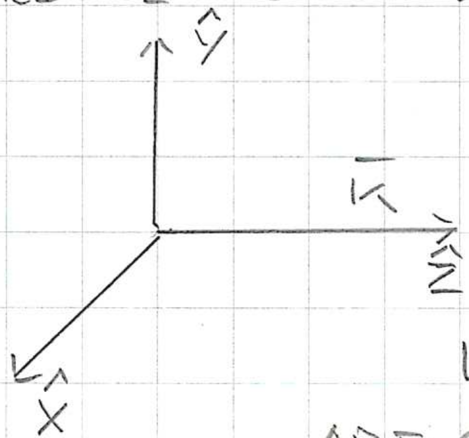
WE FIND THAT
$$\hat{M} = \sum_{k}{}' \hbar \hat{e}_k \left[\hat{a}^+_{kR} \hat{a}_{kR} - \hat{a}^+_{kL} \hat{a}_{kL} \right]$$

$\Rightarrow$ THEREFORE, SINCE $\hat{e}_k \cdot \hat{M} |\bar{k}, R/L\rangle = \pm\hbar |\bar{k}, R/L\rangle$
WE CONCLUDE THAT PHOTONS CARRY INTRINSIC ANGULAR MOMENTUM $\pm\hbar$ (KNOWN AS HELICITY) ORIENTED PARALLEL OR ANTI-PARALLEL TO THE DIRECTION OF THE MOMENTUM PROPAGATION.

- ANALOGIES BETWEEN THE LIGHT POLARIZATION AND THE S-G RESULTS

THE E.M. WAVES POLARIZATION, AS WE KNOW FROM, CAN BE DESCRIBED ON THE BASIS OF TWO VECTORS IN A 2D COMPLEX SPACE. A WAY FOR THE MATHEMATICAL REPRESENTATION IS GIVEN BY THE JONES VECTORS. BY CHOOSING A REFERENCE FRAME WITH THE WAVEVECTOR $\bar{k} \parallel \hat{z}$



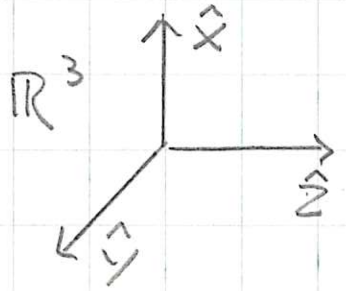
THE JONES VECTORS FOR THE LH ($\hat{x}$) (LINEAR HORIZ.) LV ($\hat{y}$) (LINEAR VERT.) ARE GIVEN BY

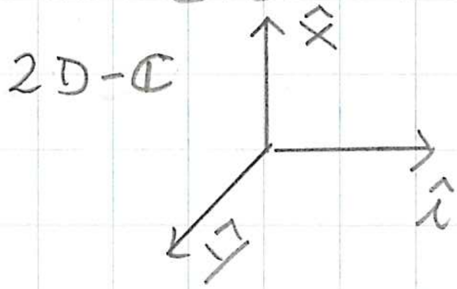
$\hat{x} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$, $\hat{y} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$, AND FOR THE RC ($\hat{R}$) RIGHT CIRCULAR AND LC ($\hat{L}$) LEFT CIRCULAR BY

$$\hat{R} = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ -i \end{vmatrix} \quad \text{AND} \quad \hat{L} = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ +i \end{vmatrix}$$

WHEREA THE JONES MATRICES FOR POLARIZER DEVICES ARE GIVEN BY FOR THE LINEAR POLARIZER HORIZONTAL ($LP\hat{H} = \hat{X}$)

$\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$, AND FOR THE LINEAR POLARIZER VERTICAL ($LP\hat{V}) = \hat{Y}$). THEREFORE, THE LINEAR POLARIZATION OF AN E.M. WAVE IS GIVEN BY TWO VECTORS $\hat{X}, \hat{Y}$, WHEREAS THE LEFT AND RIGHT CIRCULAR POLARIZATION ARE REPRESENTED BY TWO VECTORS DIRECTED PARALLEL AND ANTIPARALLEL TO THE $\hat{i}$ COMPLEX AXIS RESPECTIVELY THAT IN OUR 2D COMPLEX SPACE IS DIRECTED // TO $\hat{Z}$





THIS REPRESENTATION OF THE LIGHT POLARIZATION CONSTITUTE HAS A CLEAR ANALOGY WITH THE SPIN OF A PARTICLE (ELECTRON) WHERE THE S-G APPARATOUS PLAYS THE ROLE OF A POLARIZER. THIS ESTABLISHES THE POSSIBILITY OF USING POLARIZED PHOTONS INSTAD OF e^-, FOR EXAMPLE, TO STUDY AND TO USE SOME QUANTUM PROPERTIES OF PARTICLES, SUCH AS ENTANGLEMENT.

LET'SEE MORE CLOSER THIS ANALOGY.

AS KNOWN AND ACCORDINGY WITH OUR REFERENCE FRAME A LIGHT BEAM

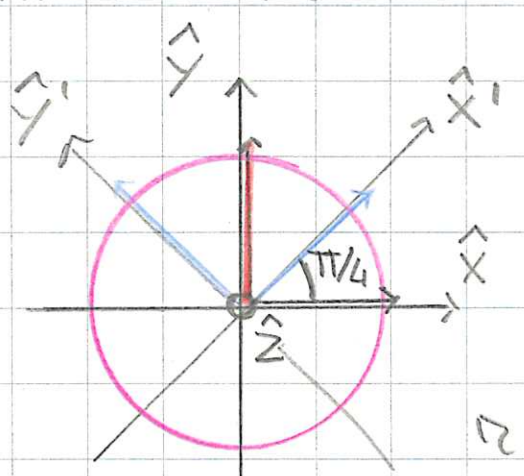
LH POLARIZED HAS AN $\bar{E}$ FIELD GIVEN BY

$$\bar{E} = E_0 \hat{x} \cos(kz - \omega t)$$

WHEREAS THE LV POLARIZATION IS

$$\bar{E} = E_0 \hat{y} \cos(kz - \omega t)$$

THIS IN A FROTAL VIEW.



POLARIZED LIGHT CAN BE OBTAINED FROM UNPOLARIZED LIGHT USING BIRIFRANGENT PLATES $(\lambda/2, \lambda/4)$ OR SIMPLY POLAROID MATERIAL FILTER. FOR EXAMPLE A $\hat{x}$-FILTER ALLOWS ONLY THE $\hat{x}$-POLARIZED LIGHT TO GO THROUGH, AND SO FOR THE $\hat{y}$-POL. OF COURSE $\hat{x}\text{-POL} \xrightarrow[\hat{z}]{\pi/2} \hat{y}\text{-POL}$. LET'S USE THE SIMBOL $\leftrightarrow \equiv \hat{x}$-POL AND $\updownarrow = \hat{y}$-POL AND $\swarrow\!\!\nearrow$ $\pi/4$-POL AND $\nwarrow\!\!\searrow$ $-\pi/4$ POL. THE FOLLOWING EXPRESSION HOLDS

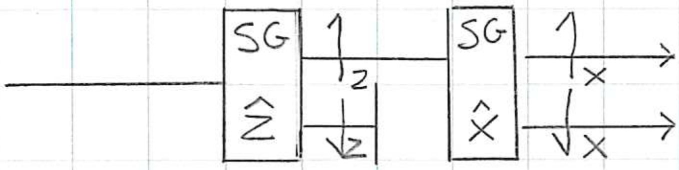
$$\text{UNPOL} + \leftrightarrow + \updownarrow = 0$$

BUT EVEN MORE INTERESTING IS

$$\text{UNPOL} + \leftrightarrow + \swarrow\!\!\nearrow + \updownarrow \neq 0.$$

IN THIS CASE SOME LIGHT, ALTHOUGH WITH LESS INTENSITY IS EMERGING FROM THE FILTERS SEQUENCE. THEREFORE, IT IS IRRELEVANT THAT THE LIGHT WAS FIRST POLARIZED IN THE $\leftrightarrow$ DIRECTION. THE ACTION OF $\swarrow\!\!\nearrow$ DELATES THE PREVIOUS INFORMATION ABOUT THE $\leftrightarrow$ POLARIZATION, THIS SITUATION IS SIMILAR TO THAT WE HAVE

SEEN WITH THE S-G DEVICE AS LONG AS WE ASSUME THE FOLLOWING CORRESPONENCE



ATOMS S_z $\uparrow\downarrow \leftrightarrow$ $\hat{x}$-POL, $\hat{y}$ POL

ATOMS S_x $\uparrow\downarrow \leftrightarrow$ $\hat{x}'$-POL, $\hat{y}'$ POL

FOLLOWING THE CLASSICAL E.D. THE $\bar{E}$ FIELDS $\hat{x}'$-POL AND $\hat{y}'$-POL ARE GIVEN BY

$$
\begin{cases}
E_0 \hat{x}' \cos(kz-\omega t) = E_0 \left[\frac{1}{\sqrt{2}} \hat{x} \cos(kz-\omega t) + \frac{1}{\sqrt{2}} \hat{y} \cos(kz-\omega t) \right] \\
E_0 \hat{y}' \cos(kz-\omega t) = E_0 \left[-\frac{1}{\sqrt{2}} \hat{x} \cos(kz-\omega t) + \frac{1}{\sqrt{2}} \hat{y} \cos(kz-\omega t) \right]
\end{cases}
$$

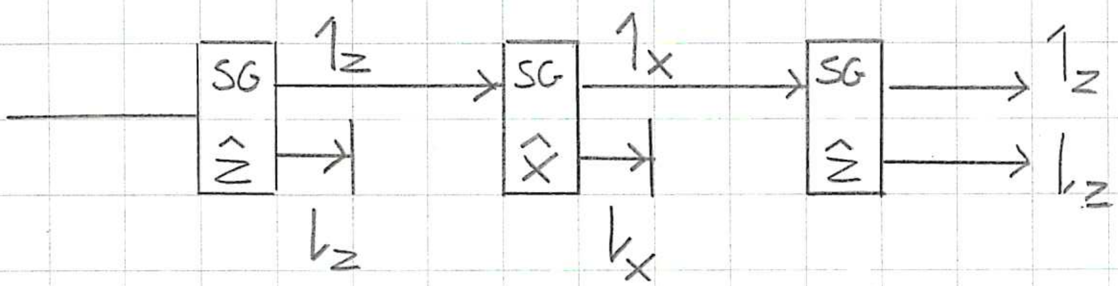
THIS IMPLIES THAT THE $\hat{x}'$ AND $\hat{y}'$ POL. BEAM CAN BE SEEN AS THE LINEAR COMBINATION OF $\hat{x}$ AND $\hat{y}$ POLARIZED BEAM WHERE THE $\frac{1}{\sqrt{2}}$ IS DUE TO THE $\pm\pi/4$ ANGLES BETWENN x, y AND x', y' AXIS.

THE USE OF THE CORRESPONDENCE BETWEEN THE SEQUENTIAL S-G EXPERIMENT AND THE POLARIZATION EXPERIMENT WITH THE OPTICAL BEAM SUGGESTS WE CAN TREAT THE SPIN STATE AS A SORT OF A VECTOR IN A NEW 2D-VECTORIAL SPACE, THAT SHOULD NOT BE CONFUSED WITH THE ORDINARY EUCLIDEAN 2D-SPACE $\mathbb{R}^2$. AS FOR THE JONES VECTORS 2D SPACE WE CAN REPRESENT THE $\uparrow_x$ WITH A VECTOR THAT ASSUMING THE DIRAC NOTATION WE CAN CALL "KET" AND WRITE $|\uparrow_x\rangle$ CAN BE SEEN AS THE LINEAR COMBINATION

OF TWO BASE VECTORS $|\uparrow_z\rangle$ AND $|\downarrow_z\rangle$ $\Rightarrow$

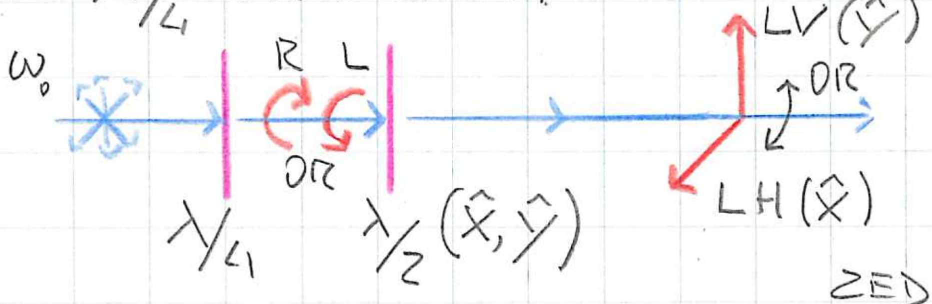
$$\begin{cases} |\uparrow_x\rangle = \frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle \\ |\downarrow_x\rangle = -\frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle \end{cases}$$

OF COURSE THESE RELATIONS CAN BE OPTAINED USING THE GENERAL FORMALISM OF THE Q.M. THEREFORE, THE NON-BLOCKED COMPONENT AT THE EXIT OF THE SECOND S-G $\hat{x}$ DEVICE IN THE PREVIOUS FIGURE MUST BE CONSIDERED AS THE OVERPOSITION OF $\uparrow_z$ AND $\downarrow_z$ STATES AND THIS IS THE REASON WHY TWO COMPONENTS ARE EMERGING FROM THE S-G $\hat{x}$ DEVICE WHEN THE FOLLOWING CONFIGURATION IS ADOPTED)



OF COURSE THE SAME ARGUMENT APPLIES IF THE $\uparrow_z$ AND $\uparrow_x$ COMPONENTS ARE BLOCKED INSTEAD OF $\downarrow_z$ AND $\downarrow_x$. A MORE COMPLEX QUESTION TO ANSWER IS: "HOW CAN WE REPRESENT THE $\uparrow_{\hat{y}}$ AND $\downarrow_{\hat{y}}$ STATE IN THE S-G CONFIGURATION GIVEN BEFORE"? SYMMETRY ARGUMENTS SUGGEST THAT IF WE OBSER A $(\uparrow_z \downarrow_z)$ BEAM MOVING IN THE $\hat{x}$ DIRECTION IS GOING THROUGH A S-G $\hat{y}$ THE RESULTING SITUATION IS SIMILAR TO THAT WHERE A $(\uparrow_z \downarrow_z)$ BEAM

MOVING ALONG THE $\hat{y}$ DIRECTION IS GOING THROUGH A $SG\hat{x}$ DEVICE. THE KETS FOR $(\uparrow_y \downarrow_y)$ COULD BE SEEN AS A LINEAR COMBINATION OF $|\uparrow_z\rangle$ AND $|\downarrow_z\rangle$ BUT THESE COMBINATIONS ARE ALREDY TAKAWEN FOR THE $\hat{z}-\hat{x}$ CASE AND THERE ARE NO MORE COMBINATIONS FOR THE $|\uparrow_x\rangle$ AND $|\downarrow_x\rangle$ STATES. HOW CAN OUR VECTORIAL-SPACE FORMALISM DISTINGUISH BETWEEN THE $|\uparrow_y\rangle, |\downarrow_y\rangle$ AND THE $|\uparrow_x\rangle, |\downarrow_x\rangle$ STATES? THE ANALOGY WITH THE LIGHT WILL HELP. LET'S CONSIDER A CIRCULARLY POLARIZED LIGHT THAT WE CAN OBTAIN USING A $\lambda/4$ WAVEPLATE.



WE OBTAIN A LINEARLY POLARIZED LIGHT $\hat{x}$ OR $\hat{y}$ (DEPENDS ON THE $\lambda/2$ OPTICAL AXIS ORIENTATION) OF EQUAL INTENSITY, ALTHOUGH WE KNOW THAT THE LC OR RC LIGHT IS DIFFERENT FROM THE $\pi/4$ LINEARLY POLARIZED LIGHT (POLARIZED ALONG $\hat{x}'$ OR $\hat{y}'$). HOW DO WE REPRESENT THE CIRCULARLY POLARIZED LIGHT? FROM THE JONES VECTORS WE KNOW THAL

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix} \text{ AND } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

THAT CAN BE TRANSFORMED USING THE COMPLEX NOTATION IN

$$\tilde{E} = E_0 \left[\frac{1}{\sqrt{2}} \hat{x} e^{i(kz-\omega t)} \pm \frac{i}{\sqrt{2}} \hat{y} e^{i(kz-\omega t)} \right]$$

WE CAN SET NOW THE ANALOGY WITH THE SPIN

$|\uparrow_y\rangle \to$ RC AND $|\downarrow_y\rangle \to$ LC

WE SEE THAT IT IS POSSIBLE TO USE COMPLEX THE PREVIOUS BASIS KET VECTORS OBTAINING

$$|\uparrow_{\hat{y}} \downarrow_{\hat{y}}\rangle = \frac{1}{\sqrt{2}} |\uparrow_z\rangle + \frac{i}{\sqrt{2}} |\downarrow_z\rangle$$

- OBSERVATION WE NOTE THAT THE 2D SPACE NEED TO DESCRIBE THE SPIN (Ag ATOMS) MUST BE A COMPLEX VECTORIAL SPACE. AN ARBITRARY VECTOR IN THIS SPACE CAN BE DESCRIBED BY A LINEAR COMBINATION OF THE TWO BASIS-SET VECTORS $|\uparrow_z\rangle$ AND $|\downarrow_z\rangle$ WITH IN GENERAL COMPLEX COEFFICIENTS. HERE WE USED THE CLASSICAL DESCRIPTION OF THE E.M. FIELD. OF COURSE, ALTHOUGH NOT NEED FOR THIS EXAMPLE, WE CAN REACH THE SAME CONCLUSIONS USING THE PHOTONS POLARIZATION FORMALISM. ACTUALLY IN THIS CASE THE ANALOGY WOULD BE MORE STRINGENT AND THE CONCEPT OF THE MEASURE OF POLARIZED LIGHT WILL BE MODIFIED IN THAT OF MEASURING THE PROBALITY OF FINDING PHOTONS IN LV, LH, RC OR LC POLARIZED LIGHT. IN ANY CASE THE FUNDAMENTAL CONCEPT GAINED IS THAT THE Q.M. SYSTEMS MUST BE REPPRESENTED BY VECTORS IN COMPLEX ABSTRACT VECTORS SPACE