MECCANICA RAZIONALE

Ing. Civile & Ambientale

Navale

12 maggio 2021

Modi normali

$$\boxed{A\,\ddot{\underline{x}} + C\,\underline{x} = \underline{0}}$$

$\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_\ell \end{pmatrix} \quad \mathbb{R}^\ell$

A matrice $\ell \times \ell$ simmetrica e definita positiva

C matrice $\ell \times \ell$ simmetrica

Linearizzazione $\longrightarrow$

$A = A(\underline{q}_E)$ matrice energia cinetica calcolata in $\underline{q}_E$

$C = \mathrm{Hess}\, V\big|_{\underline{q}_E}$ Hessiana calcolata in $\underline{q}_E$

$$A \ddot{\underline{x}} + C \underline{x} = 0 \quad \longleftrightarrow \quad \ddot{\xi}_i + \gamma_i \xi_i = 0$$

$\underline{x}$ coordinate fisiche

coordinate normali

$i = 1, \ldots, \ell$

$\gamma_i > 0$ sinusoidi con $\omega_i t$

dove $\gamma_i = \omega_i^2$

$\gamma_i < 0 \to \exp$

$\gamma_i = 0 \to (c_1 + t c_2)$

$$x \underset{S}{\longleftrightarrow} \xi$$

$$\underline{x}(t) = \underline{u}\, e^{\lambda t}$$

vettore costante

soluzioni di questo tipo

$$\left(\underline{u}^{(i)} = S(\underline{v}^{(i)}) \right)$$

λ^2 è radice dell'equazione caratteristica

$$\det(\lambda^2 A + C) = 0$$

("autovalore di C relativo ad A")

Infatti: $\det(\lambda^2 A + C) =$

$$= \det\left(S^T (\lambda^2 A + C) S \right)$$

$$\left(\begin{array}{ll} = \det S^T \det(\lambda^2 A + C) \det S & \\ S^T S = 1 & \det(S^T S) = 1 \end{array} \right)$$

$$= \det\Big(\lambda^2 \underbrace{S^T A S}_{id} + \underbrace{S^T C S}_{\begin{pmatrix} \gamma_1 & & & \\ & \gamma_2 & & \\ & & \ddots & \\ & & & \gamma_\ell \end{pmatrix}} \Big)$$

$$= (\lambda^2 + \gamma_1)(\lambda^2 + \gamma_2) \cdots (\lambda^2 + \gamma_\ell) = 0$$

Questo vuol dire che le soluzioni di $\det(\lambda^2 A + C) = 0$ sono le soluzioni di

$$\lambda^2 + \gamma_i = 0 \quad \text{al variare di } i$$

e cioè le soluzioni per

$$\ddot{\xi}_i + \gamma_i \xi_i = 0 \longrightarrow \boxed{\lambda_i^2 + \gamma_i = 0}$$

Quindi con $\det(\lambda^2 A + C) = 0$

Troviamo tutti i λ_i^2.

Quindi sappiamo che la soluzione è $\underline{x}(\tau) = \underline{u}\, e^{\lambda \tau}$

→ per trovare $\underline{u}$, sostituiamo in

$A\ddot{\underline{x}} + C\underline{x} = 0 \longrightarrow (\lambda^2 A + C)\underline{u} = 0$

→ $\underline{u}$ autovettore di C relativo ad A

$$\boxed{\underline{u}^{(i)} \cdot A \cdot \underline{u}^{(j)}} = \delta^{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

(a causa delle proprietà di $\underline{u}^{(i)}$)

condizione di ortonormalità.

Riassumendo le soluzioni di

$$A\ddot{\underline{x}} + C\underline{x} = 0$$

hanno la forma $\underline{x}(\tau) = \underline{u}\, e^{\lambda \tau}$

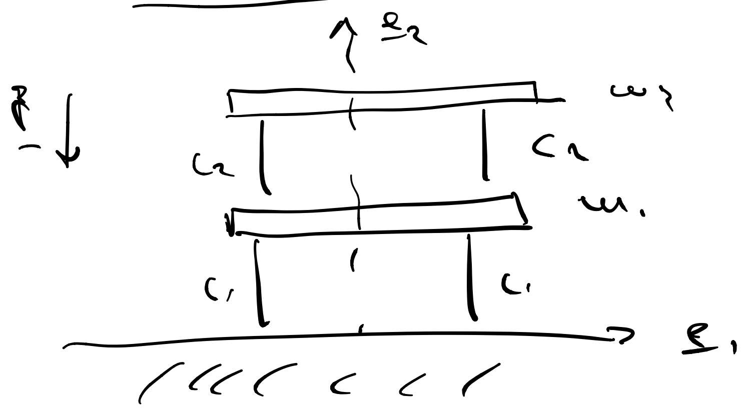
dove

- $\det(\lambda^2 A + C) = 0$
- $(\lambda^2 A + C)\underline{u} = 0$

I. vogliamo λ^2, perché $\lambda_i^2 = -\mu_i$

- non avremo bisogno della condizione di ortonormalità

Esempio Caso a due piani



$c = c_1 = c_2$

$m_1 = 2m$

$m_2 = m$

Le eq. di moto diventano

$$\begin{cases} \frac{m}{2c} \, 2 \, \frac{d^2 x_1}{dt^2} + 2x_1 - x_2 = 0 \\ \\ \frac{m}{2c} \frac{d^2 x_2}{dt^2} - x_1 + x_2 = 0 \end{cases}$$

Trucco: fa cambio variabile

$$\tau = \sqrt{\frac{2c}{m}} \, t$$

$$\frac{m}{2c} \frac{d^2}{dt^2} = \frac{d}{d\left(\frac{2c}{m} t^2\right)} = \frac{d}{d\left(\sqrt{\frac{2c}{m}} t\right)^2} =$$

$= \frac{d^2}{d\tau^2}$

le eq. diventano

$$A \frac{d^2}{d\tau^2} \underline{x} + C \underline{x} = 0$$ dove

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

ci ricordiamo che il tempo fisico

e^{-} t e non τ $\quad$ $\cos \omega\tau$ $\quad$ $\cos\left(\omega\sqrt{\frac{E_1}{L}}\,t\right)$

$\sin \omega\tau \rightarrow$

$e^{\nu\tau}$

Cerchiamo soluzioni del tipo

$$\underline{x} = \underline{u}\, e^{\lambda\tau}$$

$$\lambda^2 A + C = \begin{pmatrix} 2\lambda^2 & 0 \\ 0 & \lambda^2 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2\lambda^2 + 2 & -1 \\ -1 & \lambda^2 + 1 \end{pmatrix}$$

quindi $\det(\lambda^2 A + C) = 0$

$$= 2\left[\lambda^2+1\right]^2 - 1 = 0$$

$$\lambda^2+1 = \pm\frac{1}{\sqrt{2}} \rightarrow \lambda^2 = -1 \mp \frac{1}{\sqrt{2}}$$

quindi $\lambda^2_{(1)}$, $\lambda^2_{(2)}$ sono entrambe negative

le frequenze di oscillazione sono

$$\lambda = \pm i\omega$$

$$\omega_1 = \left(1-\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} \qquad \omega_2 = \left(1+\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Se prendiamo $\lambda^2 = \underbrace{-1+\frac{1}{\sqrt{2}}}$

$$= -\left(1-\frac{1}{\sqrt{2}}\right) = -\omega_1^2$$

$$(\lambda^2 A + C)\underline{u} = 0 \; :$$

$$\begin{pmatrix} 2\lambda^2+2 & -1 \\ -1 & \lambda^2+1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0$$

$$\begin{cases} 2(\lambda^2+1)\,u_1 - u_2 = 0 \\ -u_1 + (\lambda^2+1)\,u_2 = 0 \end{cases}$$

prendiamo la prima :

$$\rightarrow \; 2\left(-1+\frac{1}{\sqrt{2}}+1\right)u_1 - u_2 =$$

$$\frac{2}{\sqrt{2}}u_1 - u_2 = 0 \quad \rightarrow \quad u_2 = \sqrt{2}\,u_1$$

Allora $\underline{u}^{(1)} = b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \quad b_1 \in \mathbb{R}$

Se invece $\lambda^2 = -1 - \frac{1}{\sqrt{2}}$

$$\rightarrow \; 2\left(-1-\frac{1}{\sqrt{2}}+1\right)u_1 - u_2 = 0$$

$$\underline{u}^{(2)} = b_2 \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \quad b_2 \in \mathbb{R}$$

Abbiamo trovato i modi normali di vibrazione

$\rightarrow$ $\underline{u}^{(1)} \sin \omega_1 t$, $\underline{u}^{(1)} \cos \omega_1 t$

$\rightarrow$ $\underline{u}^{(2)} \sin \omega_2 t$, $\underline{u}^{(2)} \cos \omega_2 t$

La soluzione generale è

$$\underline{x}(\tau) = \underline{u}^{(1)} \left(k_1 \sin \omega_1 \sqrt{\frac{2c}{m}}\, \tau + \right.$$

$$\left. + k_2 \cos \omega_1 \sqrt{\frac{2c}{m}}\, \tau \right) +$$

$$+ \underline{u}^{(2)} \left(k_3 \sin \omega_2 \sqrt{\frac{2c}{m}}\, \tau + \right.$$

$$\left. + k_4 \cos \omega_2 \sqrt{\frac{2c}{m}}\, \tau \right)$$

Seconda Parte

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costanti arbitrarie, fissate dalle condizioni iniziali.

- detto questo

$$\underline{u}^{(1)} k_1 \sin \omega_1 \tau$$

$$\underline{u}^{(1)} k_1 \sin \omega_1 \sqrt{\frac{2c}{m}}\, t$$

IMPORTANTE TEMPO FISICO

• dettaglio

$$b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} k_1 \sin \omega_1 \sqrt{\frac{2c}{m}}\, t$$

possiamo determinare b_1 dalle condizioni di ortonormalità

$$\underline{u}^{(i)} A \, \underline{u}^{(j)} = \delta^{ij}$$

$$\underline{u}^{(1)} A \, \underline{u}^{(1)} = 1$$

$$b_1 (1 \quad \sqrt{2}) \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} =$$

$$b_1^2 (1 \quad \sqrt{2}) \begin{pmatrix} 2 \\ \sqrt{2} \end{pmatrix} = b_1^2\, 4 \overset{!}{=} 1$$

$$\longrightarrow b_1 = \frac{1}{2}$$

finito, ma non necessario:

$$\omega_1 = \left(1 - \frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}$$

$$b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} k_1 \sin \omega_1 \sqrt{\frac{2c}{m}}\, t$$

↑ costante arbitraria.

$$\underline{x}(0) = \underline{x}_0 \quad \rightarrow$$

$$\dot{\underline{x}}(0) = \tilde{\underline{x}}$$

ad exemplum $\quad \underline{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \dot{\underline{x}}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$x(T) = b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \left(k_1 \sin \omega_1 \tau + k_2 \cos \omega_1 \tau \right)$$

$$+ \; b_2 \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \left(k_3 \sin \omega_2 \tau + k_4 \cos \omega_2 \tau \right)$$

$$\tau = \sqrt{\frac{2c}{m}}\, T$$

$$\underline{x}(0) = b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} k_2 + b_2 \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} k_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} b_1 k_2 + b_2 k_4 \\ b_1 \sqrt{2} k_2 - b_2 \sqrt{2} k_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x(T) = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \left(c_1 \sin \omega_1 \tau + c_2 \cos \omega_2 \tau \right)$$

$$+ \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \left(c_3 \sin \omega_2 \tau + c_4 \cos \omega_2 \tau \right)$$

$$\tau = \sqrt{\frac{2c}{m}}\, T$$

$c_1 = k_1 b_1$ — —

in $(\lambda^2 A + C)\,\underline{u} = 0 \longrightarrow \underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

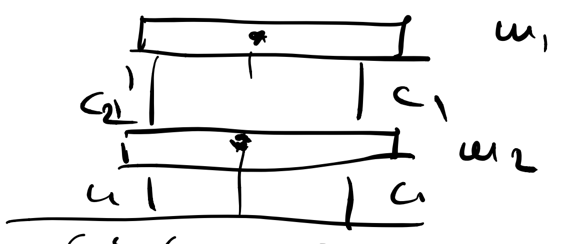
possiamo determinare $\underline{u}$ a meno di una costante moltiplicativa che viene fissata imponendo condizioni iniziali.

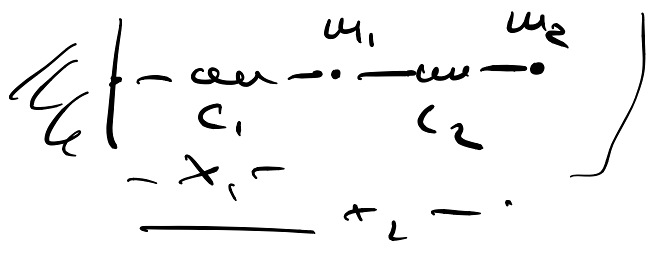
$$\underline{x}(T) = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \left(c_1 \sin \omega_1 \tau + c_2 \cos \omega_1 \tau \right) + \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \left(c_3 \sin \omega_2 \tau + c_4 \cos \omega_2 \tau \right)$$

$$\tau = \sqrt{\frac{2c}{m}}\, T$$

$$\underline{x}(T) = \begin{pmatrix} x_1(T) \\ x_2(T) \end{pmatrix}$$

$$\underline{x} \neq \begin{pmatrix} 1 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x_2$$





$$S = \left(\underline{u}^{(1)} \quad \underline{u}^{(2)} \right) = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ \sqrt{2} & -\sqrt{2} \end{pmatrix}$$

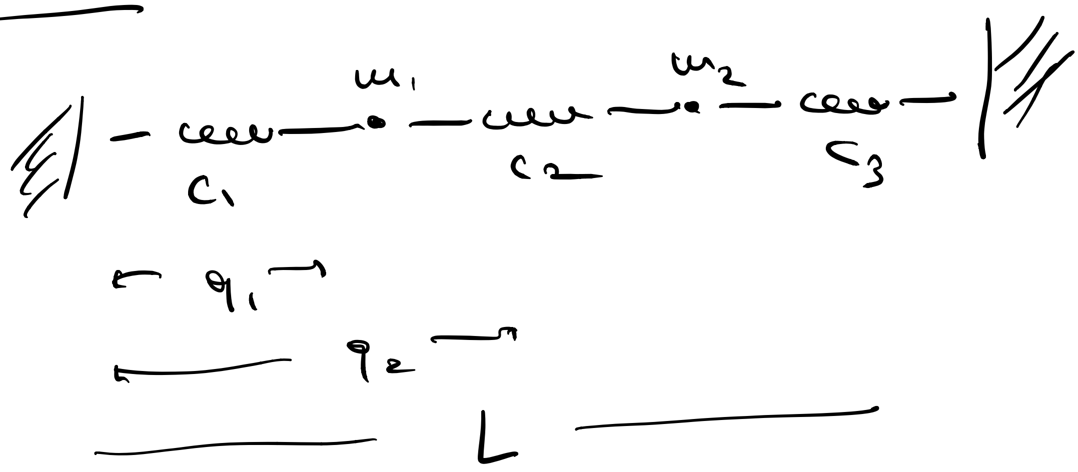
$$S(\underline{v}^{(1)}) = \underline{u}^{(1)}$$

$$\underline{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \underline{v}^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\underline{x} = S \underline{\xi} \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ \sqrt{2} & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

$$\begin{cases} x_1 = \frac{1}{2}(\xi_1 + \xi_2) \\ x_2 = \frac{\sqrt{2}}{2}(\xi_1 - \xi_2) \end{cases} \qquad \begin{aligned} \xi_1 &= \ldots \\ \xi_2 &= \ldots \end{aligned}$$

<u>Esempio</u> Torniamo



$m_1 = 3m$

$m_2 = 2m$

$c_1 = 2c$

$c_2 = 2c$

$c_3 = c$

$$A\underline{\ddot{q}} + C\underline{q} - \underline{b} = \underline{0}$$

$$A = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad C = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} 0 \\ c_3 L \end{pmatrix}$$

$$A = \begin{pmatrix} 3m & 0 \\ 0 & 2m \end{pmatrix} \qquad C = \begin{pmatrix} 4c & -2c \\ -2c & 3c \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} 0 \\ cL \end{pmatrix}$$

configurazione di equilibrio:

$$C\underline{q}_E = \underline{b} \qquad \begin{cases} 4c\, q_{1,E} - 2c\, q_{2,E} = 0 \\ -2c\, q_{1,E} + 3c\, q_{2,E} = cL \end{cases}$$

$$\rightarrow \quad q_{1,E} = \frac{L}{4}, \quad q_{2,E} = \frac{L}{2}$$

allora $x_1 = q_1 - \frac{L}{4}$, $x_2 = q_2 - \frac{L}{2}$

le equazioni del moto per le variabili linearizzate sono

$$m\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}\frac{d^2}{dt^2}\underline{x} + \begin{pmatrix}4c & -2c\\ -2c & 3c\end{pmatrix}\underline{x} = \underline{0}$$

$$\frac{m}{c}\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}\frac{d^2\underline{x}}{dt^2} + \begin{pmatrix}4 & -2\\ -2 & 3\end{pmatrix}\underline{x} = 0$$

$$\frac{m}{c}\frac{d^2}{dt^2} = \frac{d^2}{d\left(\sqrt{\frac{c}{m}}\,t\right)^2} = \frac{d^2}{d\tau^2}$$

$$\tau = \sqrt{\frac{c}{m}}\,t$$

$$\underbrace{\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}}_{\tilde{A}}\frac{d^2\underline{x}}{d\tau^2} + \underbrace{\begin{pmatrix}4 & -2\\ -2 & 3\end{pmatrix}}_{\tilde{C}}\underline{x} = 0$$

$$\tilde{A}\,\frac{d^2\underline{x}}{d\tau^2} + \tilde{C}\,\underline{x} = 0$$

Troviamo le frequenze:

$$\det\left(\lambda^2\hat{A} + \tilde{C}\right) = 0$$

$$= \det\left[\begin{pmatrix}\lambda^2 3 & 0\\ 0 & \lambda^2 2\end{pmatrix} + \begin{pmatrix}4 & -2\\ -2 & 3\end{pmatrix}\right]$$

$$= \det \begin{bmatrix} 4+\lambda^2 3 & -2 \\ -2 & 3+2\lambda^2 \end{bmatrix}$$

$$= (3\lambda^2+4)(2\lambda^2+3)-4 = 0$$

$$6\lambda^4 + (9+8)\lambda^2 + 8 = 0$$

$$\lambda^2 = \frac{-17 \pm \sqrt{17^2 - 24 \cdot 8}}{12} = \text{---}$$

$$= \frac{-17 \pm \sqrt{97}}{12}$$

$\omega_1 \sim 1.496$

$\omega_2 \sim 0.772$

Terza parte

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3) Abbiamo $\longrightarrow \underline{u}$

$$(\lambda_{(i)}^2 \tilde{A} + \tilde{C})\, \underline{u}^{(i)} = 0 \qquad \underline{u}^{(i)} = \begin{pmatrix} u_1^{(i)} \\ u_2^{(i)} \end{pmatrix}$$

$$\begin{pmatrix} 3\lambda_{(i)}^2 + 4 & -2 \\ -2 & 2\lambda_{(i)}^2 + 3 \end{pmatrix} \begin{pmatrix} u_1^{(i)} \\ u_2^{(i)} \end{pmatrix} = 0$$

$$\begin{cases} \left(3\lambda_{(i)}^2 + 4\right) u_1^{(i)} - 2u_2^{(i)} = 0 \\ -2u_1^{(i)} + \left(2\lambda_{(i)}^2 + 3\right) u_2^{(i)} = 0 \end{cases}$$

- $\lambda_{(1)}^2 = \dfrac{-17+\sqrt{97}}{12}$

$$-2u_1^{(1)} + \left(2\left(\frac{-17+\sqrt{97}}{12}\right) + 3\right) u_2^{(1)} = 0$$

$$\frac{-17+\sqrt{97}}{6} + \frac{18}{6}$$

$$\to u_1^{(1)} = \frac{1+\sqrt{97}}{12} u_2^{(1)}$$

ωστε $\underline{u}^{(1)} = b_1 \begin{pmatrix} \frac{1+\sqrt{97}}{12} \\ 1 \end{pmatrix}$

- $\lambda_{(2)}^2 = \dfrac{-17-\sqrt{97}}{12}$

$$-2u_1^{(2)} + \left(2\,\frac{-17-\sqrt{97}}{12} + 3\right) u_2^{(2)} = 0$$

$$u_1^{(2)} = \frac{1-\sqrt{97}}{12} u_2^{(2)}$$

$$\underline{u}^{(2)} = b_2 \begin{pmatrix} \frac{1-\sqrt{97}}{12} \\ 1 \end{pmatrix}$$

Allora abbiamo trovato i modi normali

$$\underline{x}(t) = \begin{pmatrix} \frac{1+\sqrt{97}}{12} \\ 1 \end{pmatrix} \left(k_1 \sin \sqrt{\frac{17-\sqrt{97}}{12} \frac{c}{m}}\, t \right.$$

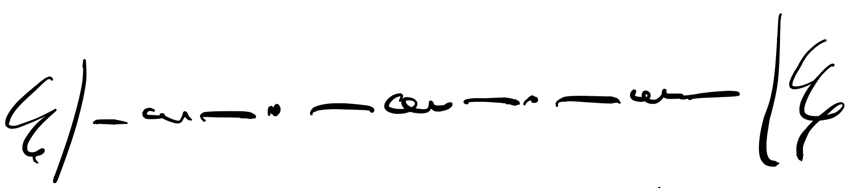
$$\left. + k_2 \cos \sqrt{\frac{17-\sqrt{97}}{12} \frac{c}{m}}\, t \right) +$$

$$+ \begin{pmatrix} \frac{1-\sqrt{97}}{12} \\ 1 \end{pmatrix} \left(k_3 \sin \sqrt{\frac{17+\sqrt{97}}{12} \frac{c}{m}}\, t \right.$$

$$\left. + k_4 \cos \sqrt{\frac{17+\sqrt{97}}{12} \frac{c}{m}}\, t \right)$$

4) Dati iniziali

$$\begin{cases} q_1(0) = 0 \\ q_2(0) = \frac{L}{2} \end{cases} \qquad \begin{cases} \dot{q}_1(0) = 0 \\ \dot{q}_2(0) = 0 \end{cases}$$



$$\underline{x}(0) = \underline{q}(0) - \underline{q}_r = \begin{pmatrix} -\frac{L}{4} \\ 0 \end{pmatrix}$$

$$\dot{\underline{x}}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\dot{\underline{x}}(0) = \begin{pmatrix} - \\ -- \end{pmatrix} k_1 \cos(\;)t +$$

$$\begin{pmatrix} \; \\ \; \end{pmatrix} k_3 \cos(\;)t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\hookrightarrow k_1 = k_3 = 0$$

$$\underline{x}(0) = \begin{pmatrix} \frac{1+\sqrt{97}}{12} k_2 + \frac{1-\sqrt{97}}{12} k_4 \\ k_2 + k_4 \end{pmatrix} = \begin{pmatrix} -\frac{L}{4} \\ 0 \end{pmatrix}$$

$$k_2 = -k_4$$

$$\frac{1+\sqrt{97}}{12} k_2 - \frac{1-\sqrt{97}}{12} k_2 = -\frac{L}{4}$$

$$\frac{\sqrt{97}}{12} k_2 + \frac{\sqrt{97}}{12} k_2 = -\frac{L}{4}$$

$$\rightarrow k_2 = -\frac{6}{4} \frac{L}{\sqrt{97}}$$

$\rightarrow$ risolto

Alcuni commenti:

$A\ddot{\underline{x}} + C\underline{x} = 0$

$\longrightarrow$ Termine forzante

$$A\ddot{\underline{x}} + C\underline{x} = \underline{F}(t)$$

Cerchiamo soluzione particolare

$$\underline{x} = \sum_{i=1}^{\ell} \underline{u}^{(i)} g_i(t)$$

$\uparrow$ determinati dal caso $\underline{F}(t) = 0$

$$A\ddot{\underline{x}} + C\underline{x} =$$

$$= \sum_{i=1}^{\ell} \left(\ddot{g}_i A \underline{u}^{(i)} + g_i C \underline{u}^{(i)} \right)$$

siccome $\lambda^2 A\underline{u} + C\underline{u} = 0$

$$C\underline{u}^{(i)} = -\lambda_{(i)}^2 A\underline{u}^{(i)} = \mu_i A\underline{u}^{(i)}$$

$$= \sum_{i=1}^{\ell} (\ddot{g}_i + \mu_i g_i) A\underline{u}^{(i)} = \underline{F}(t)$$

Adesso se moltiplichiamo scalarmente per $\underline{u}^{(j)}$ e usiamo che

$$\underline{u}^{(j)} \cdot A \cdot \underline{u}^{(i)} = \delta^{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$\underline{u}^{(j)} \cdot A \, \underline{u}^{(j)} \left(\ddot{g}_j + \gamma_j g_j \right) = \underline{F} \cdot \underline{u}^{(j)}$$

$$\hookrightarrow \quad \ddot{g}_j + \gamma_j g_j = \frac{\underline{F} \cdot \underline{u}^{(j)}}{\underline{u}^{(j)} \cdot A \, \underline{u}^{(j)}} = f_j(\tau)$$

modi normali

$$\hookrightarrow \quad \ddot{g}_j + \gamma_j g_j = f_j(\tau)$$

$\longrightarrow$ In modo simile

$$A \underline{\ddot{x}} + B \underline{\dot{x}} + C \underline{x} = \underline{0}$$

se $\quad S^T B S$ è diagonale

$$\ddot{\xi}_i + 2k_i \dot{\xi}_i + \gamma_i \xi_i = 0$$

$$S^T B S = \begin{pmatrix} 2k_1 & & \\ & 2k_2 & \\ & & \ddots \end{pmatrix}$$

ho sol $\quad \underline{x}(\tau) = \sum \underline{u}^{(i)} g(\tau)$

calcolati senza B

se invece $S^T B S$ non è diagonale

$$\lambda^2 A \underline{u} + \lambda B \underline{u} + C \underline{u} = 0 \quad \longrightarrow \quad \begin{array}{l} \lambda \leftarrow \\ \underline{u} \leftarrow \end{array}$$