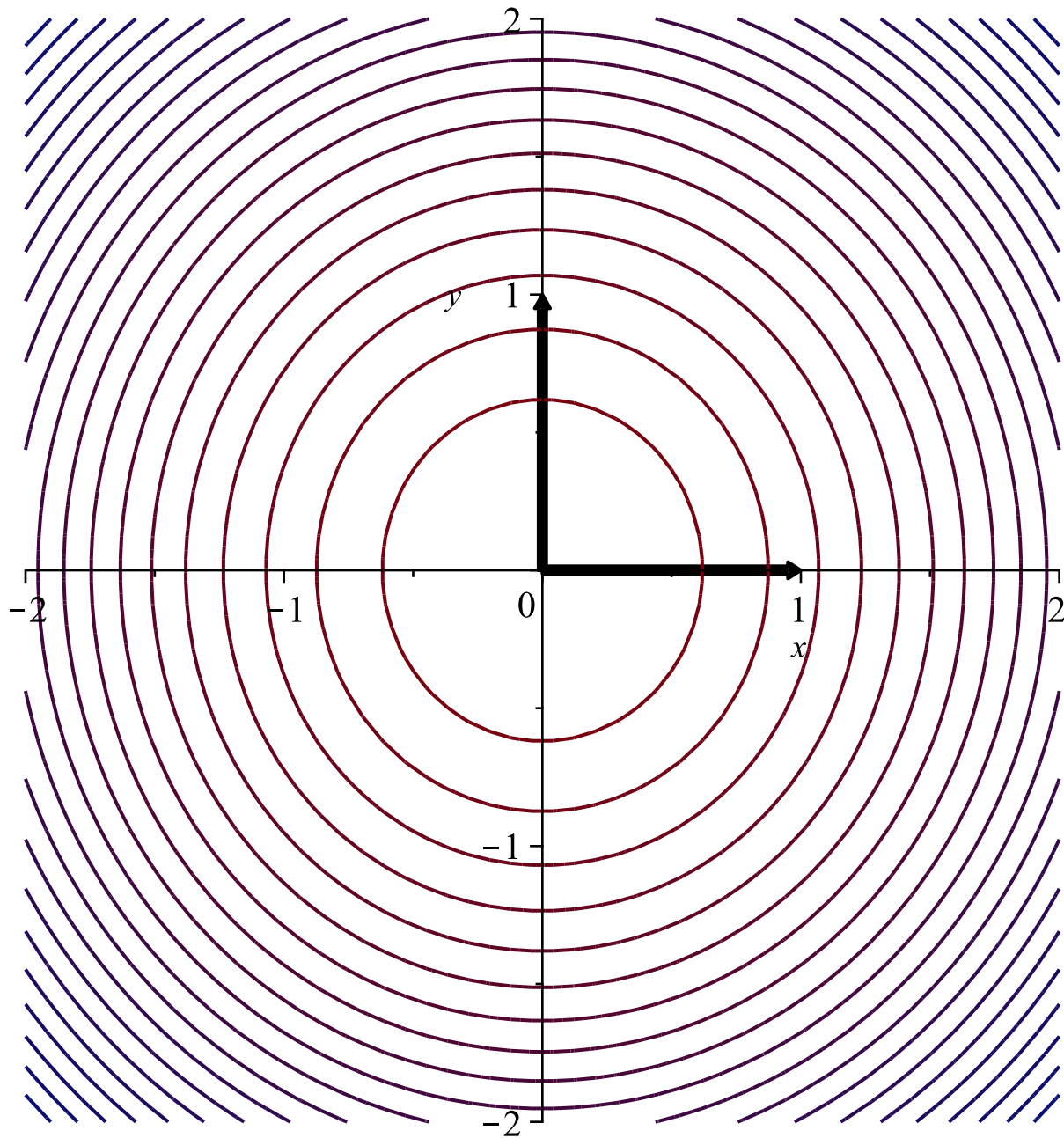
```

[>
[>
[>
[> #EX_Prot-UNCONSTRAINED OPTIMIZATION
[>
[> #Function
> f := x^2 + y^2;
                                     f := x^2 + y^2 (1)
[> #Gradient
> dfx := diff(f, x);
                                     dfx := 2 x (2)
[> dfy := diff(f, y);
                                     dfy := 2 y (3)
[> #Critical Points
> CP := solve( {dfx=0, dfy=0}, {x, y}, allsolutions, explicit);
                                     CP := {x=0, y=0} (4)
[>
[>
[> with(VectorCalculus) :
> Hf := Hessian(f, [x, y]);
                                     Hf :=  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$  (5)
[>
[> with(LinearAlgebra) :
> detHf := Determinant(Hf);
                                     detHf := 4 (6)
[> trHf := Trace(Hf);
                                     trHf := 4 (7)
[> eigf := Eigenvalues(Hf);
                                     eigf :=  $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$  (8)
[> eigvf := Eigenvectors(Hf);
                                     eigvf :=  $\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  (9)
[>
[>
[>
[>
[> with(plots) :
[> plot3d(f, x=-5..5, y=-5..5);

```



```
> p := contourplot(f, x=-2..2, y=-2..2, contours=20) : q := arrow([1, 0], [0, 1], width
= [0.03, relative], head_length=[0.05, relative], color=black) : display({p, q});
```



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```
#Function
```

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```
#Gradient
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>
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>
```

```
=
```

```
g := x2 - y2;
```

$$g := x^2 - y^2$$

(10)

```
dgx := diff(g, x);
```

$$dgx := 2x$$

(11)

```
dgy := diff(g, y);
```

$$dgy := -2y$$

(12)

```

> #Critical Points
> CP := solve( {dfg=0, dgy=0}, {x,y}, allsolutions, explicit);
CP := ( )

```

(13)

```

>
>
> with(VectorCalculus) :
> Hg := Hessian(g, [x,y]);

```

$$Hg := \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(14)

```

> with(LinearAlgebra) :
> detHg := Determinant(Hg);

```

$$\det Hg := -4$$

(15)

```

> trHg := Trace(Hg);

```

$$\text{tr} Hg := 0$$

(16)

```

> eigv := Eigenvalues(Hg);

```

$$\text{eigv} := \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

(17)

```

> eigvg := Eigenvectors(Hg);

```

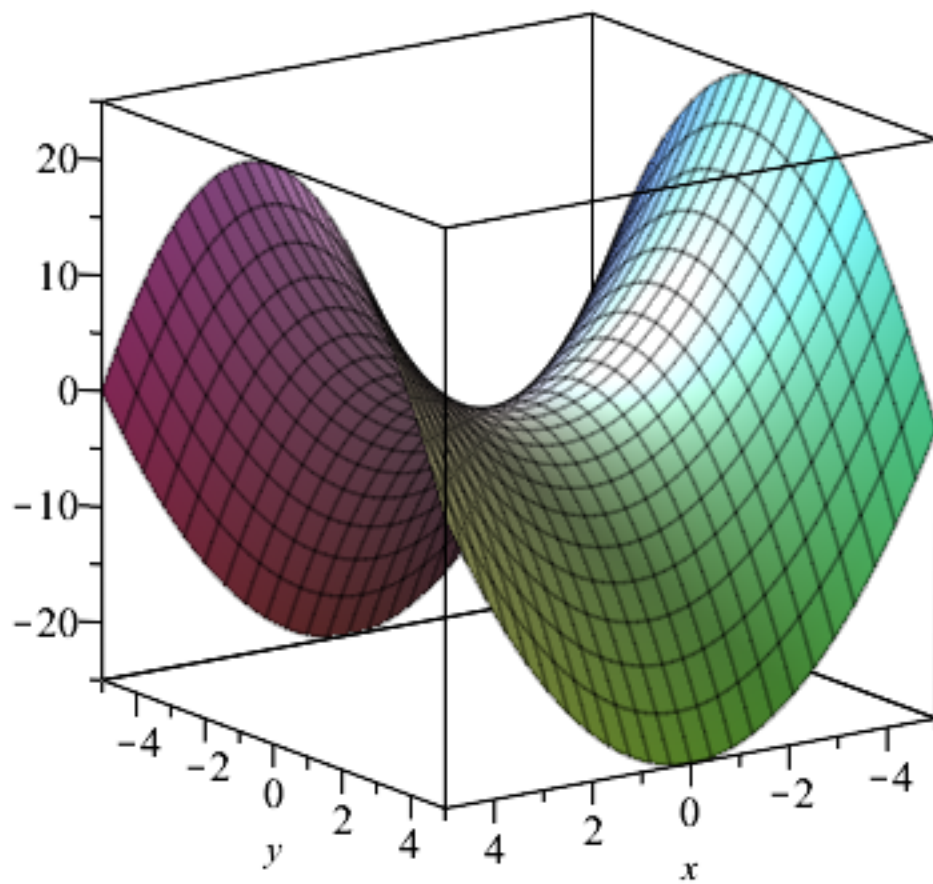
$$\text{eigvg} := \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(18)

```

> plot3d(g, x=-5..5, y=-5..5);

```



> $p := \text{contourplot}(g, x=-2..2, y=-2..2, \text{contours}=20) : q := \text{arrow}(\{[1, 0], [0, 1]\}, \text{width} = [0.03, \text{relative}], \text{head_length} = [0.05, \text{relative}], \text{color} = \text{black}) : \text{display}(\{p, q\});$

