SINCE THE SPLITTING OF THE ATOMIC BEAM IN THE S-G EXPERIMENT IS DUE TO THE SPIN OF AN UNPAIRED e^- ONE MAY WONDER WHY A BEAM OF e^- IS NOT DIRECTLY USED. THE INTERPRETATION OF THE M-G. EXPERIMENT IS BASED ON CALCULATIONS DERIVED FROM THE CLASSICAL PHYSICS. HENCE, A PARTICLE CARRING A MAGNETIC MOMENT AND PASSING THROUGH A NON-UNIFORM MAGNETIC FIELD IS DEFLECTED BY ITS ORIGINAL TRAJECTORY BY AN ANGLE PROPORTIONAL TO THE FORCE ACTING ON THE PARTICLE. HOWEVER THIS IS AN APPROXIMATION FOR THE PARTICLE MUST BE DESCRIBED BY A WAVEFUNCTION AS REQUIRED BY THE Q.M., HOWEVER, THE CLASSICAL DESCRIPTION IS A GOOD APPROXIMATION TO THE Q.M. FOR MASSIVE PARTICLE. FOR AN e^- THE WAVE PACKET DESCRIPTION (Q.M. DESCRIPTION) SHOULD BE USED AND WAVE PACKETS DISPERSE WITH TIME - THE LIGHTER THE PARTICLE THE FASTER THE DISPERSION AND THE GREATER IS THE INCERTITY ON THE POSITION OF THE PARTICLE. THE APPLICATION OF THE HEISENBERG'S UNCERNTITY PRINCIPLE TO AN e^- BEAM SHOWS THAT BECAUSE OF THE SMALL MASS OF THE e^- THE PATTERN OF THE ELECTRONS IMPINGING ON THE DETECTOR WOULD BE SO SPREAD THAT NO CONCLUSIONS COULD BE DRAWN.

SPIN OPERATORS

FROM THE S-G EXPERIMENT AND OTHER SPECTROSCOPY EXPERIMENTS WE HAVE LEARNED THAT ELECTRONS (AND OTHER PARTICLES) HAVE AN INTRINSIC ANGULAR MOMENTUM, TO WHICH 3 DEGREES OF FREEDOM ARE ASSOCIATED $\bar{S}\equiv(S_x, S_y, S_z)$ PLUS A FOURTH DEGREE OF FREEDOM WITH TWO POSSIBLE ORIENTATIONS (SPIN), SUCH THAT THE SPIN COMPONENT, IN A PREFERENTIAL DIRECTION (THAT IN EXPERIMENTS WITH MAGNETIC FIELD IS IDENTIFIED WITH THE DIRECTION OF $\bar{B}$), CAN ONLY HAVE THE VALUE $+\hbar/2$ AND $-\hbar/2$, BEHAVING AS A PURE TWO-LEVEL SYSTEM.
IN THIS RESPECT IS CORRECT TO DEFINE THESE A "SPIN STATES", USUALLY DEFINED AS SPIN-UP AND SPIN-DOWN STATES, TO WHICH WE CAN ASSOCIATE TWO "WAVE-FUNCTIONS", $\phi_\uparrow$, $\phi_\downarrow$. ACCORDINGLY TO THE QUANTUM FORMALISM IF AN EXPERIMENT MAKES OBSERVABLE THE Z COMPONENT OF THE SPIN (AS FOR IN THE S-G EXPERIMENT) THIS CORRESPONDS TO ~~APPLY~~ AN HERMITIAN OPERATOR, $\hat{S}_z$, TO THE SPIN-WF.
WE CAN ALSO CHOOSE THE WF IN SUCH A WAY THAT THE APPLICATION OF THE $\hat{S}_z$ GIVES THE OBSERVED VALUE OF THE WAVEFUNCTION. SINCE WE HAVE ONLY TWO OBSERVED VALUES $\pm\hbar/2$ WE SHOULD EXPECT

$$\hat{S}_z\phi_\uparrow = \hbar/2\,\phi_\uparrow \;;\; \hat{S}_z\phi_\downarrow = -\hbar/2\,\phi_\downarrow$$

THIS CAN BE SUMMARIZED AS

$$\hat{S}_z \phi_{m_s} = \hbar m_s \phi_{m_s}, \quad \text{WHERE } m_s = +\tfrac{1}{2} (\uparrow)$$

AND $m_s = -\frac{1}{2} (\downarrow)$. m_s IS THUS THE QUANTUM NUMBER OF THE Z-COMPONENT OF THE SPIN.

WE WILL SEARCH NOW A FORMALISM THAT WILL GIVE US "AUTOMATICALLY" THE EIGENVALUE EQs FOR THE SPIN. IN LINEAR ALGEBRA WE HAVE LEARND THAT IF $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ AND

$$\bar{V} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

WE NOW LOOK FOR A "VECTOR" ϕ AND A MATRIX M SUCH THAT $M\phi$ YIELD EXACTLY EITHER $\frac{\hbar}{2}\phi$ OR $-\frac{\hbar}{2}\phi$. A GOOD TRIAL IS TO VERIFY IF

A MATRIX OPERATOR $\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ AND THE

SPIN FUNCTIONS $\phi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ AND $\phi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

COULD WORK. BY APPLY THE SIMPLE MATRIX ALGEBRA SHOWN ABOVE WE DIRECTLY OBTAIN

$$M\phi_\uparrow = (\hbar/2)\,\phi_\uparrow \quad \text{AND} \quad M\phi_\downarrow = -(\hbar/2)\,\phi_\downarrow .$$

THE MOST GENERAL SUPERPOSITION SPIN FUNCTION IS $\phi = a\phi_\uparrow + b\phi_\downarrow = \begin{pmatrix} a \\ b \end{pmatrix}$ WITH a AND b ARE COEFF. TO GET THE NORMALIZATION CONDITIONS WE MUST USE THE SCALAR PRODUCT OF $\phi_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ AND $\phi_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

$$\phi_1^* \phi_2 = \left(a_1^*, b_1^*\right) \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \left(a_1^* a_2 + b_2^* b_2\right)$$ SO

WE OBTAIN $\phi_\uparrow^* \phi_\uparrow = 1$ AND $\phi_\downarrow^* \phi_\downarrow = 1$ AND $\phi_\downarrow^* \phi_\uparrow = 0$

SO THE WAVEFUNCTION $\phi_\downarrow^*$ AND $\phi_\uparrow$ ARE NORMALIZED AND ORTHOGONAL. OF COURSE THE REPRESENTATION FOR THE OPERATORS $\hat{S}_x$ AND $\hat{S}_y$ IS STILL OPEN. SINCE THESE MUST BE ANGULAR MOMENTA OPERATOR IT IS REASONABLE TO REQUIRE THE USUAL COMMUTATION RELATIONS FOR THE ANGULAR MOMENTA

$$[\hat{l}, l_j] = 0 \quad j = x, y, z$$

$$[\hat{l}_x, \hat{l}_y] = i\hbar \hat{l}_z$$

$$[\hat{l}_y, \hat{l}_z] = i\hbar \hat{l}_x$$

$$[\hat{l}_z, \hat{l}_x] = i\hbar \hat{l}_y$$

BY APPLING THESE COMMUTATIONS IT IS POSSIBLE TO DERIVE (NOT DONE HERE) THE $\hat{S}_x$ AND $\hat{S}_y$ OPERATORS

$$\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

THESE MATRICES WITH THE

$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

ARE KNOWN AS

THE DIRAC EQUATION AND SPINORS

- LET'S START WITH A NOTATION REVIEW.

THE 3D DIFFERENTIAL OPERATOR $\bar{\nabla}$ IS

$$\bar{\nabla} = (\partial_x, \partial_y, \partial_z) \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

WE CAN GENERALIZE THIS TO 4D ($\mathbb{R}^{1,3} \equiv \mathbb{M}$)

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}; \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- CLASSICAL NON-RELATIVISTIC SCHRÖDINGER EQ.. THE ENERGY OF A PHYSICAL SYSTEM

$$E = \frac{p^2}{2m} + U$$ THAT BY REPLACING

THE CLASSICAL PHYSICAL QUANTITIES

$$\hat{E} \to i\hbar\partial_t \; ; \; \hat{p} \to -i\hbar\bar{\nabla} \Rightarrow$$

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + U\right)\psi$$

FOR $U = 0$ (FREE PARTICLE SOLUTION) WE OBTAIN

$$\psi(\bar{x}, t) \propto e^{-iEt}\psi(\bar{r})$$

AND THE CHARGE DENSITY PROBABILITY ρ ALONG WITH THE CURRENT DENSITY PROBABILITY $\bar{J}$ ARE GIVEN BY

$$\rho = |\psi(x)|^2 \quad \bar{J} = -\frac{i}{2m}\left(\psi^*\bar{\nabla}\psi - \psi\bar{\nabla}\psi^*\right)$$

WITH THE CONSERVATION OF PROBABILITY GIVING THE CONTINUITY EQUATION

$$\partial_t\rho + \bar{\nabla}\cdot\bar{J} = 0$$

THAT IN THE COVARIANT NOTATION IS

$$\partial_\mu J^\mu = 0 \quad \text{WITH} \quad J^\mu \equiv (\rho, \bar{J})$$

WE ALSO NOTE THAT THE SCHRÖDINGER EQ. IS A FIRST ORDER IN ∂_t AND A SECOND ORDER IN ∂_x. WHEN DEALING WITH RELATIVISTIC PARTICLE THE ct AND SPACE COORDINATES ARE TREATED ON THE SAME FOOTING. (243)

THE RELATIVISTIC THEORY OF THE ELECTRON: THE DIRAC EQUATION

STARTING FROM THE SCHÖDINGER EQUATION WE SHELL NOW TO ATTEMP TO REFORMULATE THIS EQ. APPLAING THE ENERGY-MOMEUTUM RELATION AS DERIVED FROM THE SPECIAL RELATIVITY

$$E^2 - p^2c^2 = m^2c^4 \Rightarrow E = \pm\sqrt{p^2c^2 + m^2c^4}$$

IF WE REPLACE $\bar{E}$ AND $\bar{p}$ BY $E \to i\hbar\partial_t$ AND $\bar{p} \to \frac{\hbar}{i}\bar{\nabla} = -i\hbar\nabla$ WE OBTAIN A NEW OPERATOR THAT CAN ACT ON ψ SO WE OBTAIN THE EQ.

$$(i\hbar\partial_t)\psi = \left(\sqrt{-c^2\hbar^2\nabla^2 + m^2c^4}\right)\psi$$

THIS EQUATION CONTAINS THE LAPLACE OPERATOR (∇^2) UNDER THE SQUARE-ROOT. THIS APPROACH FAILS IN DESCRIBING $S \neq 0$ PARTICLES BUT FOR $S=0$ PARTICLES. FURTHERMORE, THE INTERPRETATION WITH THE $\pm E$ IS ANOTHER PROBLEM.
SINCE THE NEGATIVE ENERGY SOLUTION HAVE NEGATIVE PROBABILITY. IN ADDITION THIS APPROACH FAILED WHEN THE ATTEMP WAS MADE TO INCLUDE THE EFFECTS OF ELECTRIC AND MAGNETIC FIELDS ON THE e^- IN SUCH A WAVE EQUATION.

THE KLEIN-GORDON EQUATION.

ALL THE DIFFICULTIES STEAM FROM THE SQUARE ROOT OF THE EQ. ... IT IS MANDATORY TO FIND A WAY TO SOLVE THIS PROBLEM.

TO THIS END WE SQUARE BOTH SIDES OF

$E = (p^2c^2 + m^2c^4)^{1/2}$ AND WE OBTAIN

$$\left(-\hbar^2 \partial_t^2 + c^2\hbar^2\nabla^2\right)\psi = (m^2c^4)\psi$$

THIS EQ. IS KNOWN AS KLEIN-GORDON EQ.

BY DIVIDING BOTH SIDES BY $\hbar^2 c^2$ WE OBTAIN

$\left(-\frac{1}{c^2}\partial_t^2 + \nabla^2\right)\psi = \left(\frac{m^2c^2}{\hbar^2}\right)\psi$. WE CAN RECOGNIZE

IN THE LEFT SIDE OPERATOR THE D'ALAMBERTIAN OPERATOR $\Box^2 = \nabla^2 - \frac{1}{c^2}\partial_t^2$ SO THE K-G EQ.

CAN BE REWRITTEN AS $\Box^2\psi = \frac{mc^2}{\hbar^2}\psi$.

LET'S CONSIDER THE SOLUTION FOR THIS EQ.

- OBSERVATION.

A MORE ELEGANT WAY COULD BE SET BY $p \cdot p = p_\mu p^\mu = E^2 - |\bar{p}|^2 = m^2$. BY REPLACING THE Q.M. OPERATORS FOR E AND p WE OBTAIN

$$\Box^2\psi = \frac{mc^2}{\hbar^2}\psi$$

SINCE FOR A FORCE FREE PARTICLE WE EXPECT THE SOLUTION TO BE "de BROGLIE" WAVES, WE USE THE TRIAL FUNCTION

$$\psi = e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

IN WHICH, AS USUAL $\omega = E/\hbar$ AND $p = \hbar k$

BY SUBSTITUTING THIS ψ INTO THE K-G WAVE

EQ. WE GET $E^2 = p^2c^2 + m^2c^4$, WHICH IS THE ALGEBRIC EQ OF THE SECOND ORDER SPACE-TIME K-G NON-HOMOGENEOUS DIFFERENTIAL EQ.
THE ENERGY IS THAN OBTAIN FROM THE SQUARE ROOT OF THE ENERGY-MOMENTUM RELATIVISTIC RELATION. AS WE HAVE SEEN AT THE BEGINNING WE HAVE A POSITIVE SOLUTION AND A NEGATIVE SOLUTION, BUT FREE PARTICLES MAY HAVE ONLY POSITIVE ENERGY. HOWEVER, THE K-G EQ. WAS REINTERPRED BY PAULI AND WEISSKOPF, WHO USED THE CHARGE DENSITY INSTEAD OF THE MASS DENSITY FINDING THAT THE K-G EQUATION CAN BY USED IN QUANTUM FIELD THEORY FOR S=0 PARTICLES. HOWEVER, FURTHER DEVELOPMENT OF THIS TOPIC GOES BEYOND OUR SCOPES.

- THE DIRAC EQUATION

DIRAC CONSIDERED THE QUESTION AS TO WHETHER THE ROOT COULD NOT BE EXTRACTED FROM

$$E = \sqrt{p^2c^2 + m^2c^4}$$

IN THE LIMIT FOR $p = 0$ $\sqrt{p^2c^2 + m^2c^4} \rightarrow mc^2$
AND FOR $m = 0$

$$\sqrt{p^2c^2 + m^2c^4} \rightarrow pc$$

LET'S CONSIDER FIRST THE 1D CASE AND GENERALIZED TO

$$\sqrt{p_x^2c^2 + m^2c^4} = \alpha c p_x + \beta mc^2$$

THIS EQUATION CANNOT BE FULFILLED IN THE

GENERAL CASE $p_x \neq 0$ AND $m \neq 0$ BY ORDINARY NUMBER α AND β, HOWEVER IT CAN BE WHEN α AND β ARE MATRICES AS WE WILL DEMONSTRATE.

WE SQUARE FIRST THE PREVIOUS EQ.

$$p_x^2 c^2 + m^2 c^4 = \alpha^2 c^2 p_x^2 + (\alpha\beta + \beta\alpha) m c^3 p_x + \beta^2 m^2 c^4$$

THIS EQUALITY REQUIRES

$$\alpha^2 = 1 \quad \alpha\beta + \beta\alpha = 0 \quad \beta^2 = 1$$

THES RELATIONS ARE FAMILIAR FROM THE PAULI SPIN MATRICES BUT WE CANNOT USE THEM HERE FOR WE WANT TO DESCRIBE NOT A 1D SYSTEM BUT A 3D (x, y, z) SYSTEM. THUS WE REQUIRE

$$\sqrt{(p_x^2 + p_y^2 + p_z^2) c^2 + m^2 c^4} = \alpha_1 c p_x + \alpha_2 c p_y + \alpha_3 c p_z + \beta m c^2$$

BY SQUARING THIS EQ. LEADS ANALOGOUSLY TO THE 1D CASE TO

$$\alpha_j^2 = 1 \quad \beta^2 = 1 \quad \alpha_j \beta + \beta \alpha_j = 0 \quad \text{AND} \quad \alpha_j \alpha_k + \alpha_k \alpha_j = 0$$

FOR $j \neq k$ AND $j = 1, 2, 3$ AND $k = 1, 2, 3$.

IN ADDITION, AS ALWAYS IN Q.M. THE OPERATORS (MATRICES) ARE HERMITIAN. THE ABOVE RELATIONS MAY BE FULFILLED IN VARIOUS (BUT PHYSICALLY EQUIVALENT) WAYS. FOR EXAMPLE

$$\alpha_j = \begin{bmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

WHERE σ_j ARE THE PAULI SPIN MATRICES

$$\left(\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right.$$

AND THE $(1)_s$ IN β REPRESENT 2X2 IDENTITY MATRICES, SO THAT β MAY BE WRITTEN IN THE CONVENTIONAL NOTATION AS

$$\beta \equiv \begin{vmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{vmatrix}$$

WE CAN GO BACK NOW TO THE DIRAC EQUATION EMPLOYING THE TRANSLATION RULES

$$E \to i\hbar\partial_t \quad , \quad p_j \to -i\hbar\partial_j \qquad (j \equiv x, y, z)$$

$$\Rightarrow \quad E = \alpha_1 c p_x + \alpha_2 c p_y + \alpha_3 c p_z + \beta m c^2$$

$$(i\hbar\partial_t)\psi = \left[-i\hbar c(\alpha_1\partial_x + \alpha_2\partial_y + \alpha_3\partial_z) - \beta m c^2\right]\psi$$

WHICH IS THE DIRAC EQUATION. SOMETIMES THE DIRAC-E IS WRITTEN AS

$$\left[\frac{1}{c}\partial_t + \sum_{j=1}^{3}\alpha_j\partial_{x_j} + \frac{imc}{\hbar}\beta\right]\psi = 0$$

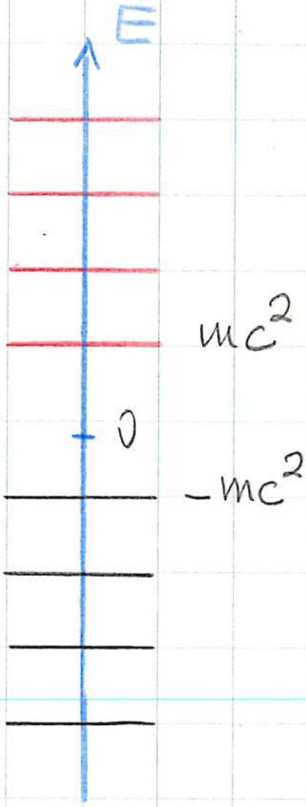
SINCE α_j AND β ARE 4X4 MATRICES THEY MUST OPERATE ON VECTORS WITH 4 COMPONENTS, i.e. ψ MUST BE

$$\psi \equiv \begin{vmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{vmatrix}$$

THIS IS A CONSEQUENCE OF THE FACT THAT IN THE DIRAC EQ. BOTH POSITIVE AND NEGATIVE SOLUTIONS ARE ALLOWED

FOR A FREE PARTICLE, IT IS POSSIBLE TO VERIFY THAT THE DIRAC EQUATION YIELD THE SAME ENERGY SPECTRUM AS THE K-G EQUATION

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$



THE SOLUTION OF THE DIRAC EQ. FOR A FORCE-FREE PARTICLE IS A PLANE WAVE HAVING THE FORM

$$\psi(\bar{r},t) = \begin{vmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{vmatrix} e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

WHERE $\psi_1 \dots \psi_4$ ARE THE AMPLITUDES.

IN THE DIRAC EQ. REPORTED ABOVE THE TIME-DERIVATIVE PLAYS A SPECIAL ROLE RELATIVE TO THE SPATIAL COORDINATES, AS ONE EXPECTS IN NON-RELATIVISTIC MECHANICS. HOWEVER, IN SPECIAL RELATIVITY TIME AND SPACE HAVE COORDINATES HAVE A SYMMETRIC POSITION AS COMPONENTS OF THE $\mathbb{R}^{1,3}$ MINKOWSKI SPACE-TIME FOUR VECTORS. THEREFORE, IN THE LITERATURE A SYMMETRISED FORM OF THE DIRAC EQ. IS OFTEN USED. THIS CAN BE OBTAINED BY MUTIPLING BOTH SIDES FROM THE LEFT,

$$i\hbar\left[\partial_t + c(\alpha_1\partial_x + \alpha_2\partial_y + \alpha_3\partial_z\right]\psi = (\beta mc^2)\psi$$

BY $\gamma^0 = \beta$ AND INTRODUCING NEW MATRICES

$$\gamma^j = \beta\alpha_j \quad \text{WITH } j = 1,2,3 \ (x, y, z)$$

THE RESULTING EQ IS

$$i\hbar\left(\gamma^0 \partial_{x^0} + \gamma^1 \partial_{x^1} + \gamma^2_{x^2} + \gamma^3_{x^3}\right)\psi = (mc)\psi$$

WITH $x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (ct, x, y, z)$
THE EXPLICT FORM OF THE MATRICES γ^0 AND γ^i
ARE

$$\gamma^0 \equiv \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}, \quad \gamma^j \begin{vmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{vmatrix}$$

WHERE σ^j ARE THE PAULI MATRICES.

DIRAC EQUATION WITH THE ELECTRIC AND MAGNETIC FIELD

FOR THIS PURPUSE WE MUST INTRODUCE THE POTENTIAL ENERGY $V(\bar{r}) = -e\varphi$ RESULTING FROM THE ELECTROSTATIC POTENTIAL φ. THIS CAN BE REPRESENTED BY ADOPTING THE FOLLOWING EXPRESSION

1) $$i\hbar\partial_t \to i\hbar\partial_t + e\varphi .$$

2) THE MAGNETIC FIELD IS TAKEN INTO ACCOUN BY REPLACING THE MOMENTUM OPERATOR

$$-i\hbar\bar{\nabla} \to -i\hbar\bar{\nabla} + e\bar{A}$$

WHERE $\bar{A}$ IS THE VECTOR POTENTIAL.

THE RESULTING DIRAC EQ. HAS BEEN SOLVED FOR SEVERAL CASES, IN PARTICULAR FOR THE H-ATOM, BESIDES THE QED EFFECTS (LAMB SHIFT)

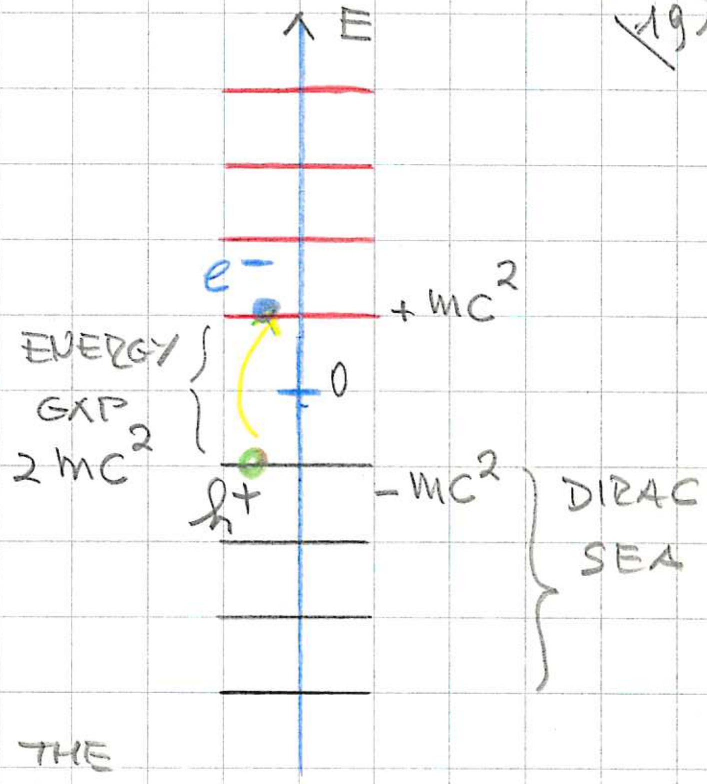
HOWEVER, THIS ISSUE IS BEYOND THE SCOPE OF THESE LECTURER

- OBSERVATION

IN SPITE OF THE ADVANCEMENT INTRODUCED BY REFORMULATING THE S-EQ ON THE BASE OF THE RELATIVISTIC MECHANICS (DIRAC-EQ), THE QUESTION OF THE MEANING OF THE NEGATIVE ENERGY VALUES FOR A FREE PARTICLES REMAINS OPEN. AT A FIRST GLANCE IT WOULD LOOK THAT AN ELECTRON WITH POSITIVE ENERGY CAN DROP DOWN TO NEGATIVE ENERGY LEVELS EMITTING LIGHT. THUS ALL THE PARTICLES WITH POSITIVE ENERGY WILL FINALLY FALL INTO THIS NEGATIVE ENERGY CHASM (ABYSS). TO SOLVE THIS QUESTION DIRAC ASSUMED THAT ALL THE NEGATIVE ENERGY STATES WHERE ALREADY OCCUPIED WITH ELECTRONS AND EACH STATE COULD BE OCCUPIED BY TWO PARTICLE WITH OPPOSITE SPIN (PAULI). SINCE POSITIVE AND NEGATIVE CHARGES MUST BE BALANCED THIS INFINITE NEGATIVE PARTICLES SEE (DIRAC SEA) MUST BE COMPENSATED BY THE INFINITE POSITIVE CHARGES OF PROTONS, THAT ALSO CAN BE DESCRIBED BY THE DIRAC THEORY. THE VACUUM WOULD, IN THIS INTERPRETATION, CONSIST OF THE TWO DIRAC SEAS.

IF WE ADD SUFFICIENT ENERGY THAT AN ELECTRON FROM THE DIRAC SEA COULD CROSS OVER THE $2mc^2$ ENERGY GAP AN e^- WITH POSITIVE ENERGY

WOUD APPEAR, LEAVING BEHIND A HOLE (e^+) IN THE DIRAC SEA.



SINCE THE HOLE IS A MISSING NEGATIVE CHARGE, BUT THE DIRAC SEA WAS PREVIOUSLY NEUTRAL THE HOLE ACTS LIKE A POSITIVE CHARGE WITH THE SAME PROPERTIES OF ITS OPPOSITE CHARGE PARTICLES, FOR THE CASE OF THE e^- ITS ANTIPARTICLE IS THE POSITRON e^+. THE ANTI-PARTICLES NOT ONLY HAVE BEEN FOUND - GIVING RISE TO THE FIELD OF THE ANTI-MATTER BUT ALSO THEIR INTERACTION IS AN IMPORTANT SECTOR OF THE MODERN PARTICLE PHYSICS.

HERE WE CAN BRIEFLY REVIEW TWO CASES, THE LOW ENERGY $e^- + e^+ \to \gamma + \gamma$ COLLISION WITH THE CREATION OF TWO PHOTONS γ, EACH OF ENERGY 0.511 MeV EQUIVALENT TO $m_e c^2$ THE REST MASS OF THE e^- AND e^+, AND THE HIGH ENERGY CASE.

IN THIS LAST CASE IF e^- AND e^+ HAVE A KINETIC ENERGY THAT OVERTAKES THE REST MASS EQUIVALENT ENERGY OF OTHER HEAVIER PARTICLES THESE COULD BE PRODUCED, OR ALTERNATIVELY IT IS POSSIBLE TO PRODUCE PHOTONS OR OTHER LIGHT PARTICLES THAT WILL EMERGE WITH A KINETIC ENERGY AND

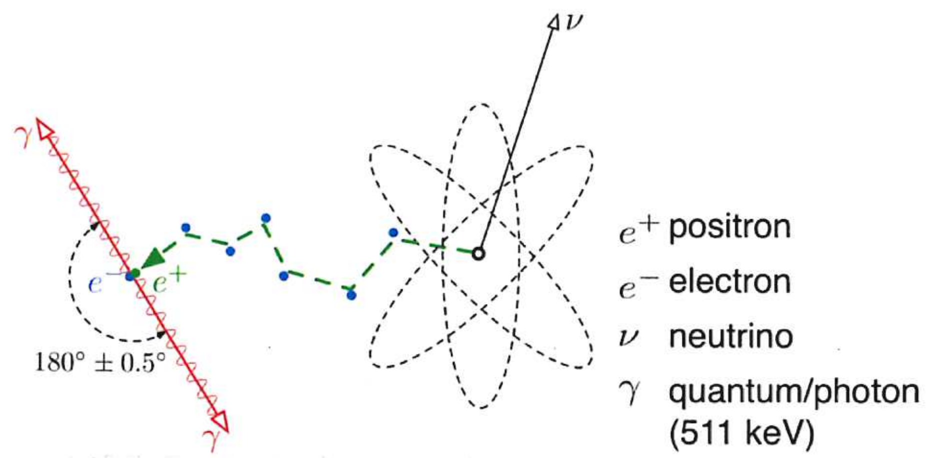
TO MATCH THE INITIAL ENERGY AND MOMENTUM BY THE CONSERVATION LAWS.

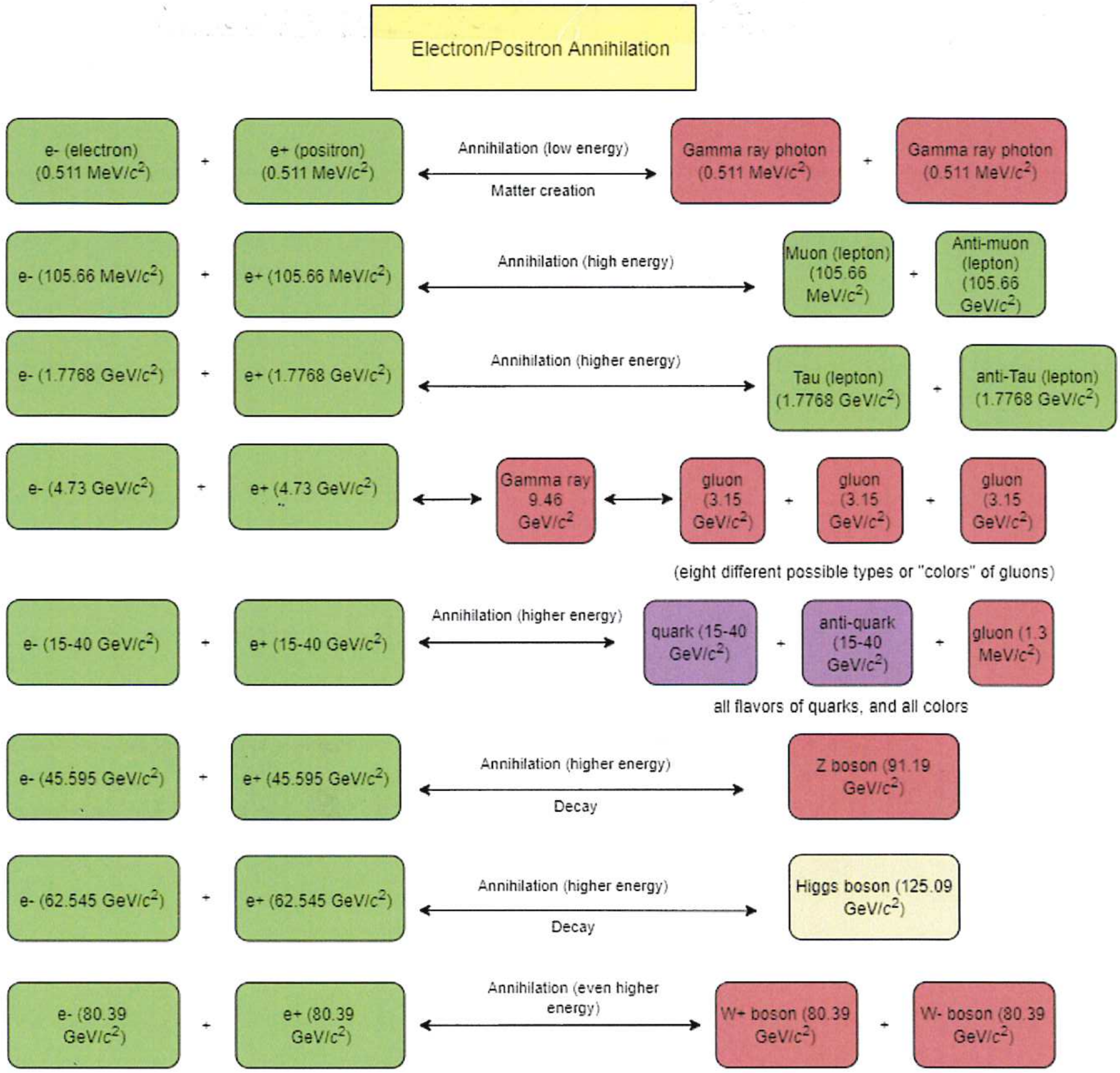
IN THESE PROCESSES (ALL) THE CONSERVATION LAWS MUST BE FULFILLED:

1 - CHARGE CONSERVATION
2 - CONSERVATION OF MOMENTUM AND ENERGY
3 - CONSERVATION OF ANGULAR MOMENTUM
4 - CONSERVATION OF TOTAL LEPTON NUMBER.

FINALLY IT IS INTERESTING TO POINT OUT THAT AT ENERGIES ABOVE THE MASS OF THE CARRIERS OF THE ELECTRO-WEAK FORCE, THE W AND Z BOSONS, THE STRENGTH OF THE WEAK FORCE BECOMES COMPARABLE TO THE ELECTROMAGNETIC FORCE.

RECENTLY THE HEAVIEST PARTICLE PRODUCED WAS THE HIGGS BOSON $m \simeq 125.09\ GeV/c^2$, WHILE BEFORE W^+-W^- PAIRS ($m \simeq 80.385\ GeV/c^2$) AND CHARGED Z BOSON ($m \simeq 91.188\ GeV/c^2$) HAVE BEEN OBSERVED




Electron/Positron Annihilation
e- (electron) (0.511 MeV/c²)
+
e+ (positron) (0.511 MeV/c²)
Annihilation (low energy)
Matter creation
Gamma ray photon (0.511 MeV/c²)
+
Gamma ray photon (0.511 MeV/c²)
e- (105.66 MeV/c²)
+
e+ (105.66 MeV/c²)
Annihilation (high energy)
Muon (lepton) (105.66 MeV/c²)
+
Anti-muon (lepton) (105.66 GeV/c²)
e- (1.7768 GeV/c²)
+
e+ (1.7768 GeV/c²)
Annihilation (higher energy)
Tau (lepton) (1.7768 GeV/c²)
+
anti-Tau (lepton) (1.7768 GeV/c²)
e- (4.73 GeV/c²)
+
e+ (4.73 GeV/c²)
Gamma ray 9.46 GeV/c²
gluon (3.15 GeV/c²)
+
gluon (3.15 GeV/c²)
+
gluon (3.15 GeV/c²)
(eight different possible types or "colors" of gluons)
e- (15-40 GeV/c²)
+
e+ (15-40 GeV/c²)
Annihilation (higher energy)
quark (15-40 GeV/c²)
+
anti-quark (15-40 GeV/c²)
+
gluon (1.3 MeV/c²)
all flavors of quarks, and all colors
e- (45.595 GeV/c²)
+
e+ (45.595 GeV/c²)
Annihilation (higher energy)
Decay
Z boson (91.19 GeV/c²)
e- (62.545 GeV/c²)
+
e+ (62.545 GeV/c²)
Annihilation (higher energy)
Decay
Higgs boson (125.09 GeV/c²)
e- (80.39 GeV/c²)
+
e+ (80.39 GeV/c²)
Annihilation (even higher energy)
W+ boson (80.39 GeV/c²)
+
W- boson (80.39 GeV/c²)