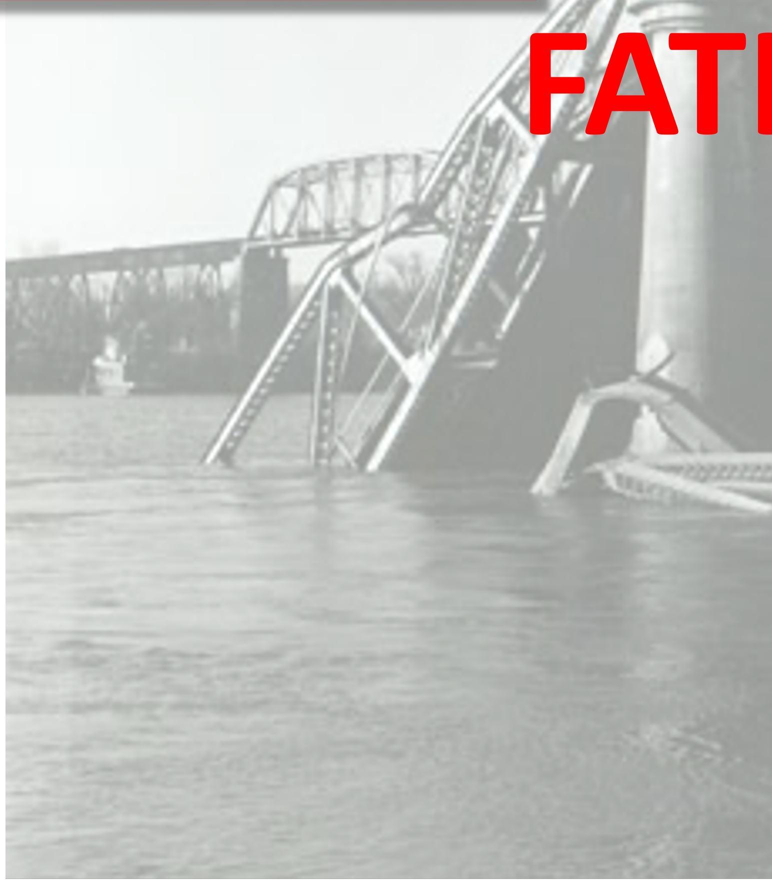
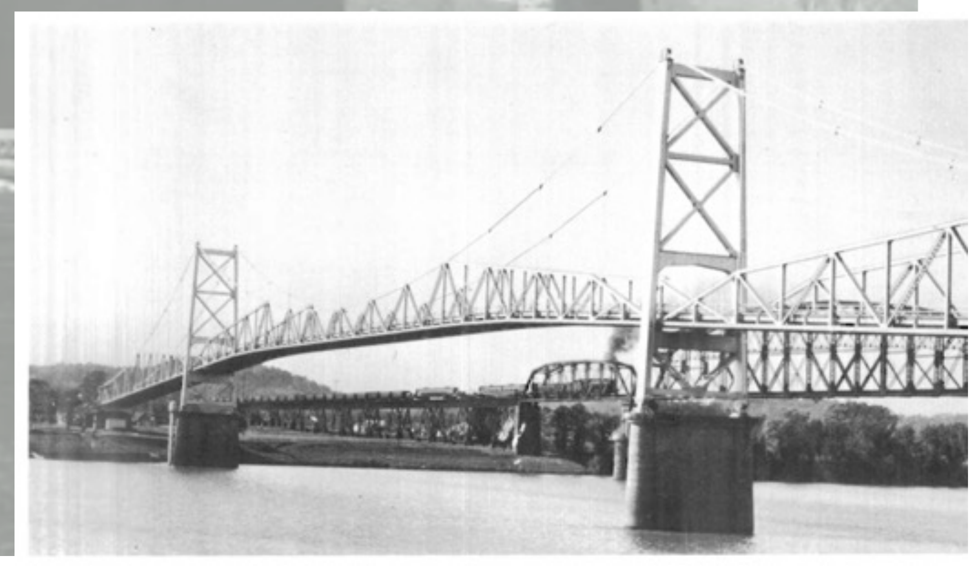
FATIGUE





Limiti di sensibilità (approssimati) di tecniche NDT

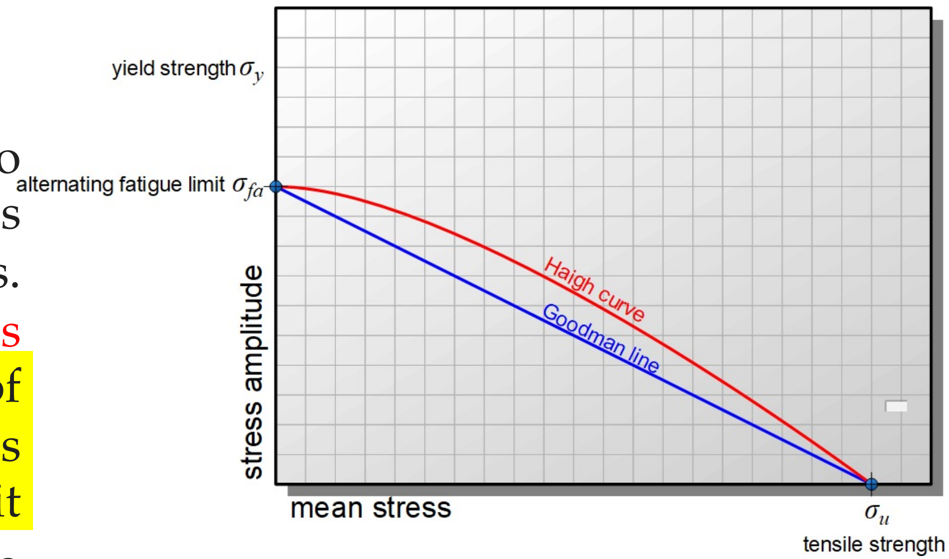
Method		Location	Dimension	Size (in.)
Eddy Current	Manual	Near Surface	Length	0.030-0.040
	Semi-Automated	Near Surface	Length	0.020-0.030
	Automated	Near Surface	Length	0.005-0.010
Ultrasonic	Manual	Subsurface	FBH*	0.032-0.064
	Automated	Subsurface	FBH*	0.016-0.032
Fluorpenetrant	Manual	Surface	Length	0.075-0.100
	Automated	Surface	Length	0.060-0.075
Magnetic Particle	Manual	Near Surface	Length	0.010-0.020

*FBH – capability based on flat bottom holes

Fatigue limit diagrams

Although the Wöhler curve is suitable for the evaluation of fatigue tests, it is not informative for the engineer. For example, a Wöhler curve only applies to a certain mean stress. With dynamically loaded components, however, the mean stress often changes in practice, which in turn infuences the fatigue strength. In order to show the infuence of the mean stress on the fatigue strength, many further curves for the most different mean stresses would have to be included in the stress-cycle diagram. The clarity would suffer greatly from this. For this reason, special diagrams are used to illustrate the infuence of the mean stress in a clearer way.

Haigh Diagram

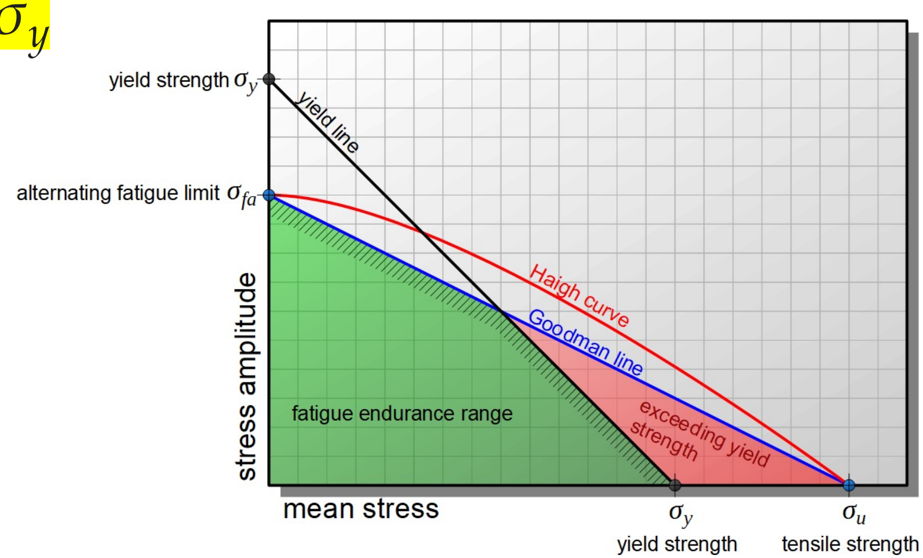


In the fatigue limit diagram according to Haigh, the bearable stress amplitude is applied directly against the mean stress. Such a Haigh diagram is also referred to as a Goodman diagram. The intersection of the curve with the vertical axis corresponds to the alternating fatigue limit σ_{fa} , since the mean stress there is zero (stress ratio $R = -1$).

The intersection of the curve with the horizontal axis, however, can be interpreted as **ultimate tensile strength** σ_u , since the specimen would theoretically fracture without an existing stress amplitude, i.e. solely due to the applied mean stress (stress ratio $R = 1$).

The Haigh curve ("fatigue limit curve") runs between these aforementioned points. **Below the fatigue limit curve are the permissible stress amplitudes for a given mean stress.** **To simplify the construction of the diagram, the Haigh curve is approached by a straight line, the so-called Goodman line.** This approximation results in a kind of "natural safety factor", as the Goodman line runs below the Haigh curve.

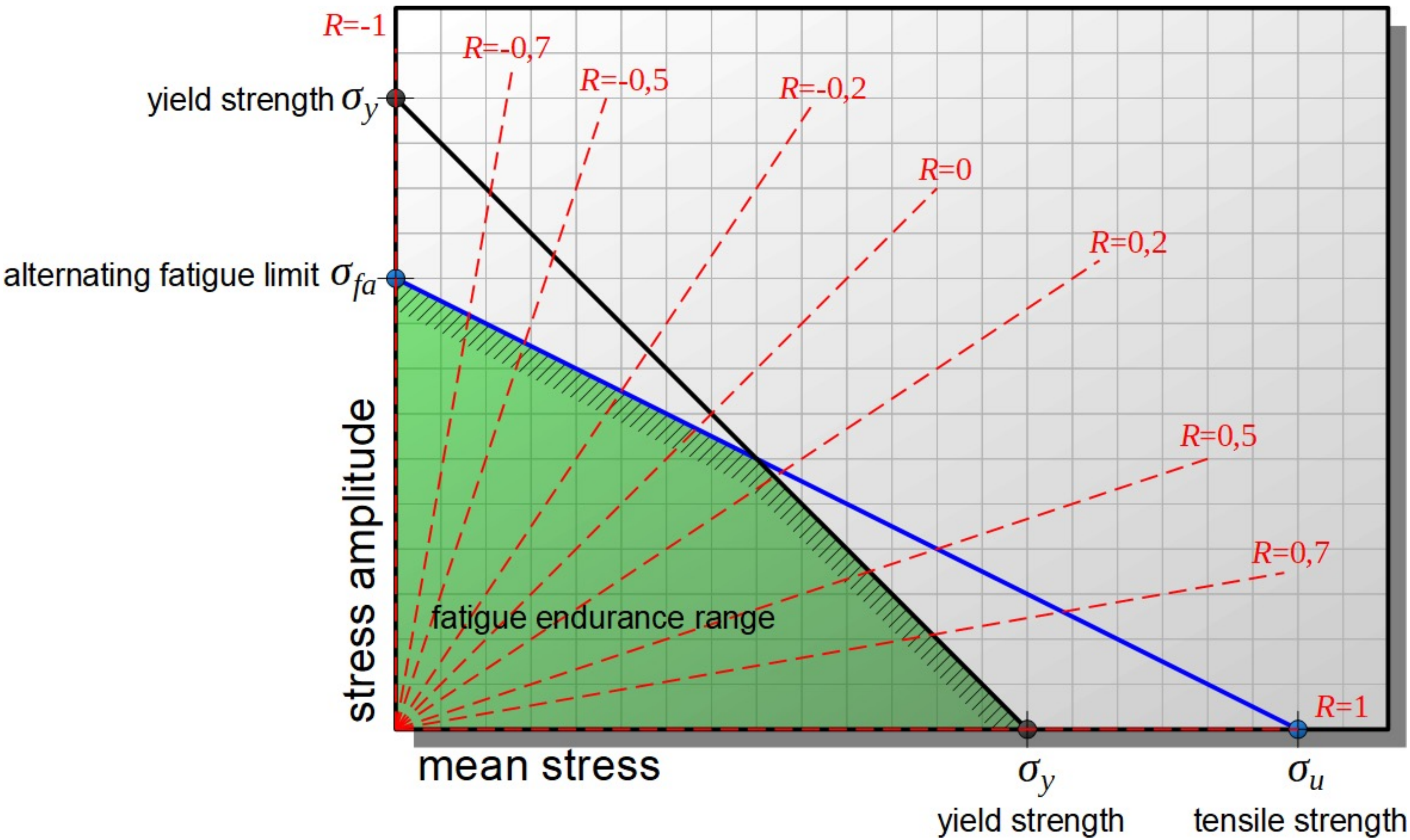
Especially with high mean stresses, however, the Haigh curve or Goodman line is only of theoretical significance, as the yield point is already exceeded in this range, resulting in unacceptable plastic deformations. The practical limit therefore does not extend up to the tensile strength σ_u but rather only the to yield strength σ_y



In principle, the sum of mean stress and stress amplitude must never exceed the value of the yield strength. Based on the yield strength value, the stress amplitude can theoretically be increased to the same extent as the mean stress is reduced. For this reason, an additional limitation is a straight line under an angle of 45°, which is also referred to as the **yield line**.

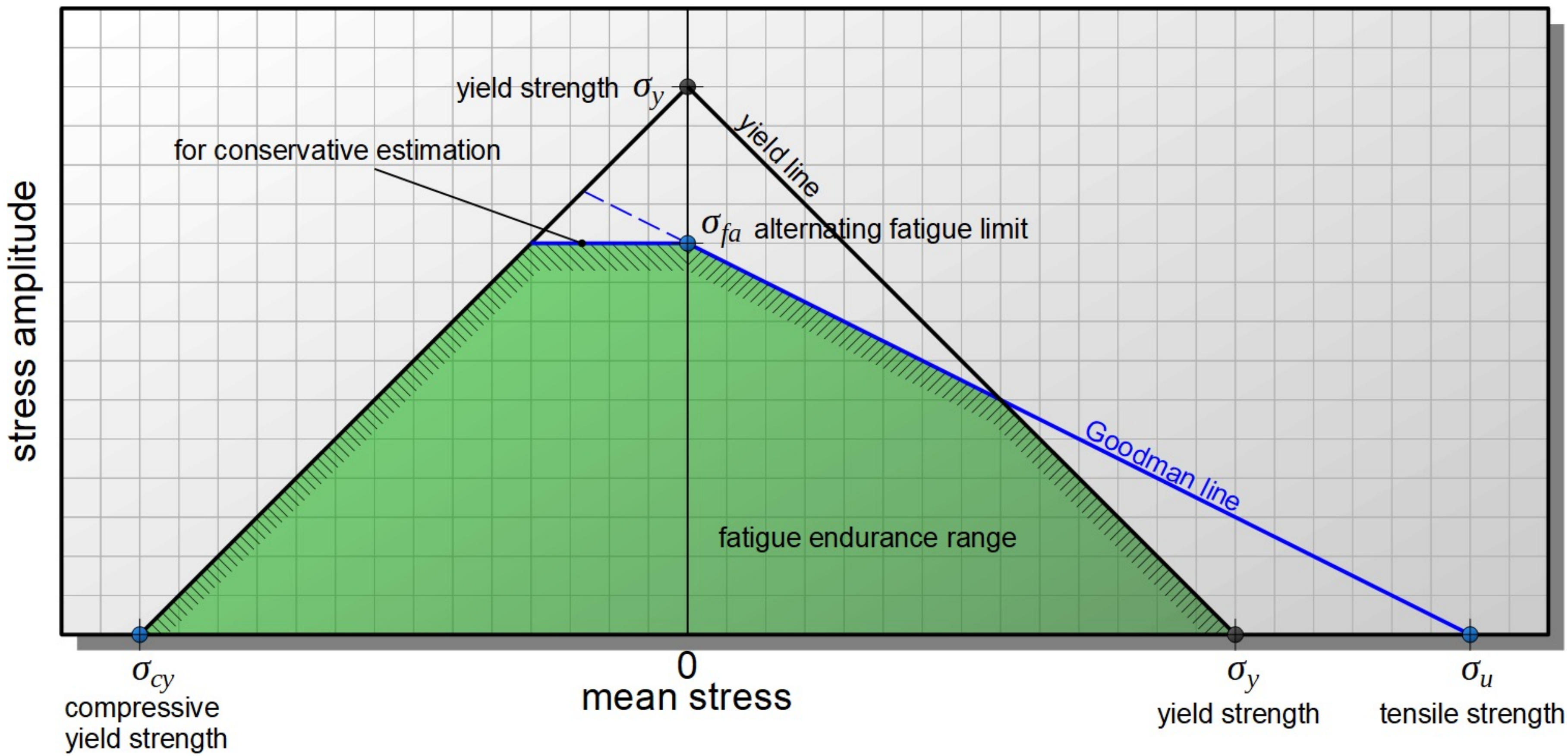
In practice, therefore, only knowledge of the alternating fatigue limit σ_{fa} , the tensile strength σ_u and the yield strength σ_y (or the o"set yield strength) is sufficient to produce a simplified Haigh diagram.

To facilitate reading the fatigue limit for a given stress ratio *R*, selected stress ratios are often included in the diagram.



Extension of the Haigh diagram for compressive mean stresses

In principle, the Haigh diagram can also be extended to negative mean stresses and thus the fatigue limits for compressive loads can be shown. The **compressive yield strength** σ_{cy} is then used as the limiting value for the mean stress. This value is entered on both axes in the same way as for tensile stress and then connected to each other by a straight line. The extrapolation of the Goodman straight limits the area. Whether this extrapolation of the Goodman line is always permissible, however, must be checked separately! For a conservative estimation, this line segment can also be assumed to be horizontal in first approximation.



Note, that due to the effect of "crack closing", fatigue limits for compressive loadings are in general higher than for tensile loadings.

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \qquad A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

Haigh e Goodman-Smith diagrams

The pairs of values σ_m, σ_a to which a certain life N_f corresponds, can be represented on a plane having the value σ_m on the abscissa and the value σ_a on the ordinate. The starting point is always the value $\sigma_a = \sigma_f$ for $\sigma_m = 0$ which comes from the Wöhler diagram.

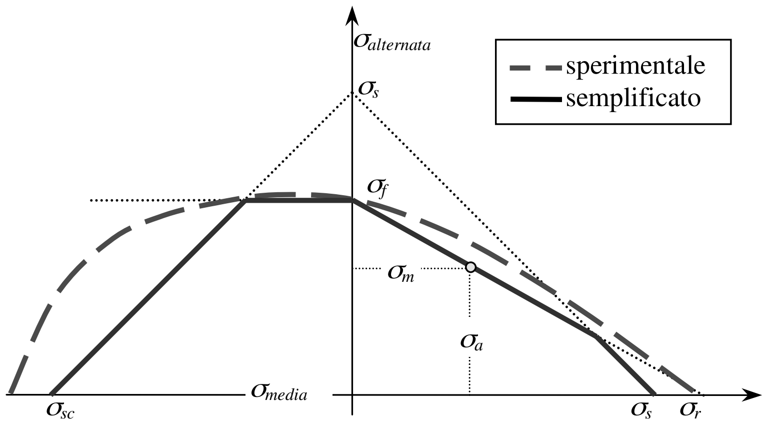


Fig.16.2 - Diagrammi di Haigh sperimentale e semplificato.

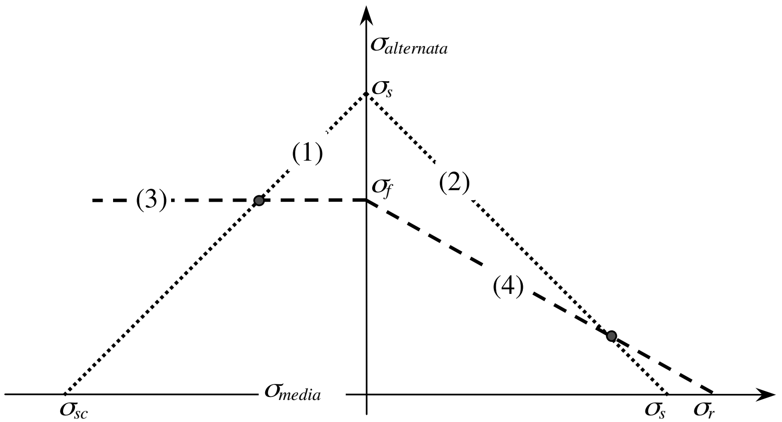


Fig.16.3 - Costruzione del Diagramma di Haigh semplificato.

The stress σ_f is the fatigue strength for the life N_f of the real element which takes into account all the effects related to finish, gradient and dimensions. In the case of infinite life, of course $\sigma_f = \sigma_l$. It is observed that the experimental points can be interpolated from a curve which is called the Haigh diagram. Since the realization of diagrams of this type for various values of N_f requires an excessive amount of experimental data, simplified representation methods have been proposed that can be carried out by the simple knowledge of the corresponding σ_f, of the yield stress σ_s and of the failure stress σ_r

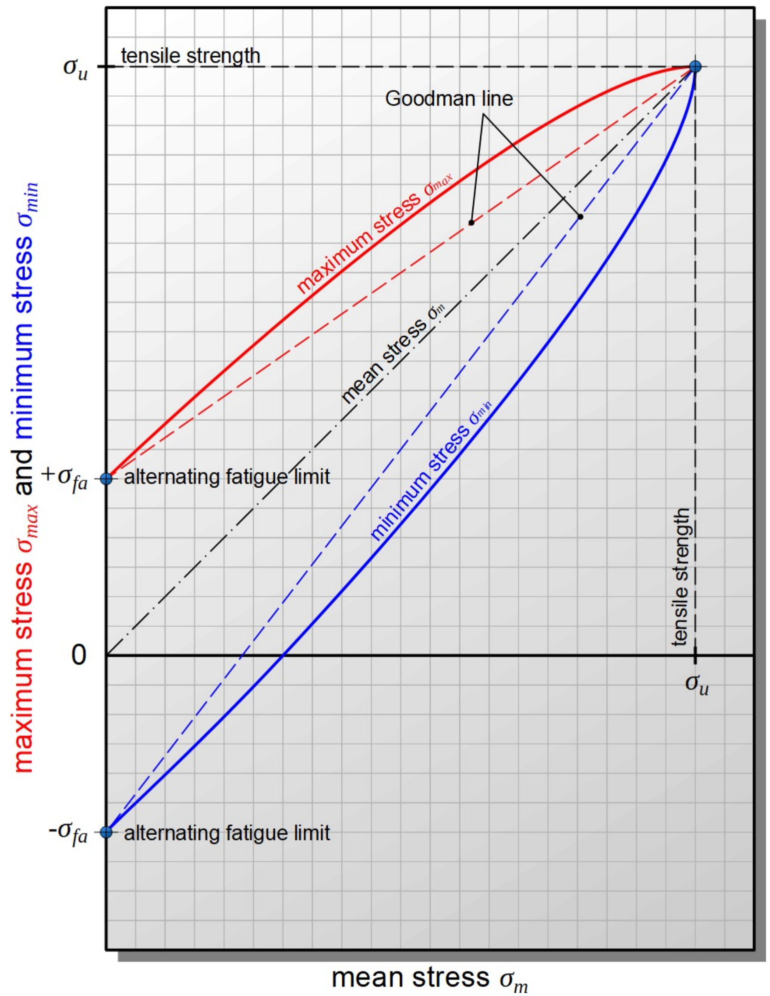
The construction of the diagram is carried out in the following steps:

1. On the «x (σ_{media})» axis, the yield stress for traction σ_s the yield stress for compression σ_{sc} and the failure σ_r are recorded. On the «y ($\sigma_{alternata}$)» axis, both σ_s and the value of alternating stress σ_f, corresponding to the Nf cycles, are reported (from the Wohler curve)
2. a line (1) is drawn from $\sigma_{alternata} = \sigma_{sc}$ to $\sigma_{media} = \sigma_{sc}$ for average compressive stresses,
3. a line (2) is drawn from $\sigma_{alternata} = \sigma_s$ to $\sigma_{media} = \sigma_s$, for mean tensile stresses,
4. a horizontal line (3) is drawn from $\sigma_{alternata} = \sigma_f$ for average compressive stresses,
5. a line (4) is drawn from $\sigma_{alternata} = \sigma_f$ to $\sigma_{media} = \sigma_r$ for mean tensile stresses.
6. The resulting limit curve is the broken line

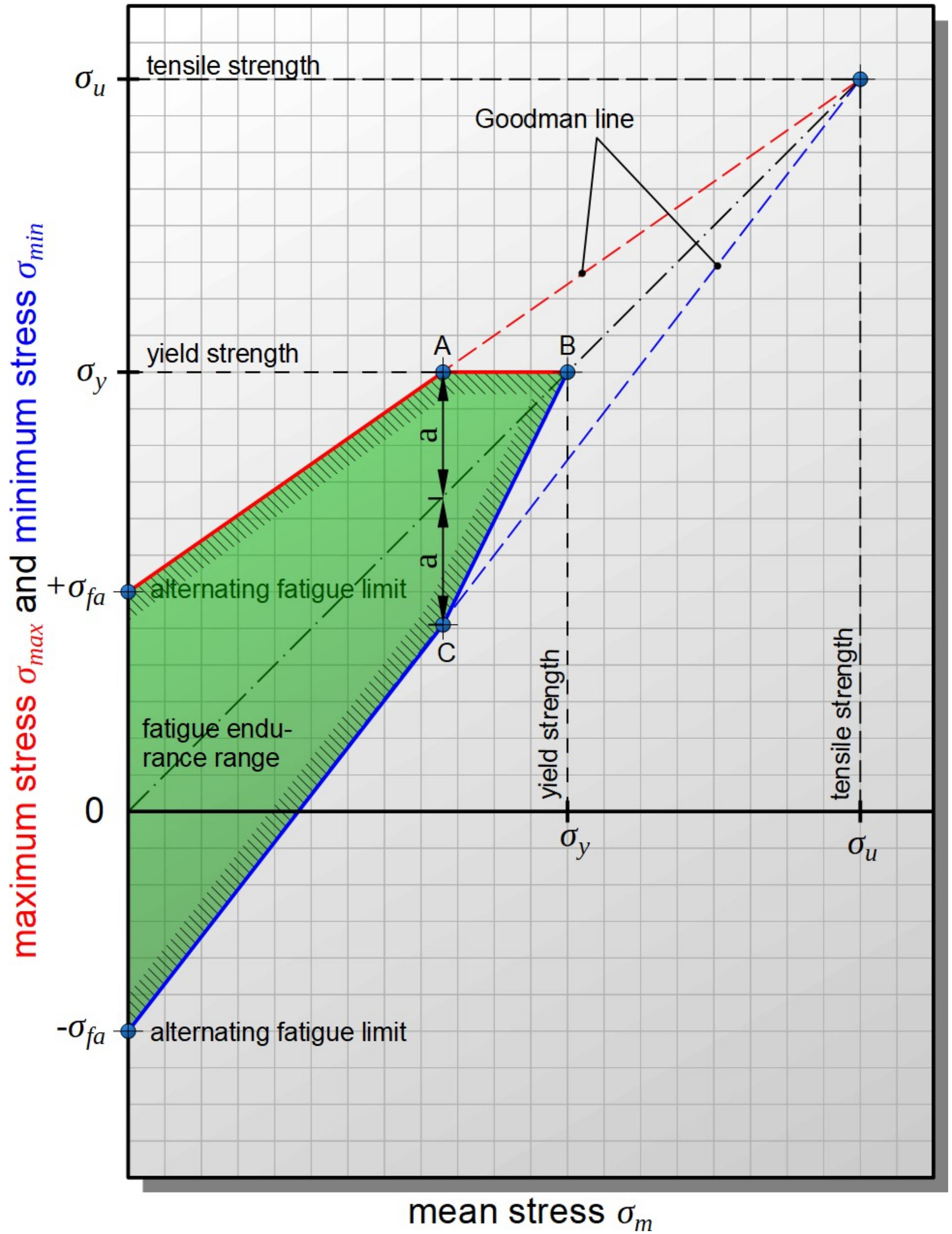
Smith diagram

In such a Smith diagram, the minimum stress σ_{min} and the maximum stress σ_{mas} are plotted against the mean stress σ_m . Based on fundamental considerations, certain fixed points arise again.

With a mean stress of $\sigma_m = 0$, the alternating fatigue limit σ_{fa} of the material is obtained, so that the value of the maximum stress is $+\sigma_{fa}$ and the minimum stress is $-\sigma_{fa}$ Another fixed point results when the mean stress just reaches the ultimate tensile strength ($\sigma_m = \sigma_u$). In this case, the material can no longer withstand stress amplitude, as otherwise the tensile strength would be exceeded.



For practical reasons, there is also an additional limitation, since so far the diagram only takes into account the fracture criterion. Usually, however, inadmissible deformations which occur at stresses above the yield point are decisive. Therefore, the Goodman lines only run up to the maximum value of the yield strength. This applies to both the maximum stress (point A) and the mean stress (point B), since of course the mean stress must not exceed the yield point too.



Alternatively, it is possible to represent the values σ_m, σ_{max} and σ_{min} which correspond to a certain life N_f on a plane having the value σ_m on the abscissa and the values σ_{max} and σ_{min} on the ordinate. The curves interpolating the experimental results constitute the Goodman-Smith diagram.

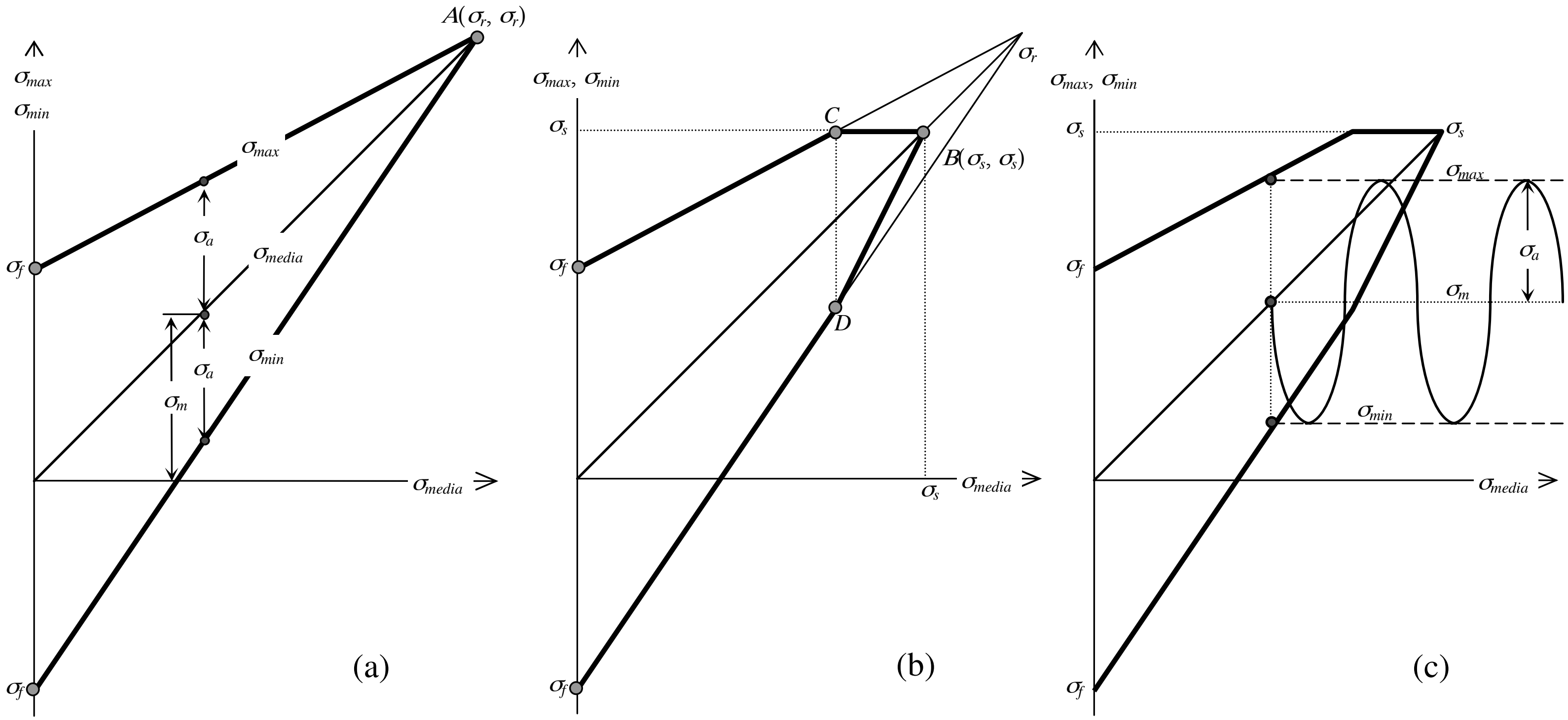
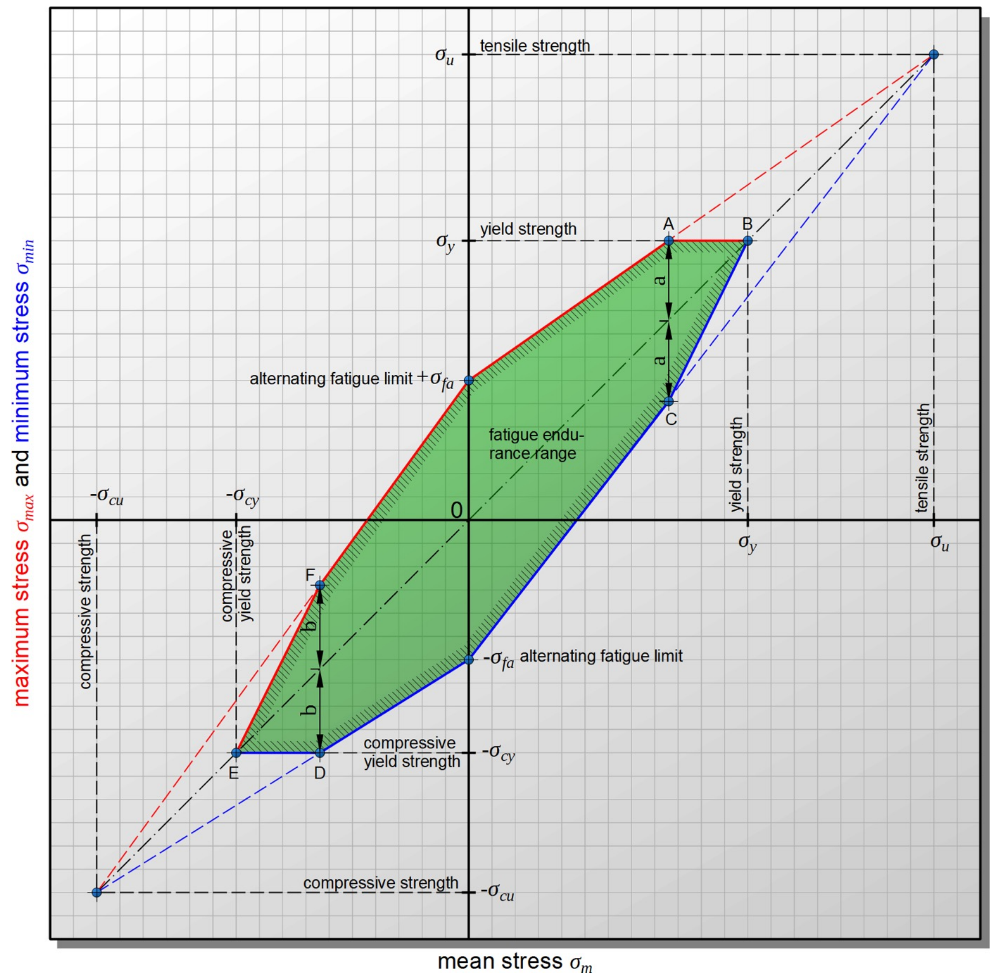


Fig.16.4 - Costruzione del diagramma di Goodman-Smith: (a) i passi da 1 a 3, (b) passi da 4 a 5, (c) esempio.

Extension of the Haigh diagram for compressive mean stresses

Instead of limiting the maximum stress by the tensile yield strength, the compressive yield strength σ_{cy} is used to limit the minimum stress (point D). The mean stress is also limited by the compressive yield strength (point E). The symmetrical distribution of the minimum and maximum stress values around the mean stress is again obtained by mirroring the point D around the mean stress line (point F).



Resistenza a cicli di ampiezza variabile: regola di Miner

Se n_1 è il numero di cicli di semiampiezza S_1, n_2 il numero di cicli ad ampiezza S_2 ed N_1, N_2 sono i numeri dei cicli che portano al collasso con ampiezze S_1 e S_2 rispettivamente, la condizione di collasso è

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1$$

O, più in generale

$$\sum \frac{n_i}{N_i} = 1$$

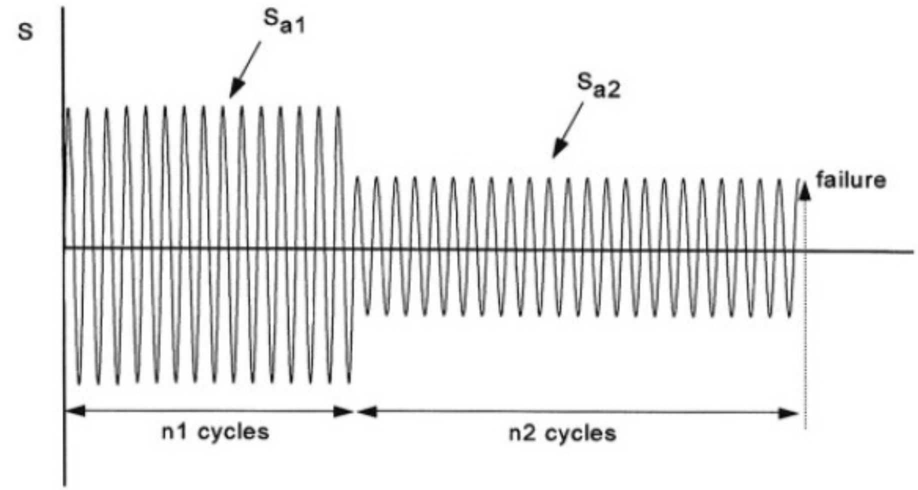
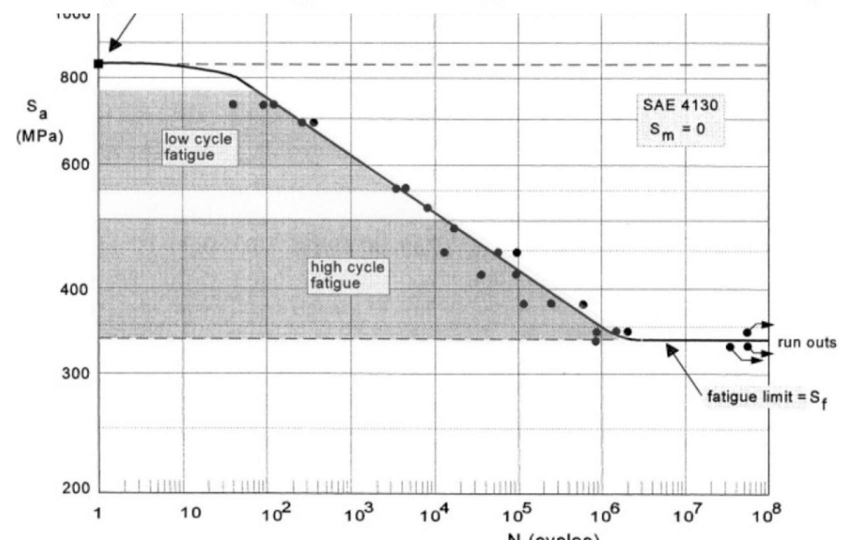


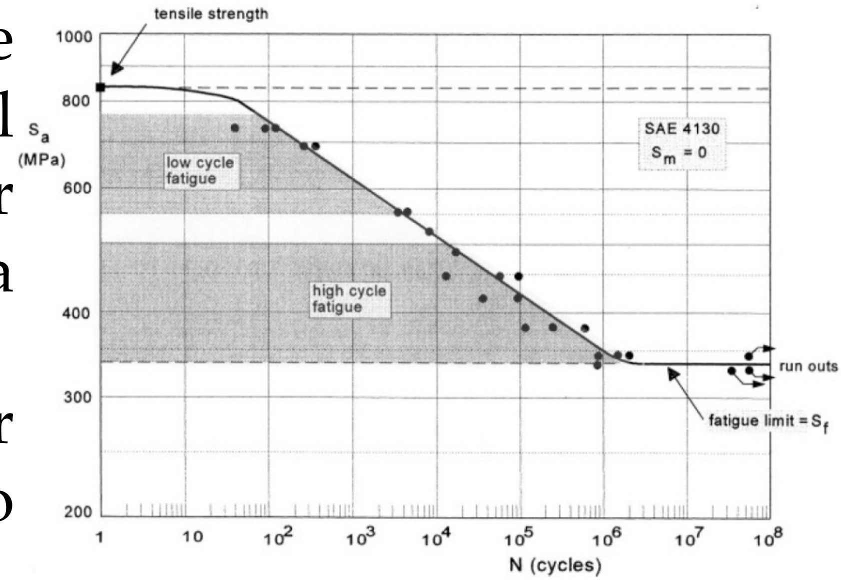
Fig.10.1: A simple Variable-Amplitude load sequence with two blocks of cycles.

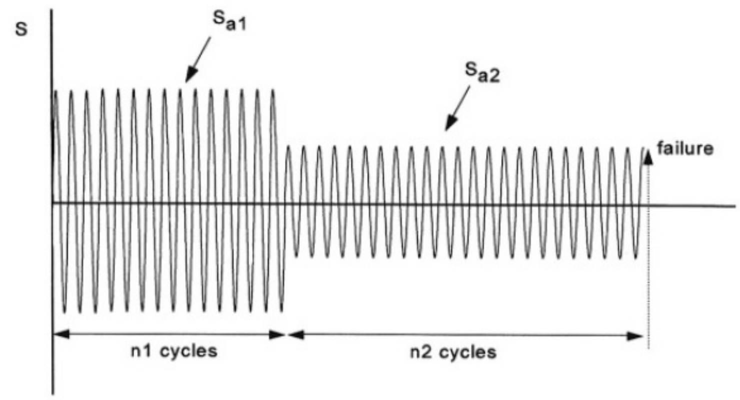


Limiti della regola di Miner

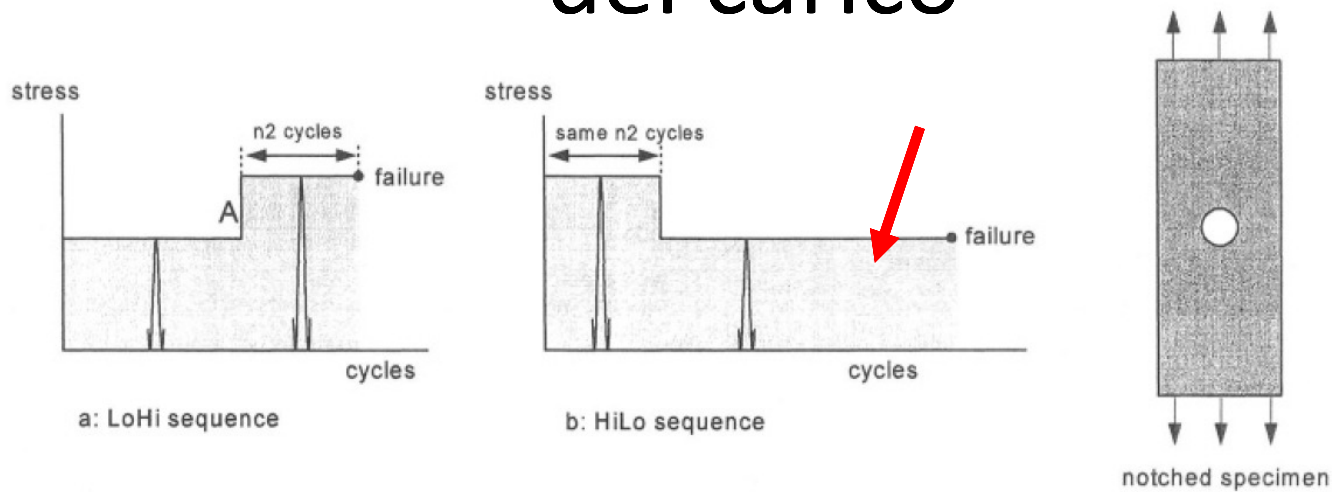
Per la regola di Miner, se $n_1<N_1$ e $\sigma_2<\sigma_f$ (limite di fatica), poiché in tal caso $N_2=\infty$, ne segue che $n_2/N_2=0$ per qualsiasi n_2 e quindi non si verifica mai la rottura del pezzo.

Questo è sbagliato, perché se per $\sigma<\sigma_f$ non si ha rottura, ciò è dovuto al fatto che tensioni inferiori a σ_f non sono in grado di innescare la fessura. Ma se $\sigma_1>\sigma_f$ la fessura può essere attivata ed i cicli di ampiezza S_2 possono portare il pezzo a rottura.





Influenza dell’ordine di applicazione del carico



L’ordine di applicazione dei cicli ha influenza sulla vita a fatica del pezzo. Se un pezzo è soggetto a n_1 cicli di ampiezza σ_1 e poi a cicli di ampiezza $\sigma_2>\sigma_1$ e dopo n_2 cicli raggiunge la rottura, lo stesso pezzo, soggetto prima ad n_2 cicli di ampiezza σ_2 e quindi a cicli di ampiezza σ_1 raggiungerà il collasso dopo un numero di cicli $> n_1$.

Questo è dovuto agli effetti positivi delle tensioni plastiche residue.

The phenomenon of crack closure was first discovered by Elber in 1970. The crack closure effect helps explain a wide range of fatigue data, and is especially important in the understanding of the effect of stress ratio (less closure at higher stress ratio) and short cracks (less closure than long cracks for the same cyclic stress intensity

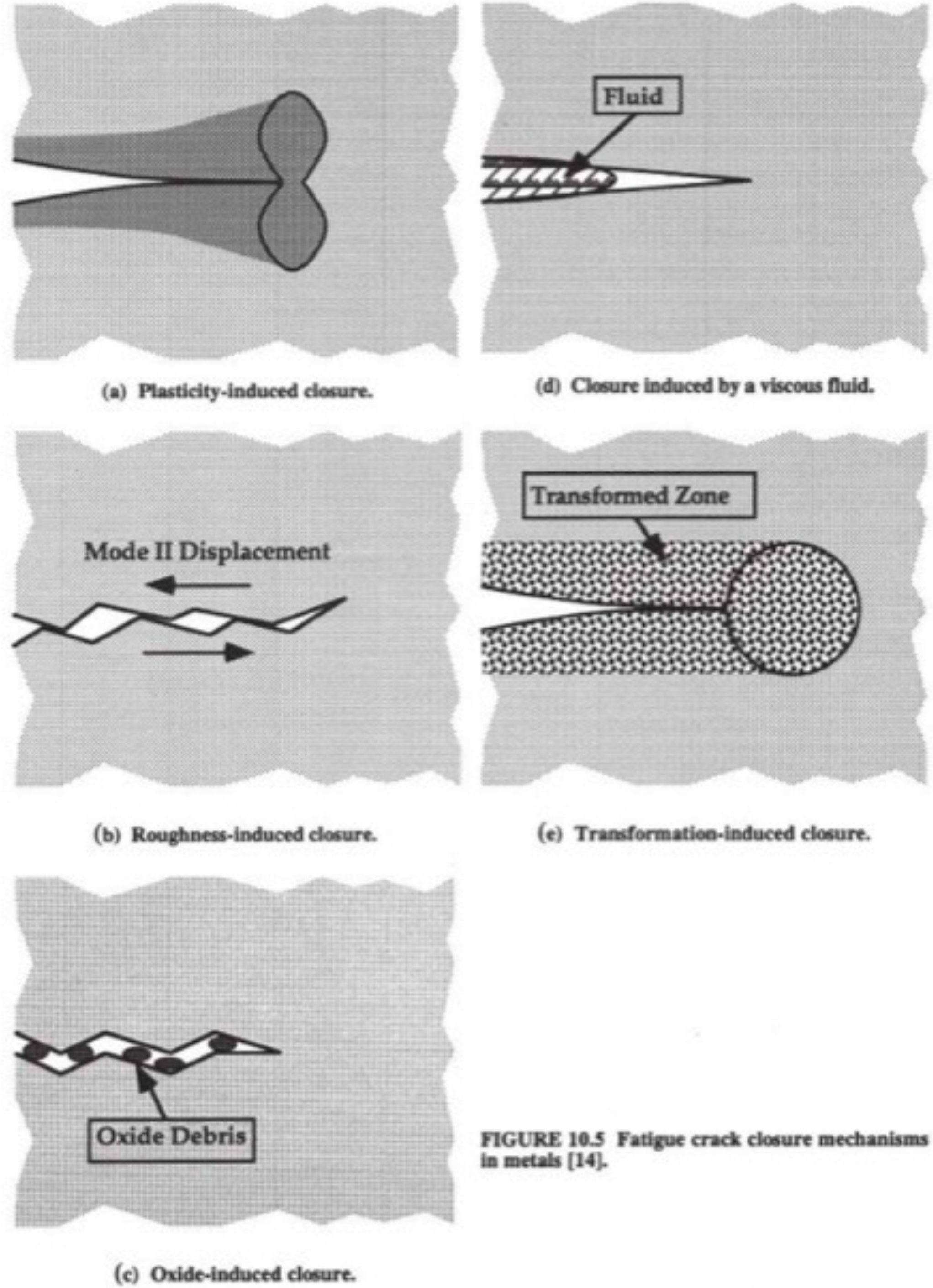


FIGURE 10.5 Fatigue crack closure mechanisms in metals [14].

EXAMPLE 10.1

Derive an expression for the number of stress cycles required to grow a semicircular surface crack from an initial radius a_o to a final size a_f, assuming the Paris-Erdogan equation describes the growth rate. Assume that a_f is small compared to plate dimensions, and that the stress amplitude, $\Delta\sigma$, is constant.

Solution: The stress intensity amplitude for a semicircular surface crack in an infinite plate (Fig. 2.19) can be approximated by

$$\Delta K = \frac{1.04}{\sqrt{2.464}} \Delta\sigma \sqrt{\pi a} = 0.663\, \Delta\sigma \sqrt{\pi a}$$

If we neglect the ϕ dependence of λ_s. Substituting this expression into Eq. (10.5) gives

$$\frac{da}{dN} = C\left(0.663\Delta\sigma\right)^m \left(\pi\, a\right)^{m/2}$$

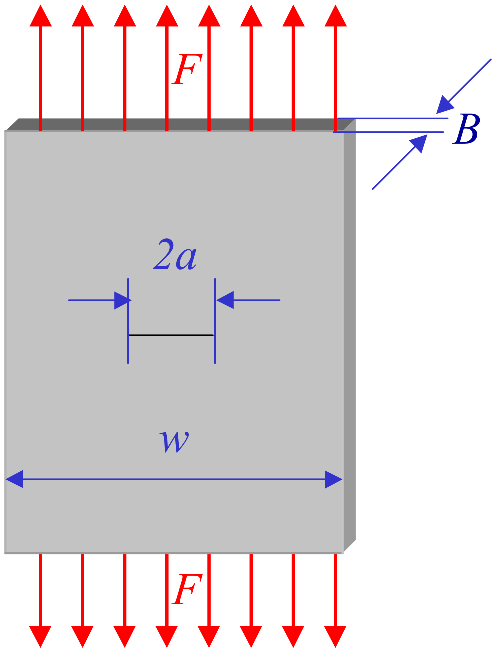
which can be integrated to determine fatigue life:

$$N = \frac{1}{C\left(0.663\sqrt{\pi}\,\Delta\sigma\right)^m} \int_{a_o}^{a_f} a^{-m/2}\, da$$

$$= \frac{a_o^{1-m/2} - a_f^{1-m/2}}{C\left(\frac{m}{2} - 1\right)\left(0.663\sqrt{\pi}\,\Delta\sigma\right)^m} \qquad (\text{for } m \neq 2)$$

Closed-form integration is possible in this case because the K expression is relatively simple. In most instances, numerical integration is required.

Esercizio 1



Si determini la durata della piastra mostrata in figura sapendo che presenta una cricca centrale passante e che è sollecitata da un carico ciclico F.

$F_{min} = 0\ kN$
$F_{max} = 300\ kN$

$w = 100\ mm$
$2a = 2\ mm$
$B = 10\ mm$

$\sigma_S = 1800\ MPa$
$K_{IC} = 57\ MPa\ m^{1/2}$

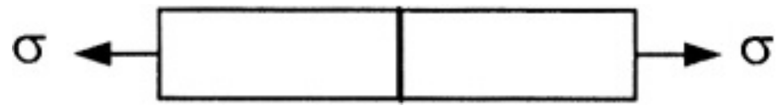
Coefficienti legge di Paris:
$C = 2.33\ 10^{-11}$
$n = 3$

Esercizio 1

Dati:	a0	w	b	K1c	DeltaS	SigmaY		
	0.001	0.1	0.01	57	300	1800		
Coeff. Paris:	C	n	DeltaK0					
	2.33E-11	3	2.7					
Parametri:	Deltaa							
	0.0005							

							Sigma limite:	
a	am	Y	DeltaK	paris	DeltaN	Ntot	a frattura	a coll. plast.
0.0010		1.7729	16.8189			0		
0.0015	0.0013	1.7731	18.8067	1.55E-07	3226	3226	909.25	1777.50
0.0020	0.0018	1.7738	22.2606	2.57E-07	1945	5171	768.17	1768.50
0.0025	0.0023	1.7746	25.2536	3.75E-07	1332	6504	677.13	1759.50
0.0030	0.0028	1.7757	27.9362	5.08E-07	984	7488	612.11	1750.50
0.0035	0.0033	1.7771	30.3924	6.54E-07	764	8253	562.64	1741.50
0.0040	0.0038	1.7786	32.6750	8.13E-07	615	8868	523.34	1732.50
0.0045	0.0043	1.7804	34.8197	9.84E-07	508	9376	491.10	1723.50
0.0050	0.0048	1.7824	36.8522	1.17E-06	429	9805	464.02	1714.50
0.0055	0.0053	1.7846	38.7915	1.36E-06	368	10172	440.82	1705.50
0.0060	0.0058	1.7870	40.6524	1.57E-06	319	10492	420.64	1696.50
0.0065	0.0063	1.7897	42.4466	1.78E-06	281	10772	402.86	1687.50
0.0070	0.0068	1.7926	44.1836	2.01E-06	249	11021	387.02	1678.50
0.0075	0.0073	1.7958	45.8712	2.25E-06	222	11244	372.78	1669.50
0.0080	0.0078	1.7992	47.5161	2.50E-06	200	11444	359.88	1660.50
0.0085	0.0083	1.8028	49.1239	2.76E-06	181	11625	348.10	1651.50
0.0090	0.0088	1.8067	50.6994	3.04E-06	165	11789	337.28	1642.50
0.0095	0.0093	1.8108	52.2469	3.32E-06	150	11940	327.29	1633.50
0.0100	0.0098	1.8152	53.7702	3.62E-06	138	12078	318.02	1624.50
0.0105	0.0103	1.8198	55.2725	3.93E-06	127	12205	309.38	1615.50
0.0110	0.0108	1.8247	56.7570	4.26E-06	117	12322	301.28	1606.50
0.0115	0.0113	1.8299	58.2263	4.60E-06	109	12431	293.68	1597.50
0.0120	0.0118	1.8353	59.6831	4.95E-06	101	12532	286.51	1588.50

Example 9.1 *A high-strength steel string has a miniature round surface crack of* 0.09 *mm deep and a outer diameter of* 1.08 *mm. The string is subjected to a repeated fluctuating load* ($\sigma_{min} = 0$, $\sigma_{max} > 0$ *at a stress ratio* $R = 0$. *The threshold stress intensity factor is* $\Delta K_{th} = 5\ MPa\sqrt{m}$, *and the crack growth rate equation is given by*



$$\frac{da}{dN} = \left(5x10^{-14}\ \frac{MN^{-4}.m^{7}}{cycles}\right)(\Delta K)^{4}$$

Determine a) the threshold stress $\Delta\sigma_{th}$ *the string can tolerate without crack growth, b) the maximum applied stress range* $\Delta\sigma$ *and c) the maximum (critical) crack size for a fatigue life of* $N = 10^4$ *cycles. Use the following steel properties:* $K_{IC} = 25\ MPa\sqrt{m}$ *and* $\sigma_{ys} = 795\ MPa$.

Solution: *It is assumed that the plastic-zone with a cyclic range* ΔK *is smaller than that for* K_I *applied monotonically, and that the surface crack can be treated as a single-edge crack configuration. Note that* $N_o = 0$ *since* a_o already exists. *Data*:

a) *From eqs. (3.56),*

$$a = \frac{D-d}{2} \tag{3.56}$$

$$d = D - 2a = 1.08\ mm - 2\,(0.09\ mm) = 0.90\ mm$$

$$\frac{d}{D} = 0.8333 \quad \text{and} \quad \frac{D}{d} = 1.20$$

Thus, eqs. (3.54) and (9.5) yield respectively

$$f\,(d/D) = \frac{1}{2}\sqrt{\frac{D}{d}}\left[\frac{D}{d}+\frac{1}{2}+\frac{3}{8}\left(\frac{d}{D}\right)-\frac{5}{14}\left(\frac{d}{D}\right)^2+\frac{11}{15}\left(\frac{d}{D}\right)^3\right] \tag{3.55}$$

$$\alpha = f\,(d/D) = 1.198\,9$$

$$\Delta\sigma_{th} = \frac{\Delta K_{th}}{\alpha\sqrt{\pi a_o}} = \frac{5\ MPa\sqrt{m}}{(1.198\,9)\sqrt{\pi\,(0.09x10^{-3}\ m)}} = 248.02\ MPa$$

$$\Delta\sigma_{\min} < \Delta\sigma_{th}$$

$$\int_{N_o}^{N} dN = \int_{a_o}^{a} \frac{da}{A\left(\Delta K\right)^n}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{\alpha}\right)^2 \Delta\sigma^{-2}$$

$$N - N_o = \frac{2a}{A\left(n-2\right)\left(\alpha\Delta\sigma\right)^n\left(\pi^{n/2}\right)}\left[\left(\frac{a}{a_o}\right)^{n/2-1} - 1\right]$$

Substituting eq. (9.17) along with $a = a_c$ into (9.16) yields an important expression for determining the stress range $\Delta\sigma$ when the final crack size is unknown

$$C_1\left(\Delta\sigma\right)^n + C_2\left(\Delta\sigma\right)^{n-2} - C_3 = 0$$

$$\begin{aligned}
C_1 &= A\left(n-2\right)\left(N - N_o\right)\left(K_{IC}\right)^n \\
C_2 &= \frac{2}{\pi}\left(\frac{K_{IC}}{\alpha}\right)^2 \\
C_3 &= 2\left(a\right)^{1-n/2}\left(\frac{1}{\pi}\right)^{n/2}\left(\frac{K_{IC}}{\alpha}\right)^n
\end{aligned}$$

b) *Use eq. (9.19) and subsequently (9.18) to get*

$$
\begin{aligned}
C_1 &= \left(5x10^{-14}\ \frac{MN^{-4}.m^7}{cycles}\right)(4-2)\left(10^4\ cycles\right)\left(25\ \frac{MN}{m^2}\sqrt{m}\right)^4 \\
C_1 &= 3.9063\times10^{-4}\ m \\
C_2 &= \frac{2}{\pi}\left(\frac{K_{IC}}{\alpha}\right)^2 = \frac{2}{\pi}\left(\frac{25\ MPa\sqrt{m}}{1.1989}\right)^2 = 276.82\ MPa^2.m \qquad (9.19) \\
C_3 &= 2\left(0.09x10^{-3}\ m\right)^{1-4/2}\left(\frac{1}{\pi}\right)^{4/2}\left(\frac{25\ MPa\sqrt{m}}{1.1989}\right)^4 \\
C_3 &= 4.2571\times10^{8}\ MPa^4.m
\end{aligned}
$$

and

$$3.9063\times10^{-4}\Delta\sigma^4 + 276.82\Delta\sigma^2 - 4.2571\times10^8 = 0$$

Solving the above biquadratic equation yields four roots. The positive root is

$$
\begin{aligned}
\Delta\sigma &= 864.93\ MPa \\
\sigma_{\max} &= \Delta\sigma = 864.93\ MPa \qquad \text{since} \qquad \sigma_{\min} = 0 \\
\sigma_{\max} &\lessapprox \sigma_{ys}
\end{aligned}
$$

b) *The critical crack size is calculated from eq. (9.17)*

$$a_c = \frac{1}{\pi}\left[\frac{K_{IC}}{\alpha}\right]^2 \Delta\sigma^{-2} = \frac{1}{\pi}\left[\frac{25\ MPa\sqrt{m}}{(1.198\,9)\,(864.93\ MPa)}\right]^2 \qquad (9.17)$$

$$a_c = 0.185\ mm = 2.056 a_o$$

$$\Delta a = a_c - a_o = 0.095\ mm$$

Therefore, $\Delta a = 0.095\ mm$ *represents* 5.56% *increment at a maximum fluctuating stress* $\sigma_{\max} \lessapprox \sigma_{ys}$ *for a fatigue life of* 10^4 *cycles.*