Cyber-Physical Systems

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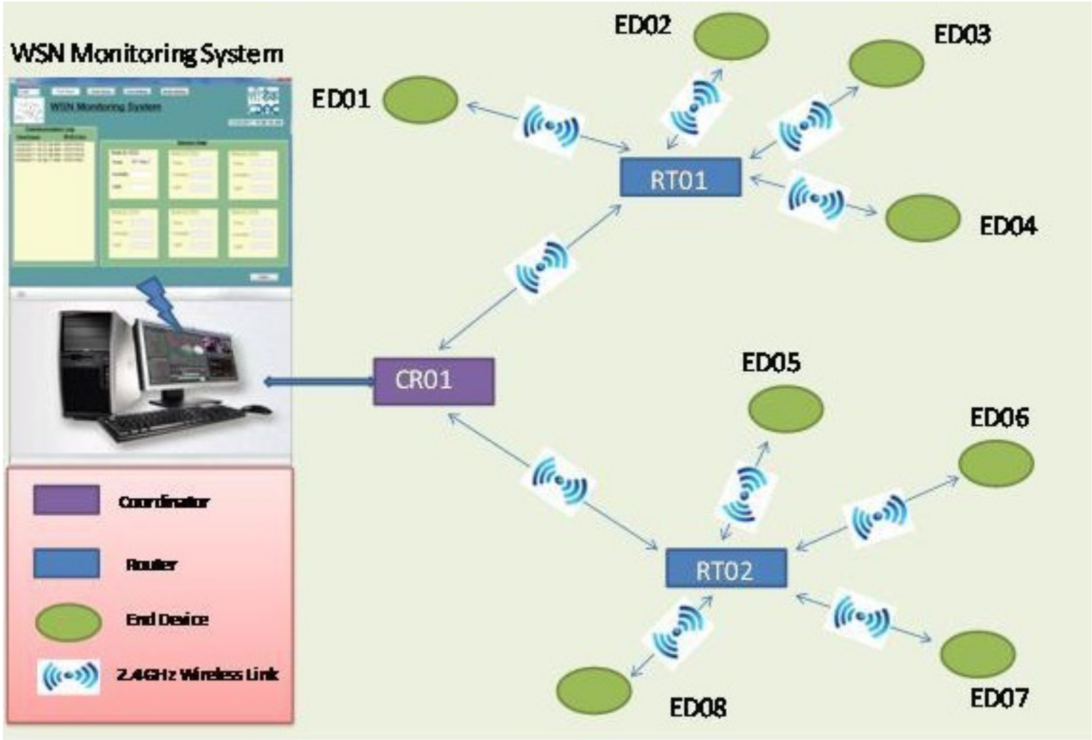
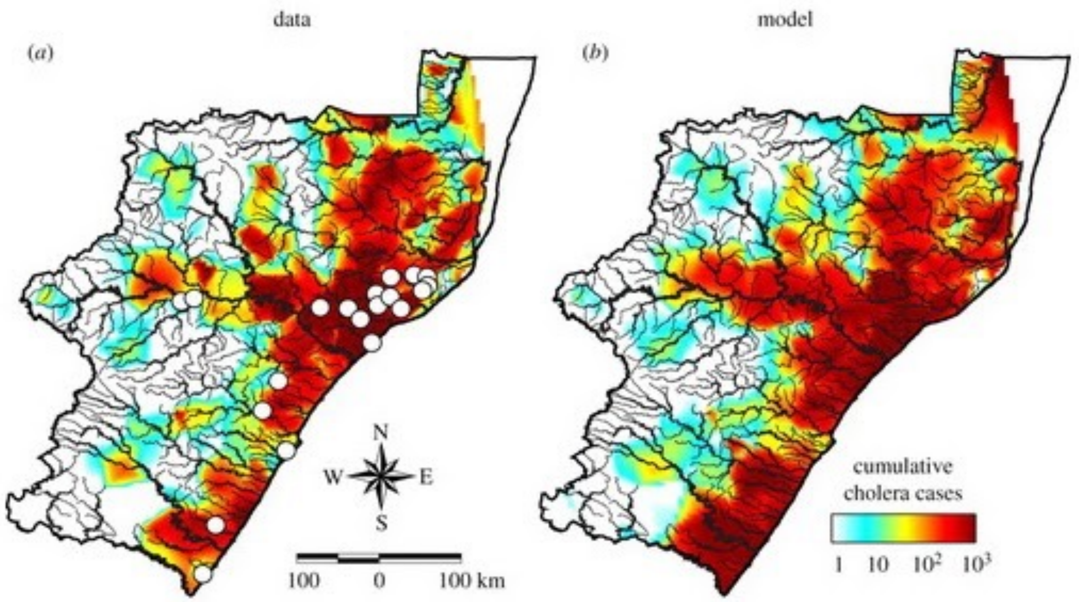
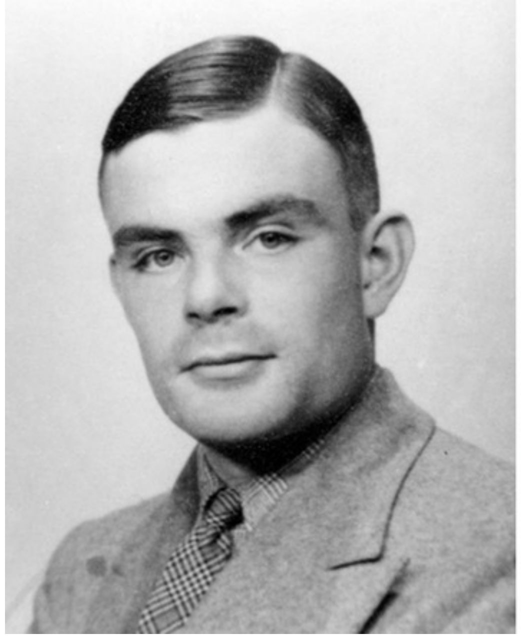
Università degli Studi di Trieste
II Semestre 2020

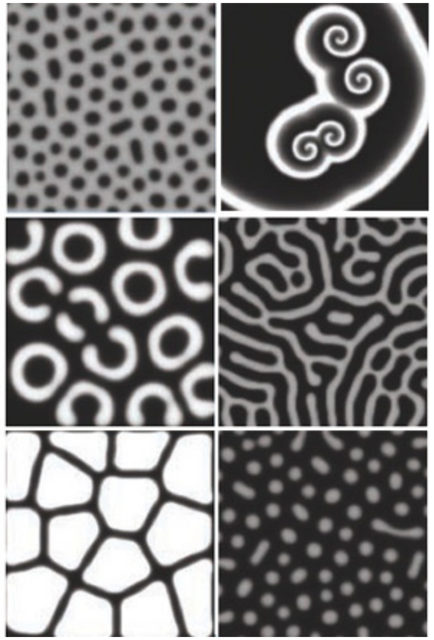
Lecture 17 STREL: Spatio-Temporal Reach and Escape Logic


WSN Monitoring System
ED01
ED02
ED03
ED04
ED05
ED06
ED07
ED08
RT01
RT02
CR01
Coordinator
Router
End Device
2.4GHz Wireless Link


MELBOURNE
$5 helmets are closer than you think

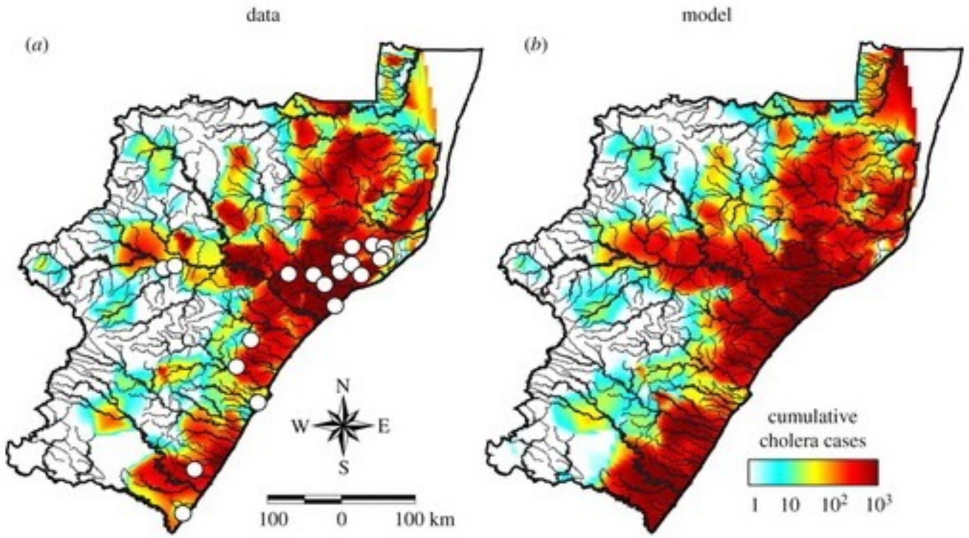

data
model
(a)
(b)
N
W
E
S
100
0
100 km
cumulative cholera cases
1
10
10²
10³

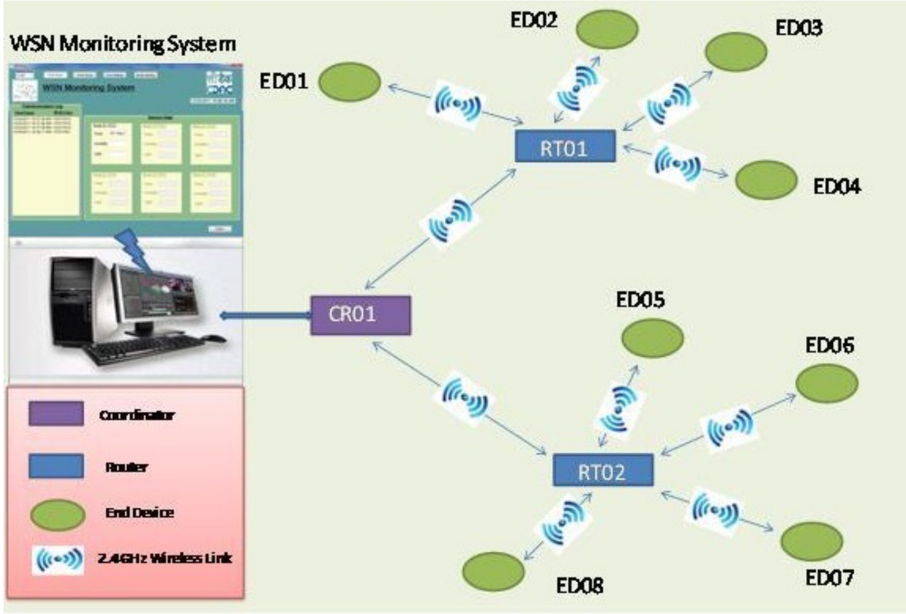




Availability: I can always find a station with at least one bike in a radius of 500 meters

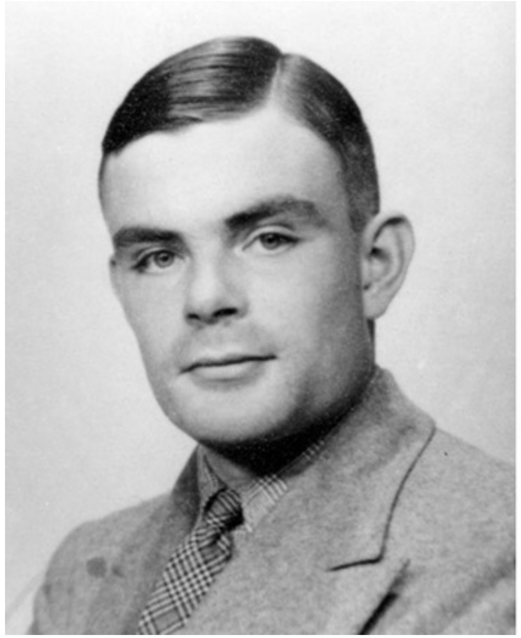
Spread: after 10 time units, there exists a location l′ at a certain distance from location l where the number of infected individuals is more than 50

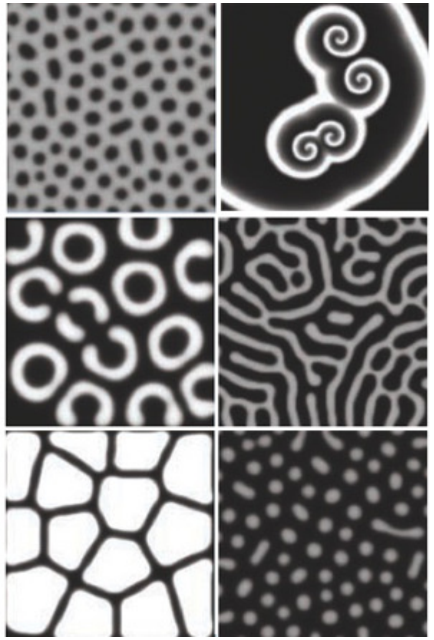




Reliability: we can always find a path of sensors such that all sensors have a battery level greater than 0.5

Spots: regions with low density of protein A are always surrounded by regions with high level of protein B





How to specify such spatio-temporal behaviours in a formal and human-understandable language ?

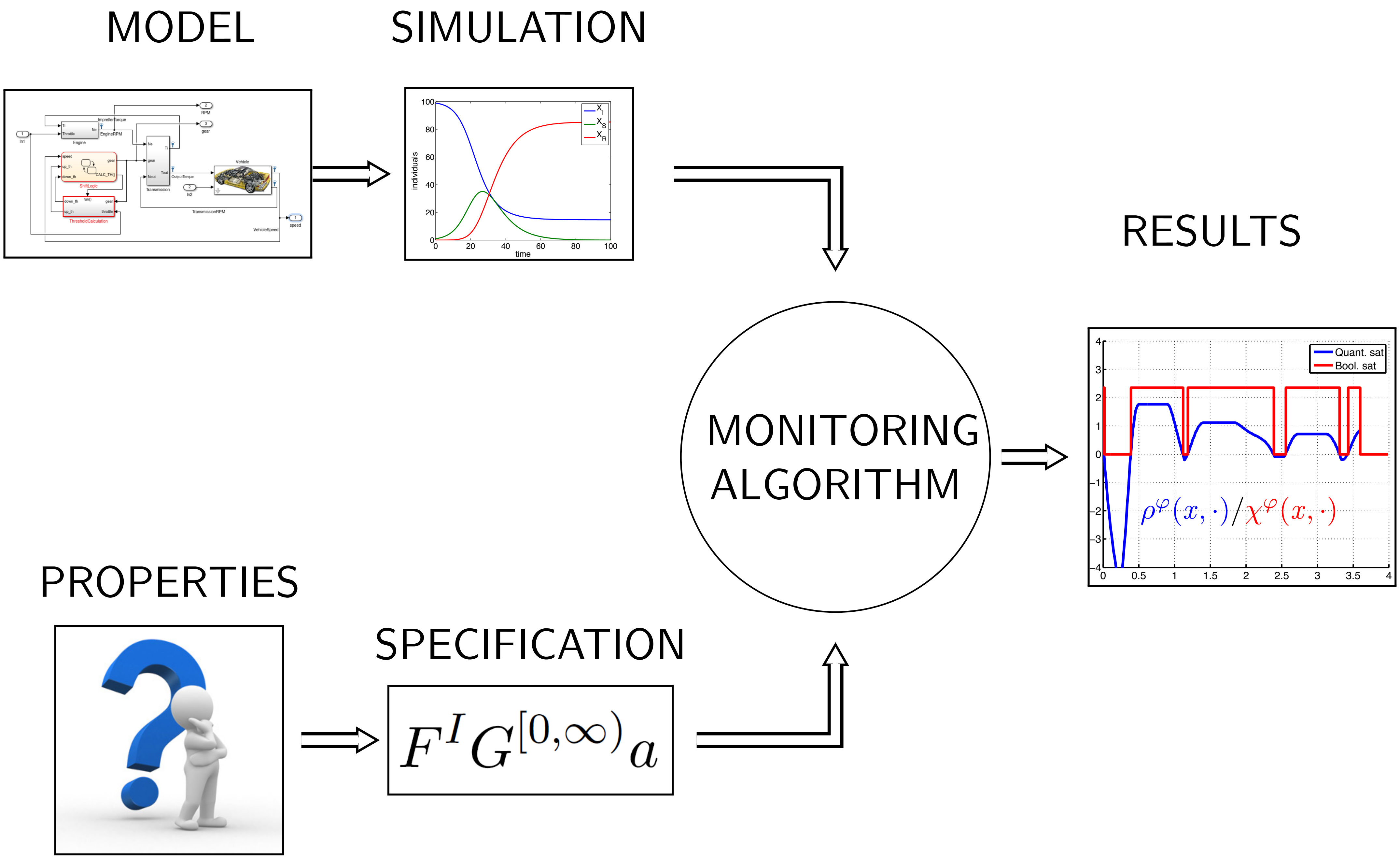
How to monitor their onset efficiently?

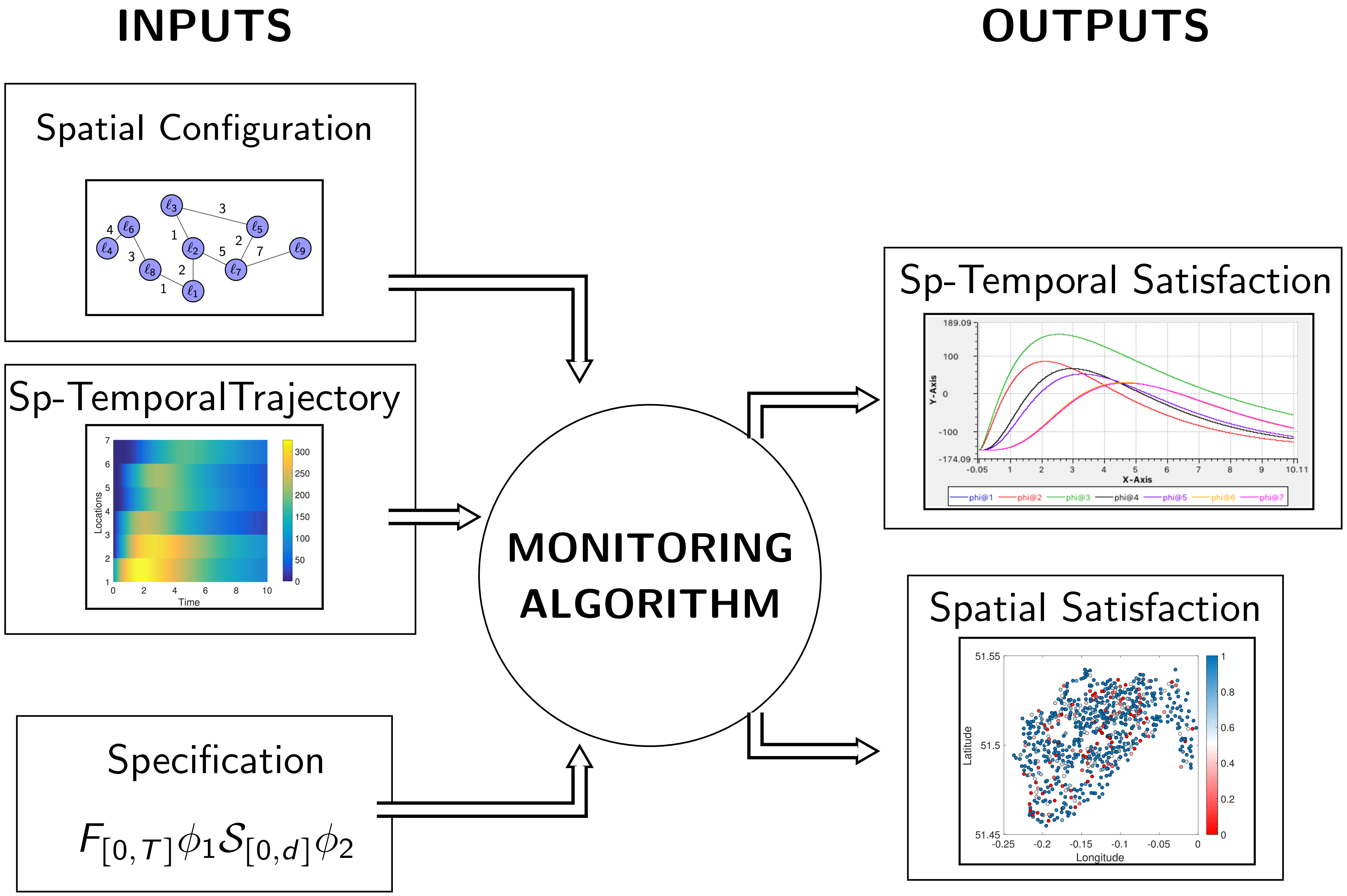
Part 1 :

- Signal Temporal Logic (STL)
- Space Model and traces
- Spatio- Temporal Reach and Escape Logic (STREL)

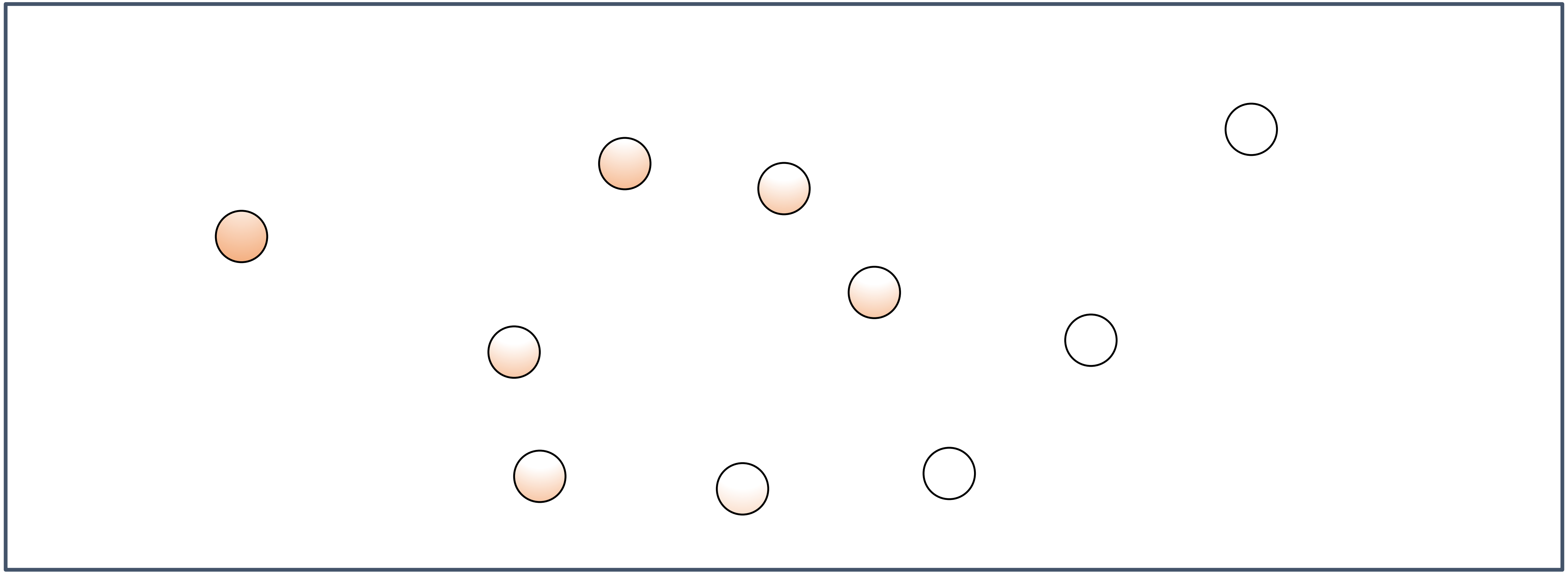
Part 2:

- Monitoring
- Applicability to different scenarios


MODEL
SIMULATION
individuals
time
RESULTS
MONITORING ALGORITHM
Quant. sat
Bool. sat
$\rho^{\varphi}(x,\cdot)/\chi^{\varphi}(x,\cdot)$
PROPERTIES
SPECIFICATION
$F^{I}G^{[0,\infty)}a$


INPUTS
OUTPUTS
Spatial Configuration
Sp-TemporalTrajectory
Specification
$F_{[0,T]}\phi_1 \mathcal{S}_{[0,d]}\phi_2$
MONITORING ALGORITHM
Sp-Temporal Satisfaction
Spatial Satisfaction
Time
Locations
X-Axis
Y-Axis
Latitude
Longitude

Running Example: Wireless Sensor Network



Space Model, Signal and Traces

Spatial Configuration

We consider a discrete space described as a weighted (direct) graph

Reasons:

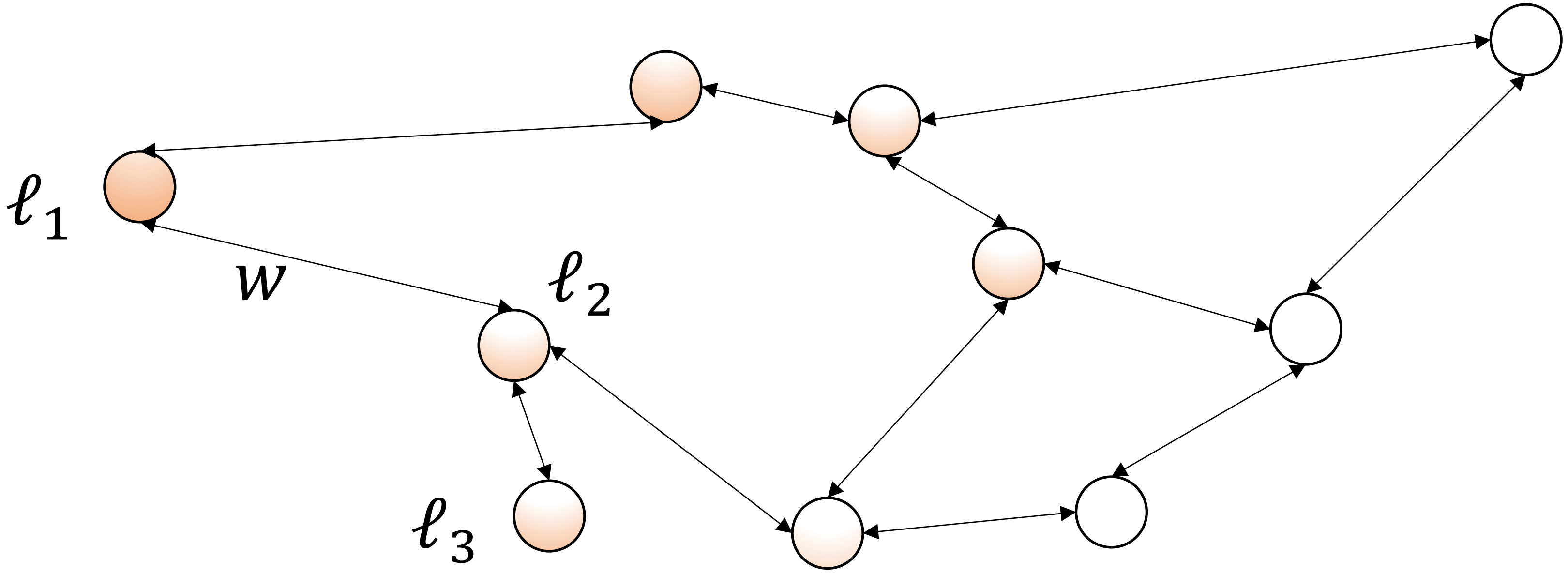
- many applications, like bike sharing systems, smart grid and sensor networks are naturally framed in a discrete spatial structure

- in many circumstances continuous space is abstracted as a grid or as a mesh, e.g. numerical integration of PDEs

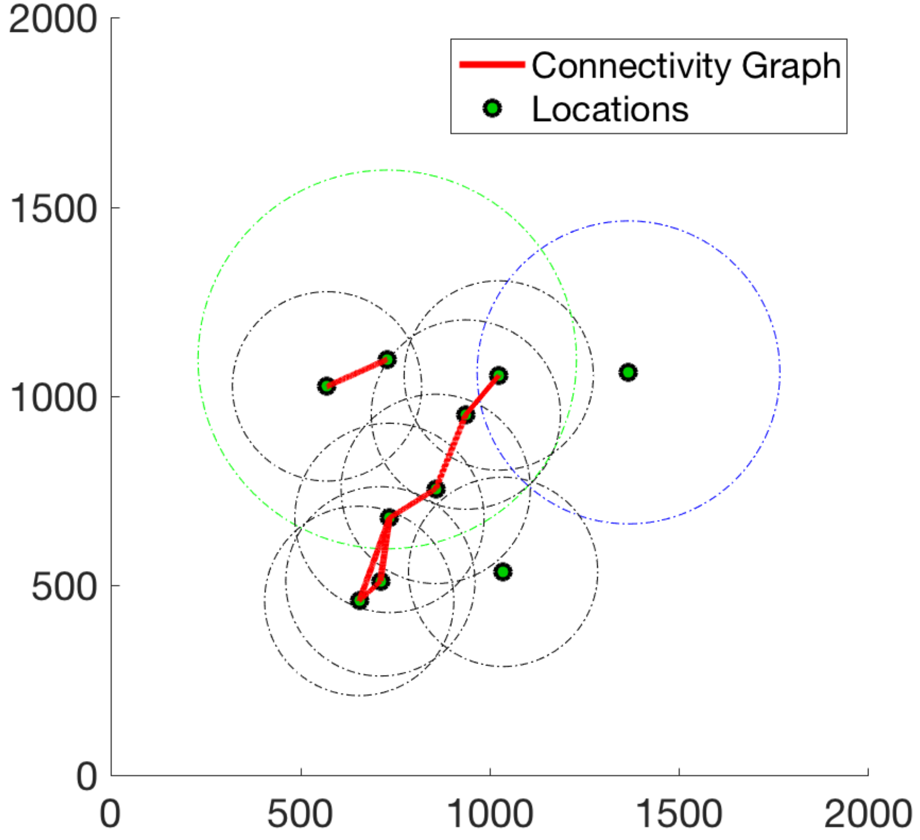
Space Model $S = \langle L, W \rangle$

– L is a set of nodes that we call locations;

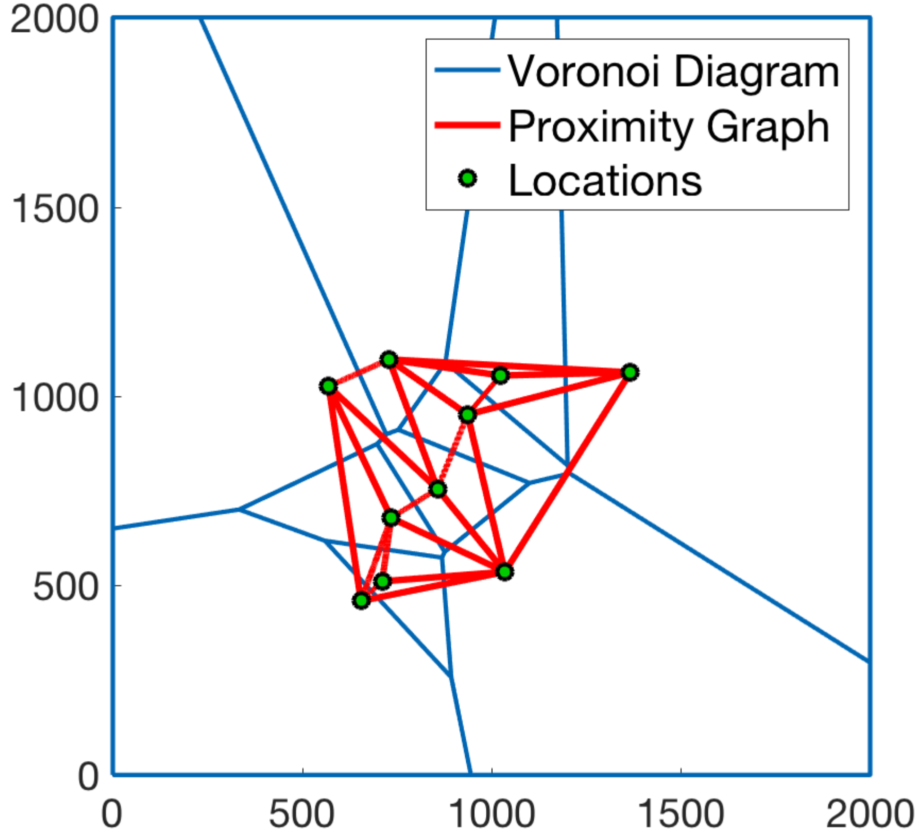
– $W \subseteq L \times \mathbb{R} \times L$ is a proximity function associating a label $w \in \mathbb{R}$ to distinct pair $\ell_1, \ell_2 \in L$. If $(\ell_1, w, \ell_2) \in W$, it means that there is an edge from ℓ_1 to ℓ_2 with weight $w \in \mathbb{R}$



Example



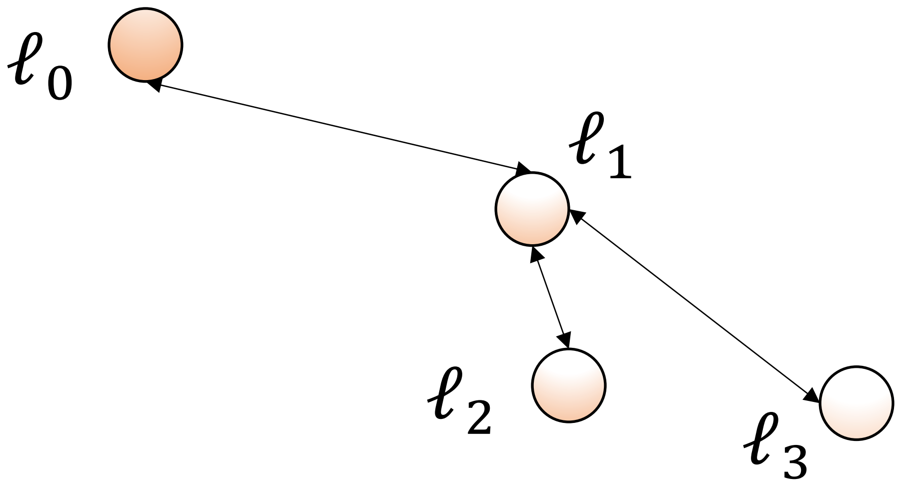
Connectivity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000



Voronoi Diagram
Proximity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000

Route $\tau = \ell_0 \ell_1 \ell_2 \dots$

It is a infinite sequence s.t. $\forall i \geq 0 \; \exists \, w \; s.t. (\ell_i, w, \ell_{i+1}) \in W$



$\ell_0 \ell_1 \ell_2 \ell_1 \dots$ is a route

$\ell_0 \ell_1 \ell_2 \ell_3 \dots$ is a not route

$\tau[i]$ to denote the $i - th$ node τ

$\tau(\ell)$ to denote the first occurrence of $\ell \in \tau$

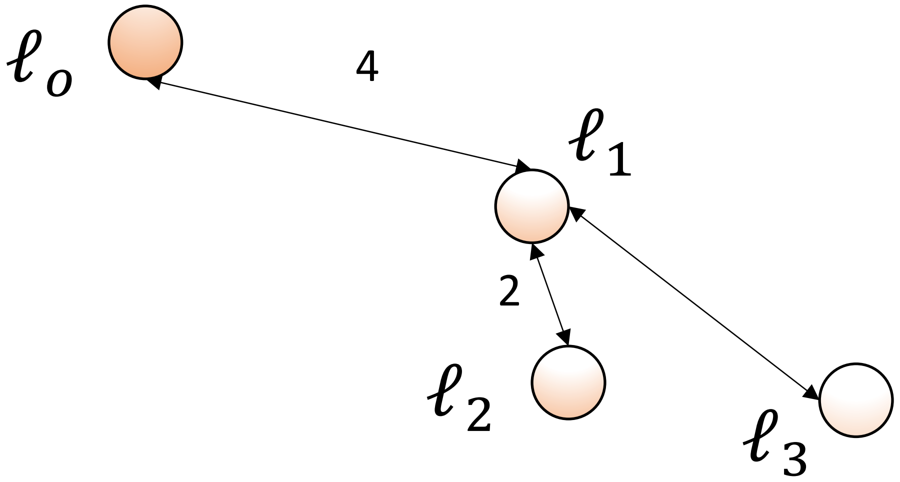
Route Distance $d_\tau^f[i]$

The distance $d_\tau^f[i]$ up to index i is:

$$d_\tau^f[i] = \begin{cases} 0 & i = 0 \\ f(d_{\tau[1..]}^f[i-1], w) & (i > 0)\ and\ \tau[0] \overset{w}{\mapsto} \tau[1] \end{cases}$$

$$d_\tau^f(\ell) = d_\tau^f[\tau(\ell)]$$

Route Distance $d_\tau^f[i]$



$weight(x, y) = x + y$

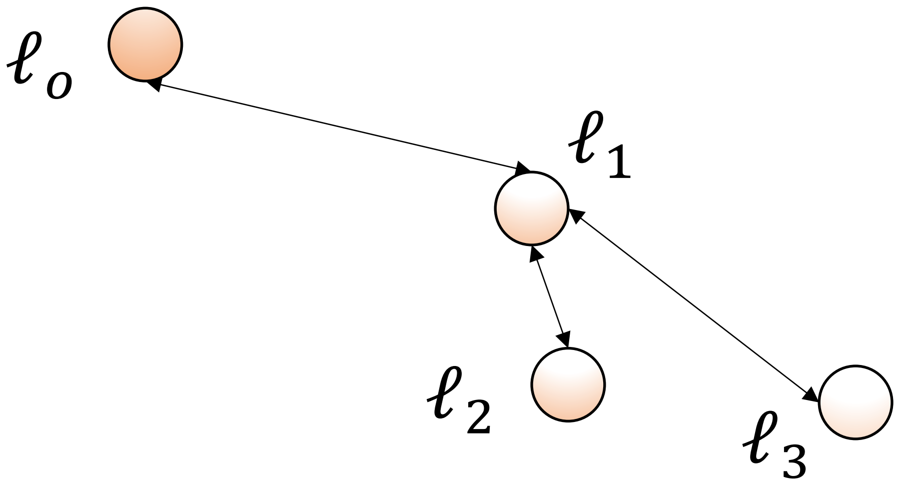
$hops(x, y) = x + 1$

$d_{\ell_0\ell_1\ell_2..}^{weight}[2] =$ weight($d_{\ell_1\ell_2..}^{weight}[1]$, 4) = $d_{\ell_1\ell_2}^{weight}[1]$ + 4 = …

$=$ weight($d_{\ell_2..}^{weight}[0]$, 2) + 4 = 6

Location Distance $d_S^f[\ell_i, \ell_j]$

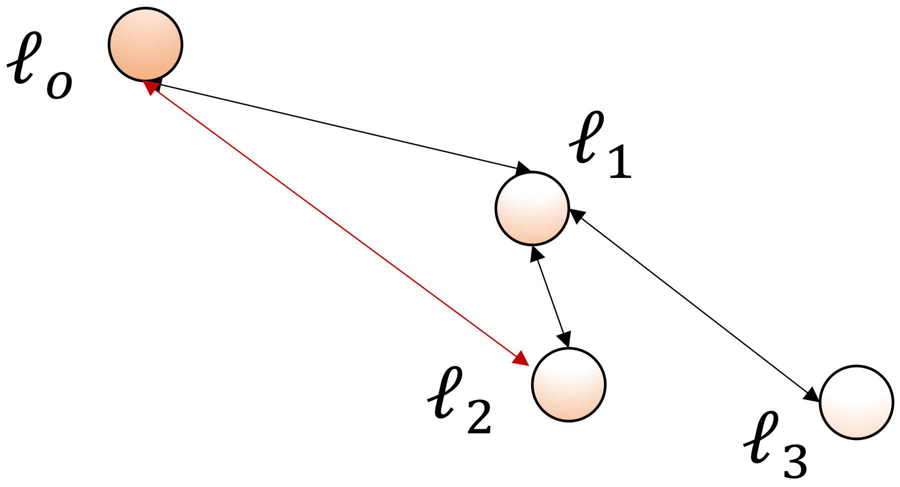
$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] \mid \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 2**

Location Distance

$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] \mid \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 1**

Signal and Trace

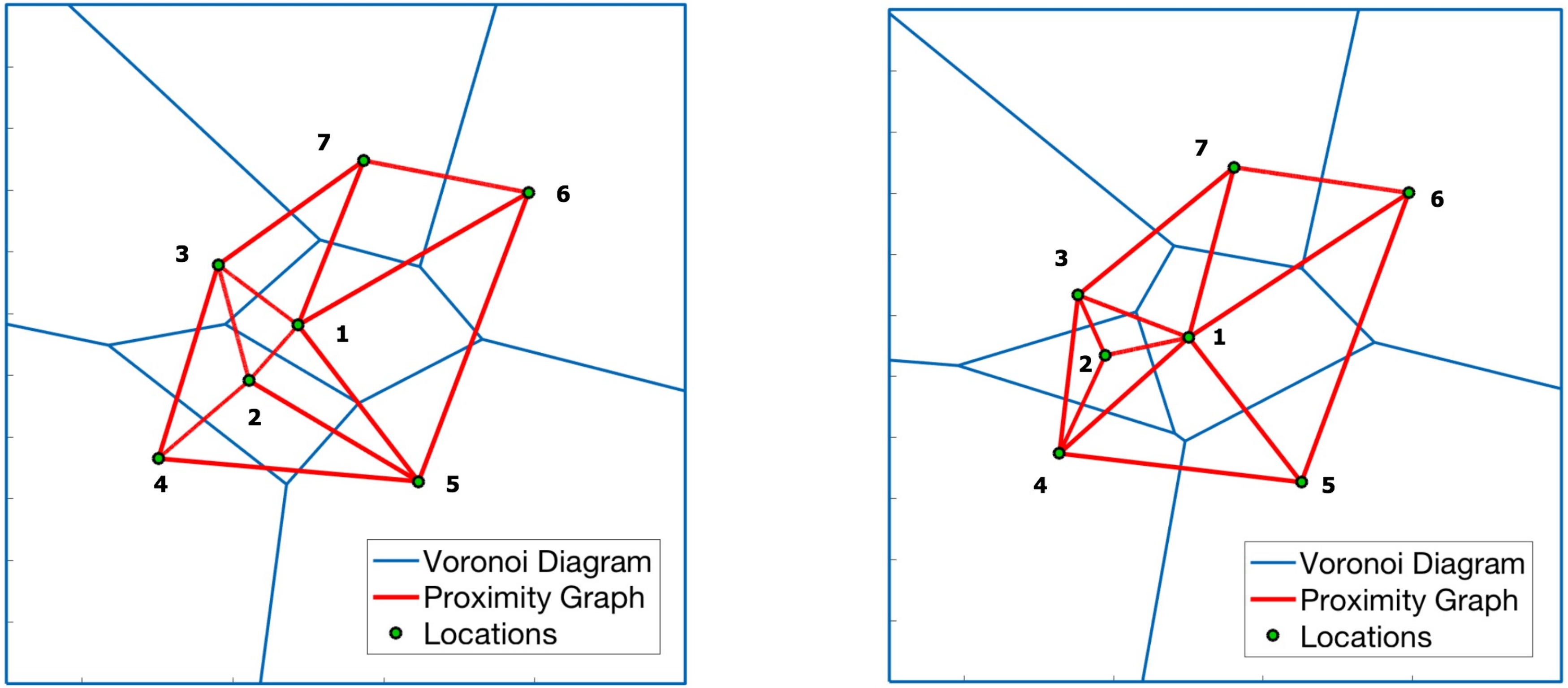
Spatio-Temporal Signals $\sigma: L \to \mathbb{T} \to D$

Spatio-Temporal Trace $\vec{\boldsymbol{x}}: L \to \mathbb{T} \to D^n$

$$x(\ell) = (\nu_B , \nu_T)$$
$$x(\ell, t) = (\nu_B(t) , \nu_T(t))$$

Dynamic Spatial Model

(t_i, S_i) for $i = 1, \ldots, n$ and $S(t) = S_i \forall t \in [t_i, t_{i+1})$



STREL

Spatio- Temporal Reach and Escape Logic (STREL)

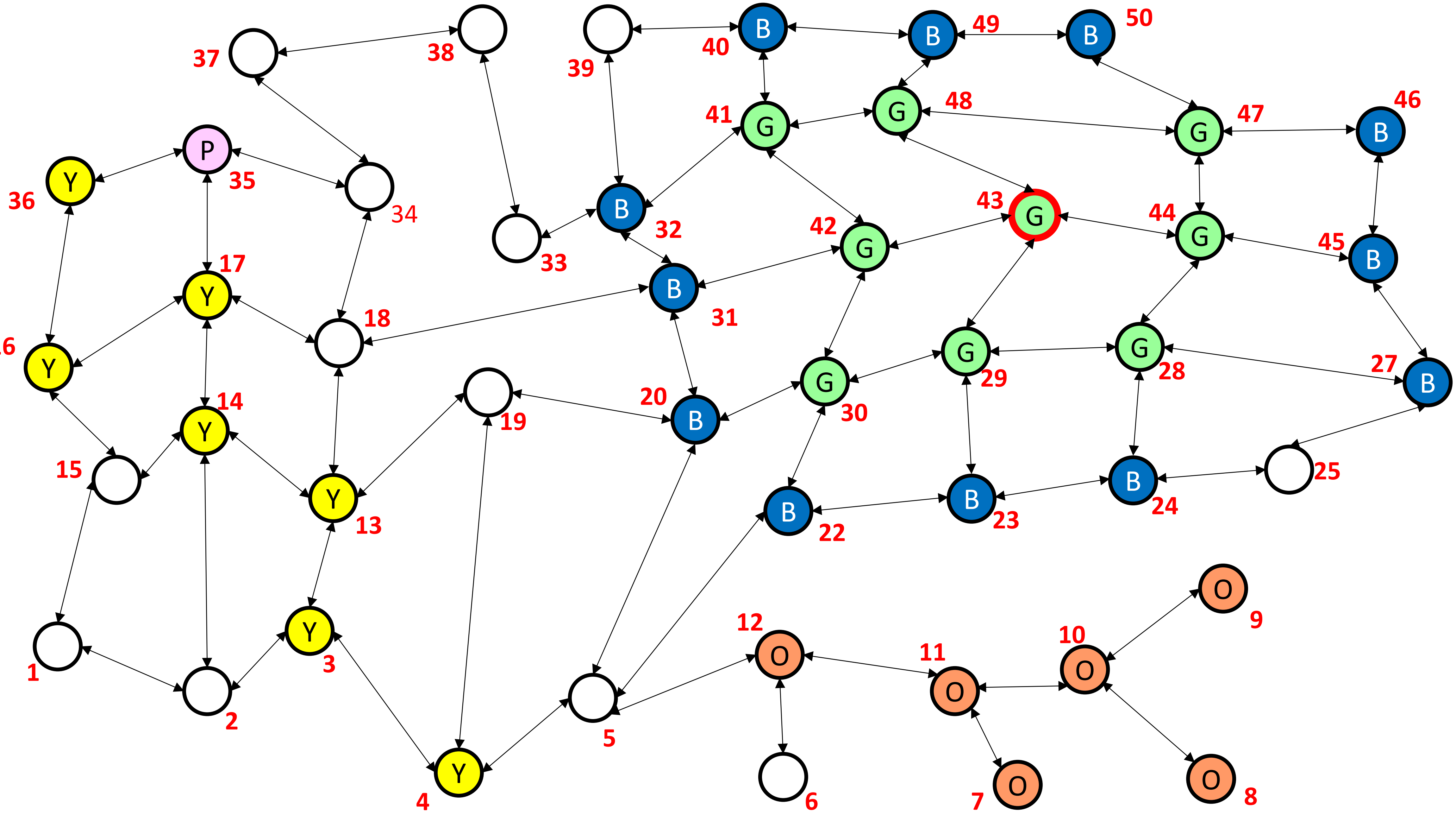
It is an extension of the Signal Temporal Logic with a number of spatial modal operators

STREL Syntax

$$\varphi := true \mid \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \, \mathrm{U}_I \, \varphi_2 \mid \varphi_1 \, \mathrm{S}_I \, \varphi_2 \mid \varphi_1 \, \mathcal{R}_d^f \, \varphi_2 \mid \mathcal{E}_d^f \, \varphi$$

In addition, we can derive:

- The disjunction operator: $\vee$
- the temporal operators: $F_I, G_I, O_I, \mathrm{H_I}$
- the spatial operators: somewhere, everywhere and surround

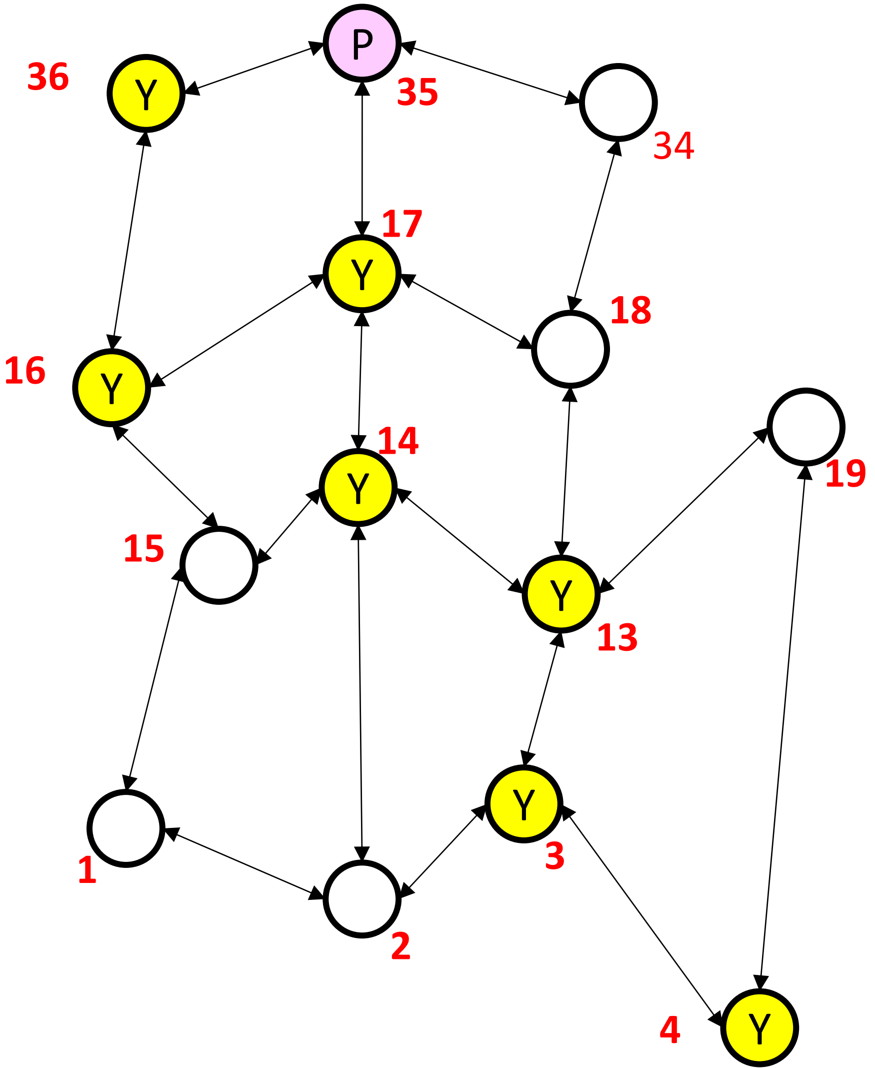


1
2
3 Y
4 Y
5
6
7 O
8 O
9 O
10 O
11 O
12 O
13 Y
14 Y
15
16 Y
17 Y
18
19
20 B
22 B
23 B
24 B
25
27 B
28 G
29 G
30 G
31 B
32 B
33
34
35 P
36 Y
37
38
39
40 B
41 G
42 G
43 G
44 G
45 B
46 B
47 G
48 G
49 B
50 B

Reach: $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$

$(S, \vec{x}, \ell, t)$ satisfies $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$ iff it satisfies φ_2 in a location ℓ' reachable from ℓ through a route τ, with a length $d_\tau^f[\ell'] \in [d_1, d_2]$ and such that $\tau[0] = \ell$ and all its elements with index less than $\tau(\ell')$ satisfy φ_1

Reach $yellow\mathcal{R}^{hops}_{[1,4]}pink$



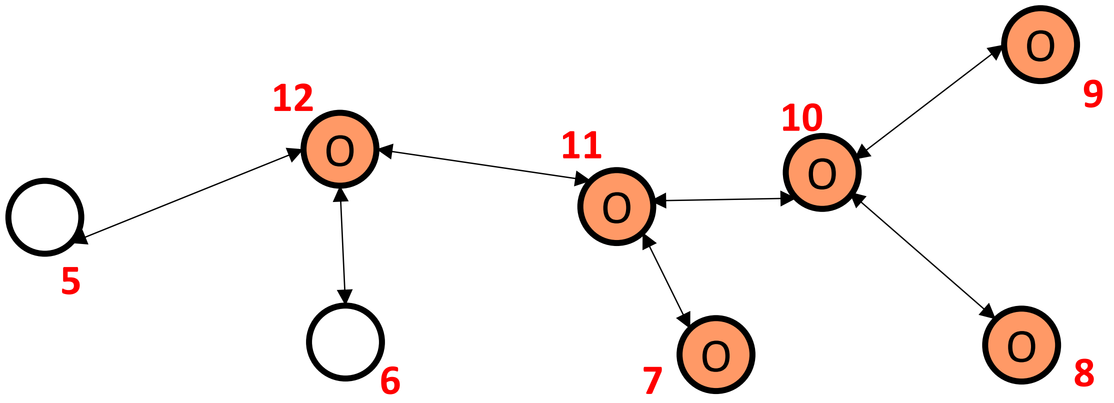
$$\tau = \ell_3 \ell_{13} \ell_{14} \ell_{17} \ell_{35}$$

$$d_\tau^{hops}[\ell_{35}] = 4, \text{ where } \tau[0] = \ell_3$$

Escape: $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$

$(S, \vec{x}, \ell, t)$ satisfies $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$ if and only there exists a route τ and a location $\ell' \in \tau$ such that $\tau[0] = \ell$, $d_S^f\,[\tau[0], \ell'] \in [d_1, d_2]$ and all elements $\tau[0], \dots \tau[k]$ (with $\tau(l') = k$) satisfy φ

Escape: $\mathcal{E}^{hops}_{[3,\infty]} orange$

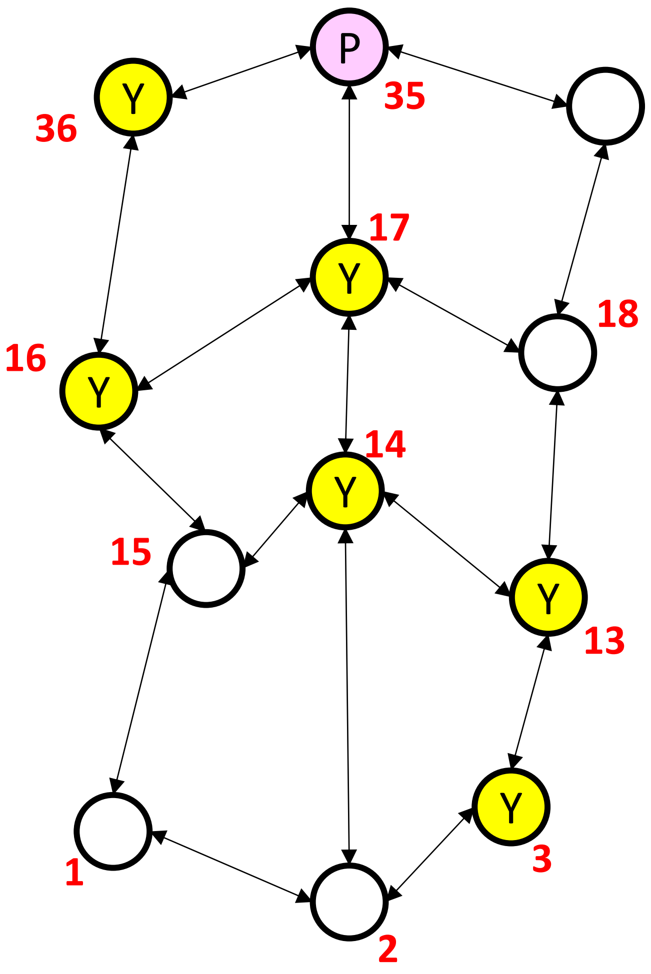


$$\tau = \ell_9 \ell_{10} \ell_{11} \ell_{12}$$

$$\tau[0] = \ell_9, \ \tau[3] = \ell_{12}$$

$$d_S^{hops}[\ell_9, \ell_{12}] = 3$$

Somewhere: $\Diamond^{f}_{[d_1,d_2]}\varphi := true\mathcal{R}^{f}_{[d_1,d_2]}\varphi$



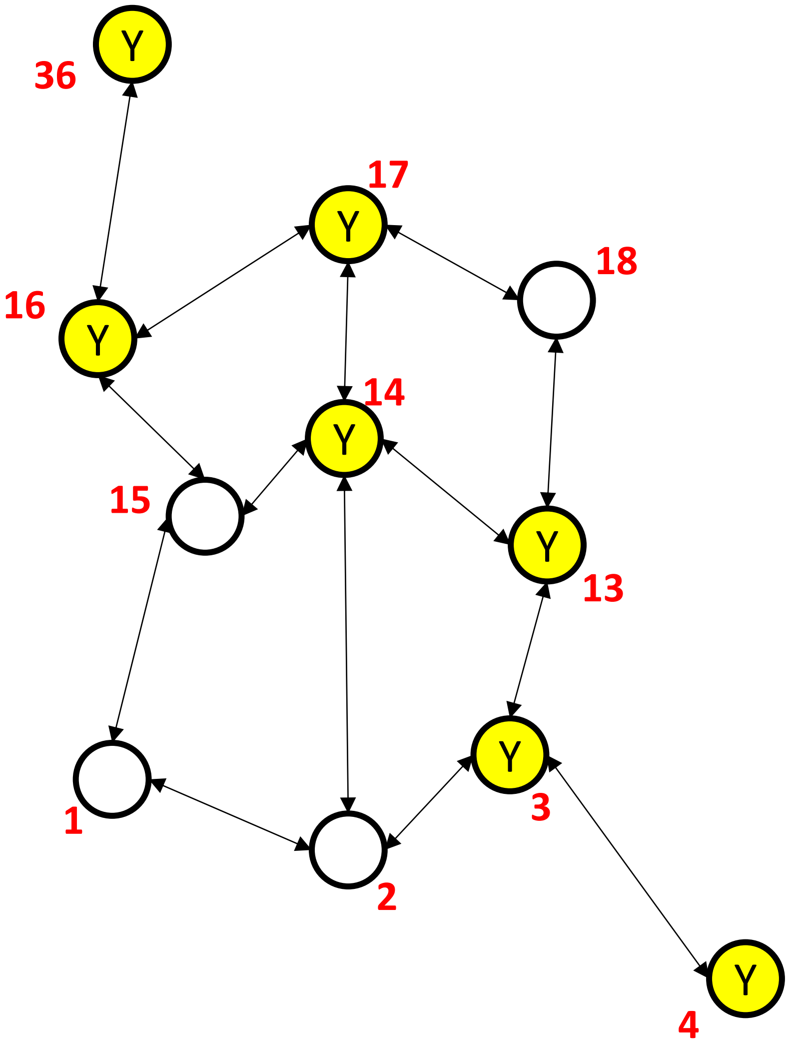
$$\Diamond^{hops}_{[3,5]} pink$$

$$\tau = \ell_1 \ldots \ell_{35}$$

$$\tau[0] = \ell_1,\ \tau[k] = \ell_{35}$$

$$d^{hops}_{\tau}(k) \in [3,5]$$

Everywhere: $\boxdot^{f}_{[d_1,d_2]}\varphi := \neg \diamondsuit^{f}_{[d_1,d_2]}\neg\varphi$



$(S, \vec{x}, \ell, t)$ satisfies iff all the locations ℓ' reachable from ℓ via a path, with length $d^{f}_{\tau}[\ell'] \in [d_1, d_2]$ satisfy φ

$$\boxdot^{hops}_{[2,3]} yellow$$

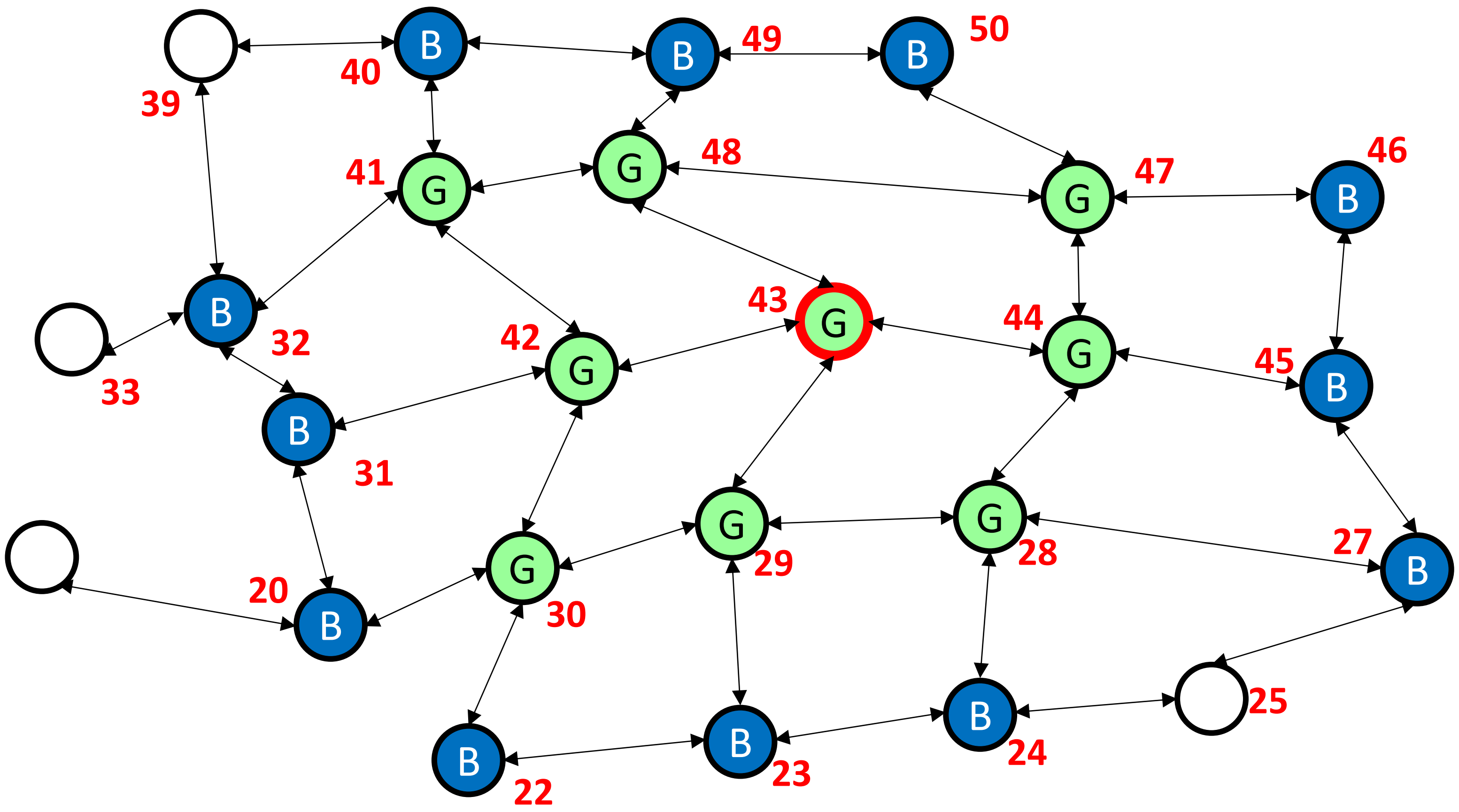
Surround: $\varphi_1 \circledcirc^{f}_{[d_1,d_2]} \varphi_2$

$$\varphi_1 \circledcirc^{f}_{[d_1,d_2]} \varphi_2 := \varphi_1 \wedge \neg(\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \neg(\varphi_1 \vee \varphi_2) \wedge \neg(\mathcal{E}^{f}_{[d_2,\infty]}(\varphi_1))$$

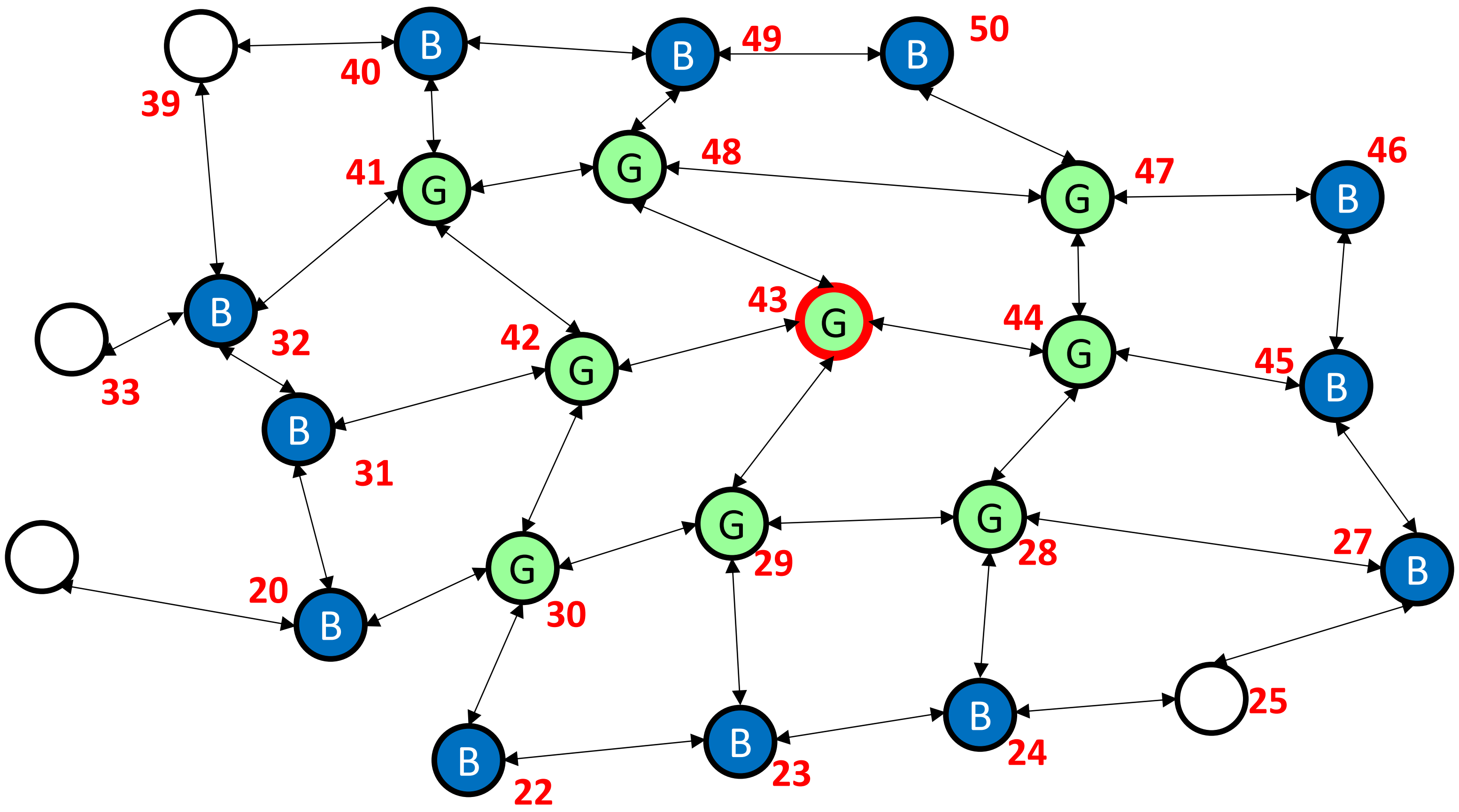
$(S, \vec{x}, \ell, t)$ iff there exists a φ_1-region that contains ℓ, all locations in that region satisfies φ_1 and are reachable from ℓ via a path with length less than d_2 .

All the locations that do not belong to the φ_1-region but are directly connected to a location of that region must satisfy φ_2 and be reached from ℓ via a path with length in the interval $[\mathrm{d}_1, d_2]$.

Surround: $green \circledcirc^{hops}_{[0,100]} blue$



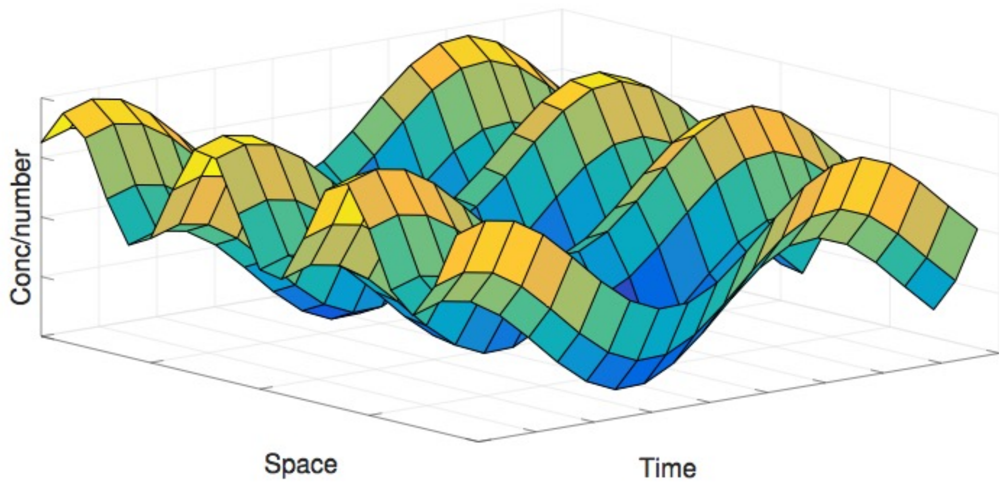
Surround: $green \circledcirc_{[2,3]}^{hops} blue$



Offline Monitoring Algorithm

Spatial Boolean Signal

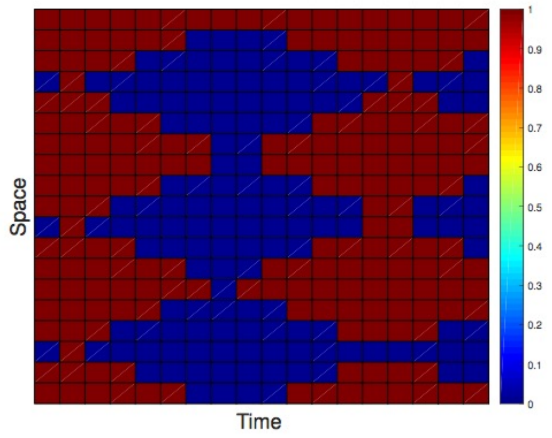
$s_\varphi : L \to [0, T] \to \{0, 1\}$ such that $s_\varphi(\ell, t) = 1 \Leftrightarrow (\mathcal{S}, \vec{x}, \ell, t) \models \varphi$



SSTL Monitor
Formula ϕ

$s_\varphi(\ell, 0)$

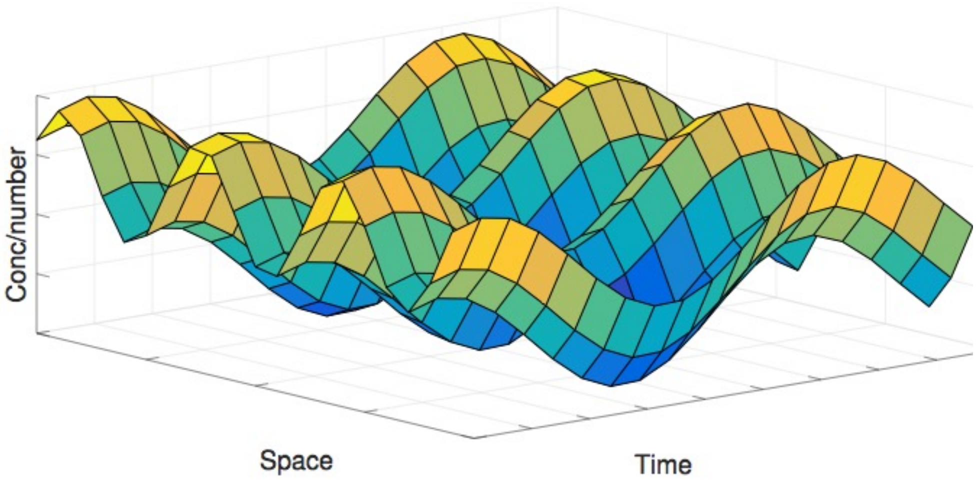
Spatial Boolean Satisfaction

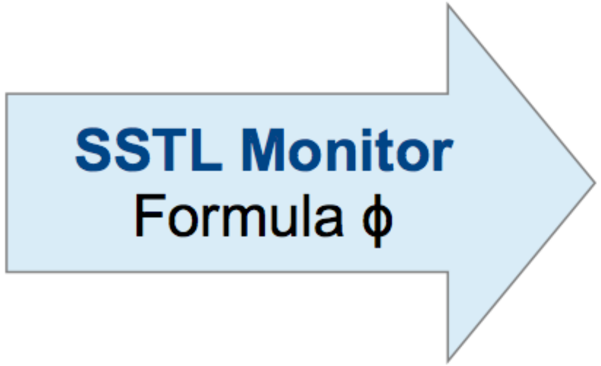


Offline Monitoring Algorithm

Spatial Quantitative Signal

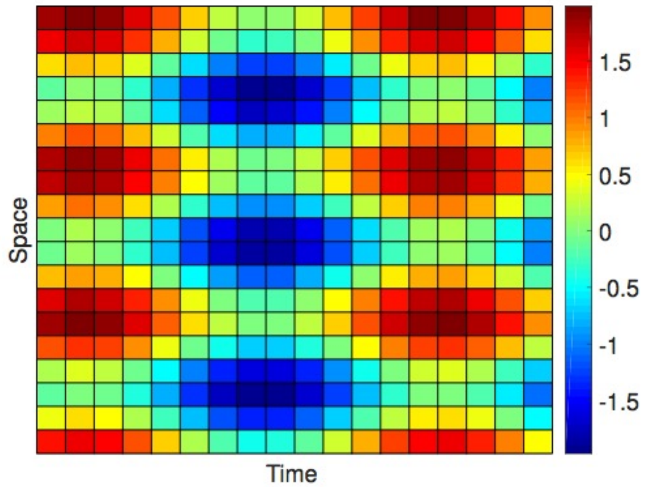
$\rho_\varphi : L \to [0, T] \to \mathbb{R} \cup \pm\infty$ such that $\rho_\varphi(\ell, t) = \rho(\mathcal{S}, \vec{x}, \ell, t)$

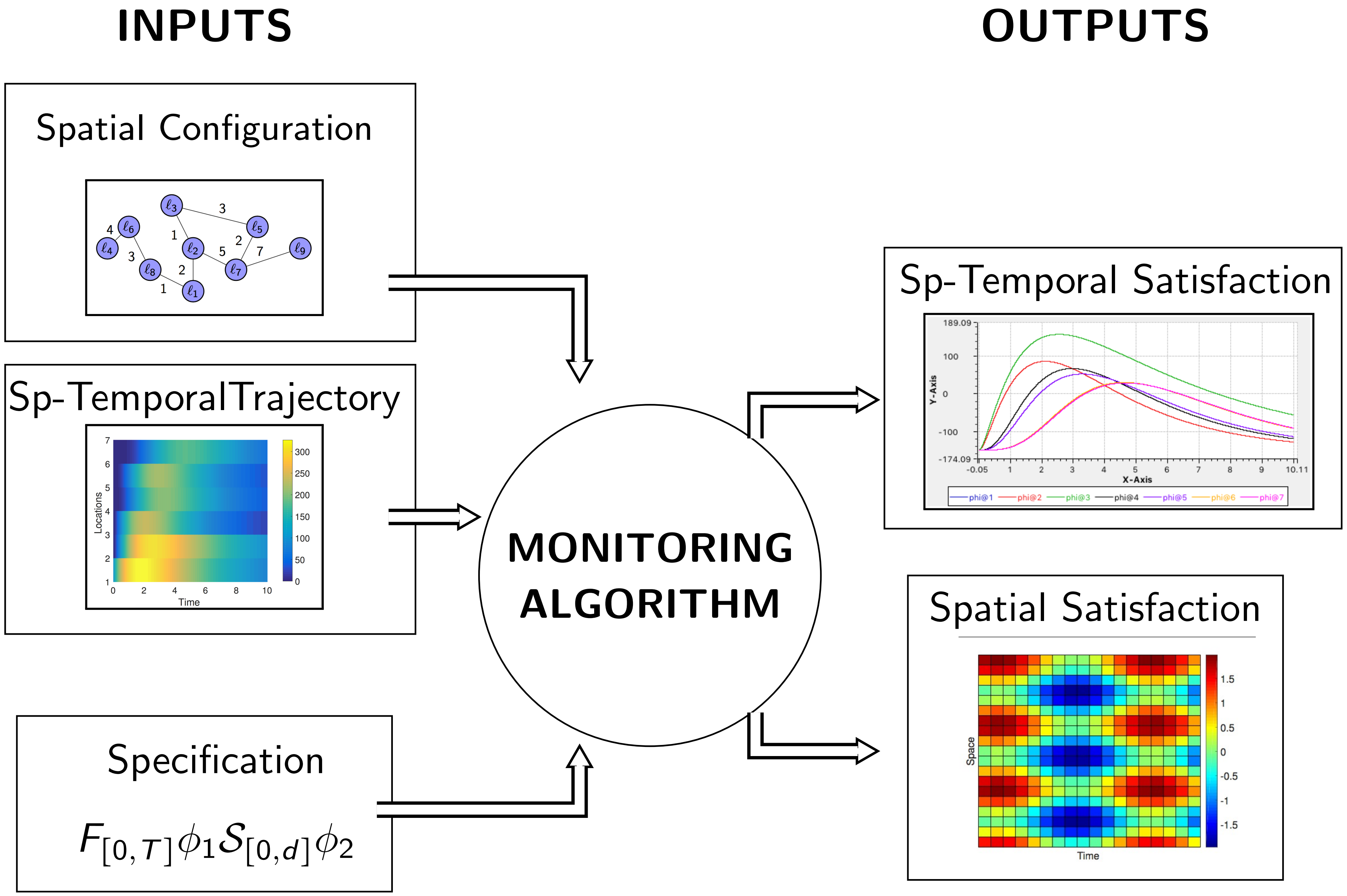




$\rho_\varphi(\ell, 0)$

Spatial Quantitative Satisfaction

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MONITORING
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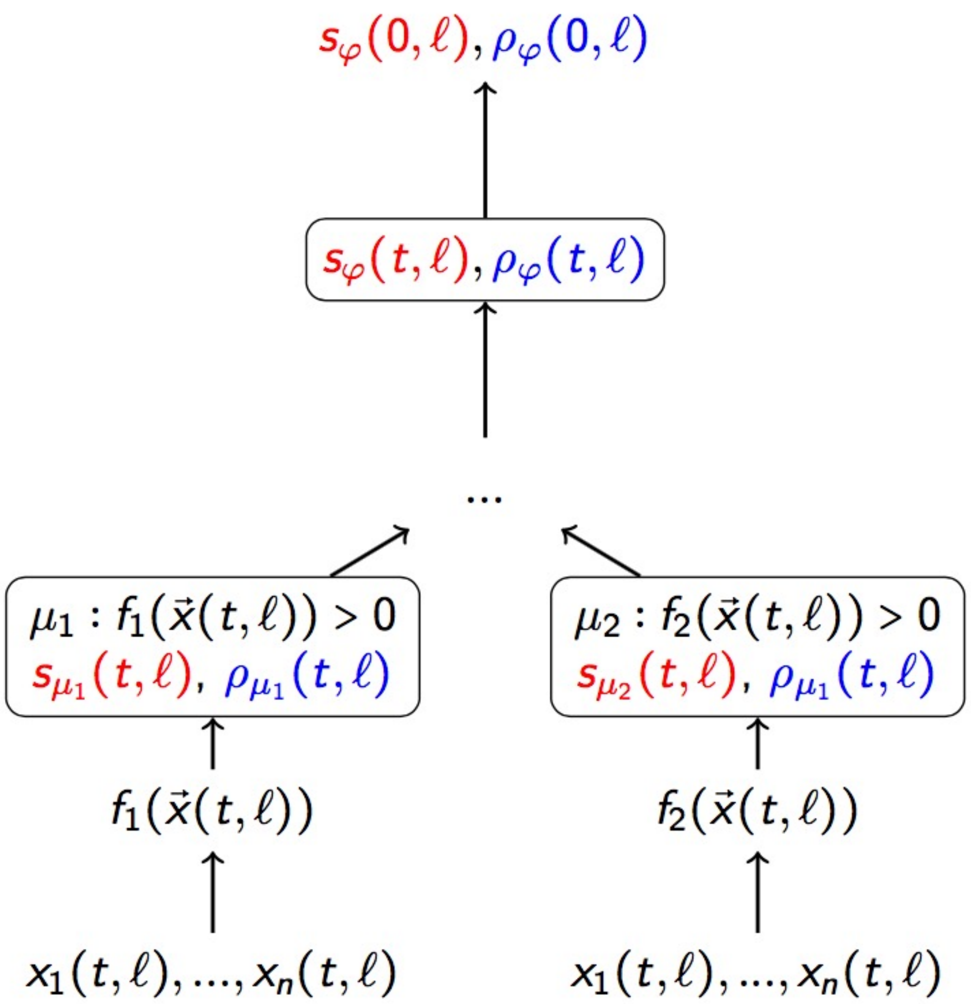
Sp-Temporal Satisfaction
Spatial Satisfaction
Locations
Time
X-Axis
Y-Axis
phi@1
phi@2
phi@3
phi@4
phi@5
phi@6
phi@7
Space

Offline Monitoring Algorithm



Spatial Boolean satisfaction

Spatial Quant. satisfaction

Spatial Boolean signals

Spatial Quant. signals

Secondary signals

Primary signals

Computational consideration

- Temporal operators: like in STL monitoring [1] is **linear** in the length of the signal times the number of locations in the spatial model.
- Spatial properties are more expensive, they are based on a variations of the classical Floyd-Warshall algorithm.
  The number of operations to perform is **quadratic** for the reach operator and **cubic** for the escape

[1] O. Maler, T. Ferrére, and D. Nickovic. Efficient Robust Monitoring for STL. In Proc.CAV 2010