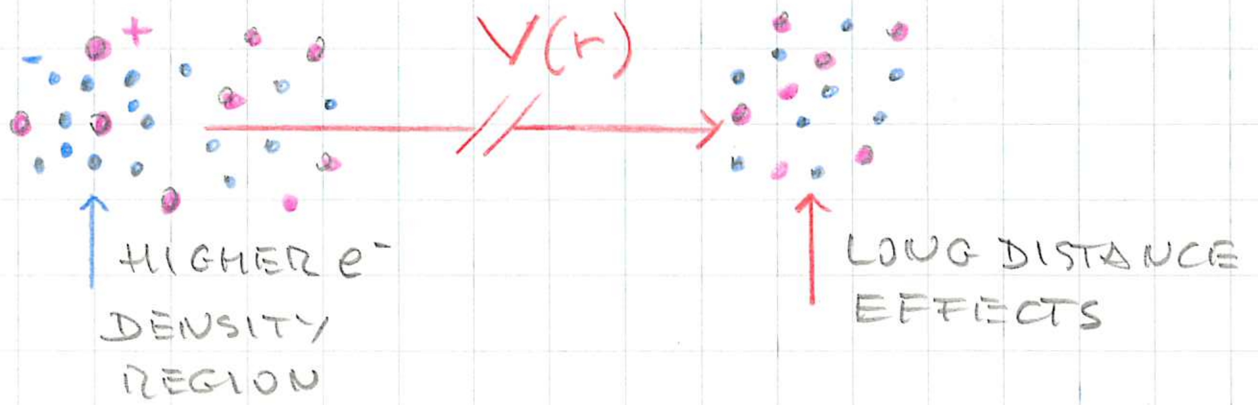
PLASMA PHYSICS BY SETTING $K_B = 1$.
FOR EXAMPLE FOR THE ITER PARAMETERS $T \sim 10\, keV$ $n \sim 10^{14}/cm^3 \Rightarrow P.E./K.E \sim 10^{-6}$.
IN SHORT WE WILL CONSIDER THE CASE OF PLASMA TREATED AS QUASI-NEUTRAL GAS OF CHARGE PARTICLES SHOWING COLLECTIVE BEHAVIOUR. HOWEVER, WE NEED TO DEFINE BETTER WHAT WE MEAN WITH "QUASI-NEUTRAL" AND COLLECTIVE BEHAVIOUR. IN A MORE MATHEMATICAL FORM IF n_e AND n_i ARE RESPECTIVELY THE NUMBER DENSITY FOR ELECTRONS AND IONS WITH A CHARGE Z THAN THESE ARE LOCALLY BALANCED $n_e \simeq Z n_i$
THE SECOND PROPERTIES "COLLECTIVE" BEHAVIOUR ARISES BECAUSE OF THE LONG RANGE CHARACTER OF THE COULOMB POTENTIAL $\left(\propto \frac{1}{r}\right) \Rightarrow$ THE LOCAL DISTURBANCE IN EQUILIBRIUM CAN HAVE A STRONG INFLUENCE ON REMOTE REGION OF THE PLASMA



IN OTHER WORD MACROSCOPIC FIELDS USUALLY DOMINATES OVER SHORT-LIVED MICROSCOPIC FLUCTUATIONS AND A NET CHARGE IMBALANCE

$$\rho = e(Z n_i - n_e)$$

WILL GIVE RISE TO AN ELECTROSTATIC FIELD ACCORDING TO $\bar{\nabla} \cdot E = \rho / \varepsilon_0$ (IN THE FREE SPACE)

LIKEWISE THE SAME SET OF CHARGES MOVING WITH VELOCITY V_e AND V_i WILL GENERATE AN ELECTRICAL CURRENT DENSITY

$$\bar{J} = e(Z n_i v_i - n_e v_e) \Rightarrow \bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

THES ARE THE FIELDS INTERNALLY DRIVEN THAT DETEMINE THE DYNAMICS OF THE PLASMA, INCLUDING THE RESPONSE TO EXTERNALLY APPLIED FIELDS.

NOW THAT WE HAVE ESTABLISHED WHAT THE PLASMA ARE WE CAN WONDER WHERE TO FIND THEM

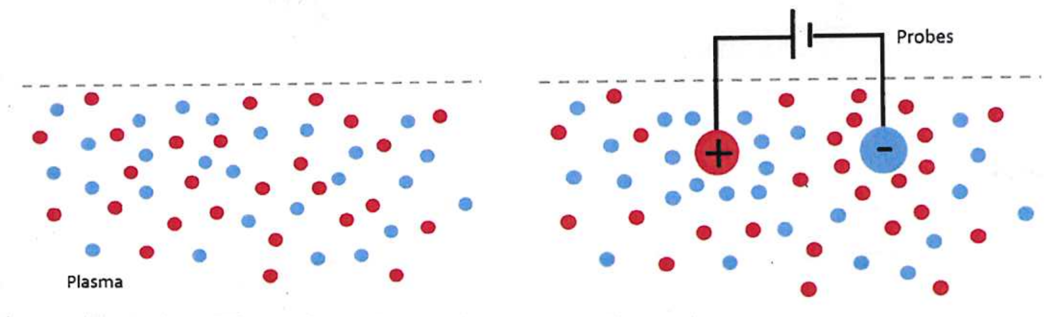
Type	Electron density n_e (cm^{-3})	Temperature T_e (eV^a)
Stars	10^{26}	2×10^3
Laser fusion	10^{25}	3×10^3
Magnetic fusion	10^{15}	10^3
Laser-produced	10^{18}–10^{24}	10^2–10^3
Discharges	10^{12}	1–10
Ionosphere	10^6	0.1
Interstellar medium	1	10^{-2}

[a] 1 eV ≡ 11 600 K.

- THE DEBYE SHILDING

IN MOST TYPES OF PLASMA QUASI-NEUTRALITY IS NOT JUST AN IDEAL STATE BUT IT IS A STATE THAT THE PLASMA ACTIVELY TRIES TO ACHIVE BY READJUSTING THE LOCAL CHARGE DISTRIBUTION IN RESPONSE TO A DISTURBANCE, LET ASSUME WE DROP A POSITIVE CHARGE IN A PLASMA SYSTEM. THE POSITIVE CHARGE WILL ATTRACT THE NEARBY e^- AND

REPEL THE POSITIVE IONS LEADING TO AN ALTERATE AVERAGE CHARGE DENSITY IN THE REGION, WE CAN CALCULATE THE POTENTIAL $\phi(\vec{r})$ OF THIS BALL AFTER SUCH A READJUSTMENT HAS TAKEN PLACE.



FIRST OF ALL WE NEED TO KNOW THE VELOCITY OF e^- AND i^+ MOVE. FOR EQUAL e^- AND i^+ TEMPERATURE WE HAVE $(T_e = T_i)$

$$\frac{1}{2} m_e \langle V_e \rangle^2 = \frac{1}{2} m_i \langle V_i \rangle^2 = \frac{3}{2} K_B T.$$

THEREFORE, FOR HYDROGEN PLASMA WHERE $Z = A = 1$

$$\frac{\langle V_i \rangle}{\langle V_e \rangle} = \left(\frac{m_e}{m_i}\right)^{1/2} = \left(\frac{m_e}{A m_p}\right)^{1/2} \simeq \frac{1}{43}$$

THIS SHOWS THA THE IONS ARE ALMOST STATIONARY ON THE e^- TIME SCALE. TO A GOOD APPROXIMATION WE OFTEN WRITE $n_i \simeq n_0$ WHERE $n_0 = N_A \rho_m / A$ IS THE MATERIAL (GAS) DENSITY, WITH ρ_m BEING THE MASS DENSITY AND N_A THE AVOGADRO NUMBER. IN THERMAL EQUILIBRIUM THE ELECTRON DENSITY FOLLOWS A BOLTZMANN DISTRIBUTION

$$\frac{n_e}{n_i} = e^{e\phi/K_B T} \Rightarrow n_e = n_i \, e^{e\phi/K_B T}$$

WHERE $\phi(\vec{r})$ IS THE POTENTIAL CREATED BY

THE EXTERNAL DISTURBANCE. FROM THE POISSON EQ WE CAN ALSO WRITE

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0} = -\frac{e}{\varepsilon_0}(n_i - n_e)$$

BY COMBINING THIS EQ. WITH THE PREVIOUS TWO IN SPHERICAL COORDINATES, $\left[\nabla^2 \to \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right)\right]$

TO ELIMINATE n_e WE OBTAIN

$$\phi_D = \frac{1}{4\pi\varepsilon_0} \frac{e^{-r/\lambda_D}}{r}$$

THIS CONDITION SUPPOSES $\phi(r) \to 0$ FOR $r \to \infty$, THE CHARACTERISTIC LENGTH-SCALE λ_D IS KNOWN AS DEBYE LENGTH AND IS GIVEN BY

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2}$$

THE DEBYE LENGTH IS A FUNDAMENTAL PROPERTIES OF NEARLY ALL PLASMA AND DEPENDS ONLY ON THE PLASMA T AND n_e. OF COURSE λ_D HAS THE DIMENSION OF A LENGTH. AN IDEAL PLASMA HAS A LARGE NUMBER OF PARTICLES PER DEBYE SPHERE: $(\lambda_D)^3$

$$N_D \equiv n_e \frac{4}{3}\pi \lambda_D^3 \gg 1$$

WHICH IS A PRE-REQUISITE FOR COLLECTIVE BEHAVIOUR. AN ALTERNATIVE PARAMETER IS THE SO CALLED "PLASMA PARAMETER"

$$g \equiv \frac{1}{n_e \Lambda_D^3}$$ WHICH IS ESSENTIALLY THE RECIPROCAL OF N_D.

- OBSERVATION CLASSICAL PLASMA THEORY IS BASE ON THE ASSUMPTION THAT $g \ll 1$. THIS IMPLIES THE DOMINANCE OF COLLECTIVE EFFECTS OVER PARTICLES COLLISIONS. PARTICLES COLLISIONS IS A COMPETING MECHANISM THAT WE NEED TO KNOW.
- COLLISIONS IN PLASMA WHEN $N_D \leq 1$ SCREENING EFFECTS ARE REDUCED AND COLLISION WILL DOMINATES THE PARTICLE DYNOMICS. IN INTERMEDIATE REGIMES THE COLLISION MECHANISMS ARE USUALLY MEASURED VIA THE ELECTRON-ION COLLISION RATE. WE ARE NOT GOING TO DERIVE IT HERE BUT JUST REPORT THE EQUATION THAT DESCRIBES THIS PARAMETER WHILE MORE DETAILS ARE GIVEN IN J.D. HUBA, NRL PLASMA FORMULARY (NAVAL RES. LAB - WHASHINGTON DC - 2007) - W. KRUER, THE PHYSICS OF PLASMA INTERACTION (ADDISON-WESLEY, BOSTON 1988).

$$\nu_{ei} = \frac{\pi^{3/2} n_e Z e^4 \ln\Lambda}{2^{1/2} (4\pi\varepsilon_0)^2 m_e^2 V_{Te}^3} \quad (s^{-1})$$

WHERE $V_{Te} = \sqrt{K_B T_e / m_e}$ IS THE e^- THERMAL VELOCITY

AND Λ IS A SLOWLY VARYING TERM, CALLED THE COULOMB LOGARITHM, WHICH TYPICALLY TAKES NUMERICAL VALUES OF THE ORDER 10-20.

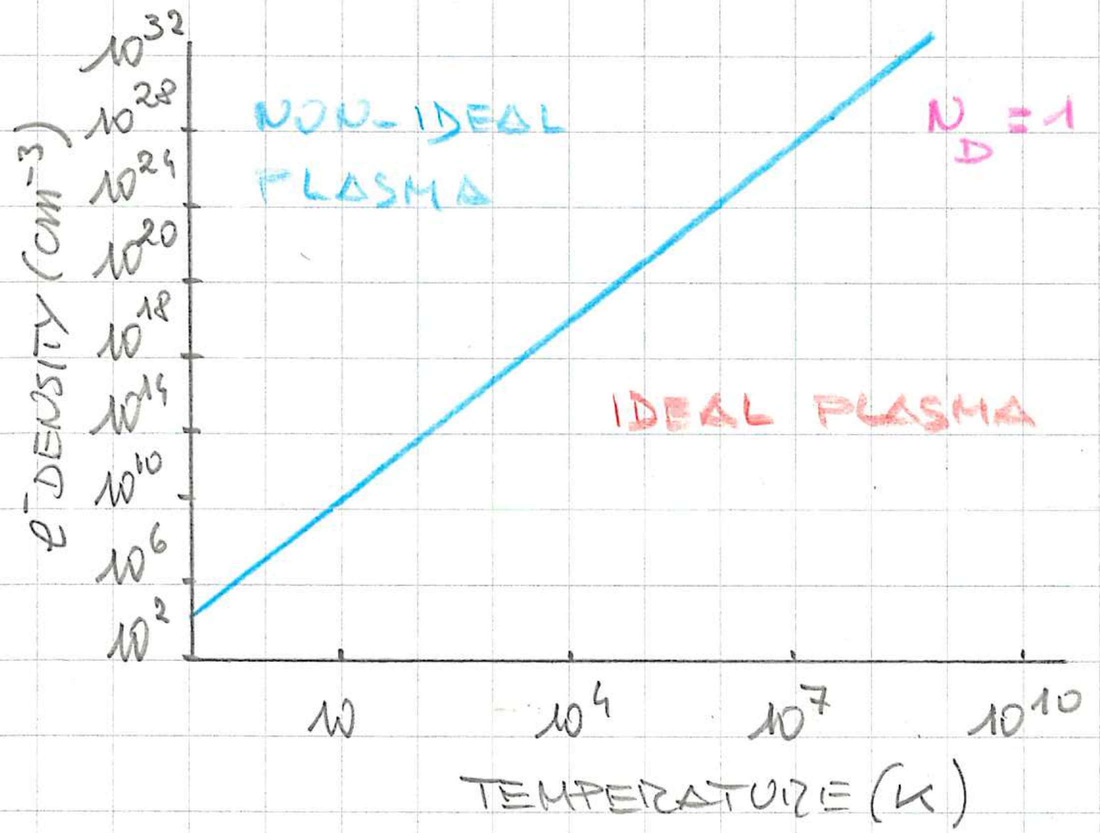
- **OBSERVATION** ν_{ei} MAY BE DIFFERENT IN OTHER TEXT BOOK DEPENDING ON THE DEFINITION GIVEN. THAT USED HERE IS CONSISTENT WITH THE REFERENCES GIVEN ABOVE. IN SUMMARY IT DEFINES THE COLLISION RATE ACCORDING TO THE AVERAGE TIME TAKEN FOR A THERMAL e^- TO BE DEFLECTED $\pi/2$ VIA MULTIPLE SCATTERING FROM FIXED IONS. THE COLLISION FREQUENCY CAN ALSO BE WRITTEN AS

$$\frac{\nu_{ei}}{\omega_p} \simeq \frac{Z \ln \Lambda}{10 N_D}$$

WITH $\Lambda \simeq 9 N_D / Z$ AND ω_p = PLASMA FREQUENCY.

- **PLASMA CLASSIFICATION**

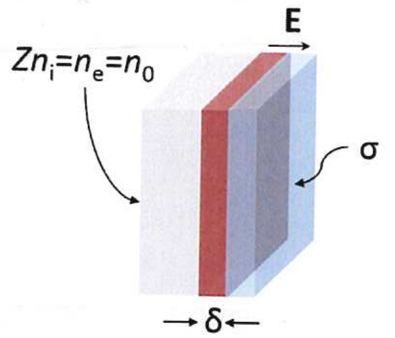
BY USING THE DEFINITION OF N_D WE CAN MAKE A CLASSIFICATION OF PLASMA TYPES IN THE DENSITY-TEMPERATURE SPACE.



PLASMA OSCILLATIONS SO FAR WE HAVE DEFINED SOME PHYSICAL PARAMETERS OF A PLASMA AT EQUILIBRIUM. WE ARE NOW GOING TO DESCRIBE HOW A PLASMA WILL RESPOND TO AN EXTERNAL PERTURBATION (LASER PULSE OR PARTICLE BEAM...).

LET'S CONSIDER THE FOLLOWING FIGURE, WHERE IN A QUASI-NEUTRAL PLASMA SLAB AN ELECTRONS LAYER IS DISPLACED FROM ITS INITIAL POSITION BY A DISTANCE δ. THIS CREATES TWO CAPACITOR PLATES WITH A SURFACE CHARGE DENSITY $\sigma = \pm e n_e \delta$ RESULTING IN AN ELECTRIC FIELD

$$\bar{E} = \frac{\sigma}{\varepsilon_0}\hat{x} = \frac{e n_e \delta}{\varepsilon_0}$$



OR $\frac{d^2\delta}{dt^2} + \omega_p^2 \delta = 0$ WHERE ω_p IS THE PLASMA FREQUENCY AS DERIVED FROM THE MOTION EQ.

$m_e \ddot{x} = (-e)\bar{E}_s$ ⇒ FOR $\bar{E}_s$ THAT INDUCES SMALL $\delta x = \delta_{x_0} e^{-i\omega t}$ AND $\bar{E}_s = \sigma / \varepsilon_0 \, \hat{x} = \frac{e n_e \delta x}{\varepsilon_0}$

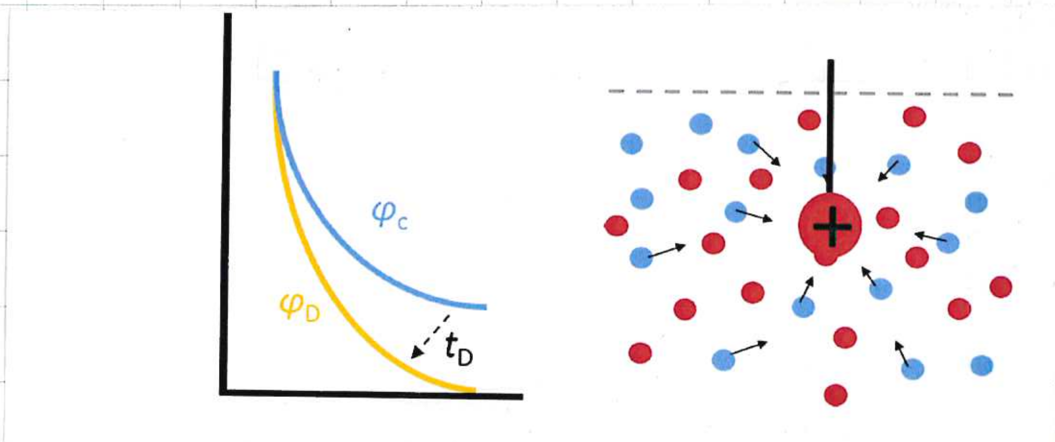
WHEN ω WILL BE $\simeq \omega_p = \sqrt{\frac{e^2 n_e}{m_e \varepsilon_0}}$ THE e^- GAS

WILL BE RESONATING AND IT WILL OSCILLATE RESPECT TO THE QUASI-STATIC POSITIVE IONS GAS. IT IS CLEAR THAT ω_p DEPENDS ONLY ON n_e BEING ALL THE OTHER TERMS CONSTANT

$$\omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m_e}} \simeq 5.6 \times 10^4 \left(n_e\right)^{1/2}$$

ω_p CAN BE DERIVED FROM THE DEBYE SCREENING MECHANISM ASKING HOW QUICKLY IT WOULD TAKE THE PLASMA AFTER A SUDDEN EXTERNAL PERTURBATION TO REACH THE QUASI-NEUTRALITY AGAIN. FOR A PLASMA AT T_e THE RESPONSE TIME TO RECOVER QUASI-NEUTRALITY IS JUST THE RATIO OF THE DEBYE LENGTH TO THE THERMAL VELOCITY $V_{Te} \equiv \sqrt{K_B T_e / m_e}$

$$\Rightarrow \quad t_D \simeq \frac{\lambda_D}{V_{Te}} = \left(\frac{\varepsilon_0 K_B T_e}{e^2 n_e} \cdot \frac{m_e}{K_B T_e}\right)^{1/2} = \omega_p^{-1}$$



- OBSERVATION IF THE PLASMA RESPONSE TIME IS SHORTER THAN THE PERIOD OF AN EXTERNAL E.M. FIELD THAN THE RADIATION WILL BE SHIELDED OUT. TO MAKE THIS STATEMENT MORE QUANTITATIVE WE CAN CONSIDER THE RATIO

$$\frac{\omega_p^2}{\omega^2} = \frac{e^2 n_e}{\varepsilon_0 m_e} \cdot \frac{\lambda^2}{4\pi^2 c^2}$$

SETTING THIS TO 1 DEFINES THE λ_μ FOR WHICH $n_e = n_c$ WHERE n_c IS THE CRITICAL DENSITY. THIS $\Rightarrow$ $n_c \simeq 10^{21} \lambda_\mu^{-2}$ (cm^{-3}). RADIATION WITH $\lambda > \lambda_\mu$ WILL BE REFLECTED. (IN THE PRE-SATELLITE/CABLE ERA THIS PROPERTIES WAS EXPLOITED TO GOOD EFFECT IN THE TRANSMISSION OF LONG-WAVE RADIO SIGNALS WHICH UTILIZES THE REFLECTION FROM THE EARTH IONOSPHERE TO EXTEND THE RANGE OF TRANSMISSION.

- OBSERVATION WE SHOULD REMIND ALSO HERE THE PLASMA FREQUENCY IN GOOD METALS AS DERIVED FROM THE DRUDE MODEL. REMEMBERING THE EXPRESSION FOR THE CONDUCTIVITY $\sigma_0 = n_e e^2 \tau / m_e$, WHERE τ IS THE AVERAGE TIME BETWEEN INELASTIC COLLISSION, FOR THE QUASI-STATIC CASE WE SEEK NOW TO FIND THE EXPRESSION OF THE CONDUCTIVITY FOR HIGH-FREQUENCY HARMONIC E.M. FIELDS (OPTICAL CONDUCTIVITY $\tilde{\sigma}$).

THE EQ. OF MOTION IS GIVEN BY

$$m\ddot{r} = q\tilde{E}_0 e^{-i\omega t} - m\dot{r}/\tau \qquad \text{BEING } \tilde{E} = \tilde{E}_0 e^{-i\omega t}.$$

IGNORING TRANSIENTS A HARMONIC SOLUTION FOR $\dot{r}(t) \equiv \bar{v}(t)$ IS

$$\bar{v}(t) = \frac{q\tilde{E}_0/m}{1/\tau - i\omega} e^{-i\omega t} = \tilde{v}(\omega) e^{-i\omega t}$$

THE AMPLITUDE OF THE TIME-HARMONIC CURRENT DENSITY THAT DEVELOP IN THE SYSTEM IS THEN

$$\tilde{J}(\omega) = -n_e e \tilde{v}(\omega) = \frac{n_e e^2 \tau}{m} \frac{\tilde{E}_0}{1 - i\omega\tau}$$

$$= \tilde{\sigma}(\omega) \tilde{E}_0(\omega)$$

THIS EQ. DEFINES A COMPLEX FREQUENCY-DEPENDENT DRUDE CONDUCTIVITY,

$$\tilde{\sigma}(\omega) = \frac{n_e e^2 \tau / m}{1 - i\omega\tau} = \frac{\sigma_0}{1 - i\omega\tau}$$

BY MULTIPLING $\tilde{\sigma}(\omega)$ FOR $\frac{\varepsilon_0}{\varepsilon_0}$ WE OBTAIN A PHYSICAL QUANTITY

$$\omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m}$$ THAT HAS

THE DIMENSION OF $\left(1/s\right)^2 \Rightarrow$ THE SQUARE OF

A FREQUENCY WHICH IS CONSISTENT WITH THE PLASMA FREQUENCY DERIVED FOR A PLASMA. HENCE ALSO IN METALS WE CAN DEFINE A PLASMA FREQUENCY. REMEMBERING THAT FROM THE ABRAHAM-LORENTZ MODEL THE COMPLES DIELECTRIC FUNCTION IS $\tilde{\varepsilon}(\omega) = \varepsilon_0 + i \frac{\tilde{\sigma}(\omega)}{\omega}$

WE CAN WRITE THE DRUDE DIELECTRIC FUNCTION

$$\frac{\tilde{\varepsilon}(\omega)}{\varepsilon_0} = \left[1 - \frac{\omega_p^2 \tau}{1 + \omega^2 \tau^2}\right] + i\left[\frac{\omega_p^2 \tau}{\omega} + \frac{1}{1 + \omega^2 \tau^2}\right]$$

- PLASMA CREATION TO GAIN SOME IDEA OF WHEN FIELD IONIZATION OCCURS WE NEED TO KNOW THE TYPICAL FIELD STRENGTH REQUIRED TO IONIZE AN ATOM. AT THE BOHR RADIUS

$$a_B = \frac{\hbar^2}{m_e e^2} \simeq 5.3 \times 10^{-9} \text{ cm}$$

THE COULOMB FIELD STRENGTH IS

$$E_C = \frac{e}{4\pi\varepsilon_0 a_B^2} \simeq 5.1 \times 10^9 \, V/m$$

THE THRESHOLD CAN BE EXPRESSED IN THE SO CALLED ATOMIC INTENSITY

$$I_C = \frac{\varepsilon_0 c E_C^2}{2} \simeq 3.51 \times 10^{16} \, W/cm^2$$

A LASER INTENSITY

$I_L > I_C$ WILL THEREFOR GUARANTEE IONIZATION FOR ANY TARGET MATERIAL ALTHOUGH IT CAN ALSO OCCOUR BELOW THIS THRESHOLD DUE TO NON-LINEAR MULTIPHOTON EFFECT.

- OBSERVATION COMMENT ON THE LINEAR PROCESSES SUCH AS THE PHOTOELECTRIC OR PHOTO IONIZATION EFFECTS AND NON-LINEAR EFFECTS.

SIMULTANEOUS FIELD IONIZATION OF MANY ATOMS CAN PRODUCE A PLASMA WITH ELECTRON DENSITY n_e AND $T_e \sim 1-10 \, eV$.

- RELATIVISTIC THRESHOLD

BEFORE DISCUSSING WAVE PROPAGATION IN PLASMA IS USEFUL TO HAVE SOME IDEA OF THE IONIZATION MECHANISM ITSELF. TO DO SO LET'S WRITE THE EQ. OF MOTION FOR A FREE ELECTRON EXPOSED TO A LINEARLY POLARIZED $\bar{E} = (E_0 \sin \omega t)\hat{y}$.

$$\frac{dv}{dt} \simeq -\frac{e E_0}{m_e} \sin \omega t$$

BY INTEGRATING WE OBTAIN THE e^- VELOCITY

$$v = \frac{e E_0}{m_e \omega} \cos \omega t$$

$$= v_0(\omega) \cos \omega t$$

THE OSCILLATION AMPLITUDE $a_0 \equiv \frac{v_0}{c} = \frac{p_0}{m_e c} = \frac{eE_0}{m_e \omega c}$

IN MANY ARTICLES a_0 IS REFERRED AS "QUIVER" VELOCITY OR MOMENTUM. THE LASER INTENSITY, WAVELENGTH λ_L, E_0 AND ω ARE RELATED THROUGH

$$I_L = \frac{1}{2}\varepsilon_0 c E_0^2 \,, \quad \lambda_L = \frac{2\pi c}{\omega} \Rightarrow$$

$a_0 \simeq 0.85\left(I_{18}\lambda_\mu^2\right)^{1/2}$ WHERE $I_{18} = \frac{I_L}{10^{18}(W/cm^{-2})}$

$\lambda_\mu = \frac{\lambda_L}{\mu m}$. FROM THIS EXPRESSION IT CAN BE SEEN THAT WE WILL HAVE RELATIVISTIC v_e, OR $a_0 \sim 1$ FOR INTENSITIES $I_L \geq 10^{18}\, W/cm^2$ AT $\lambda_L \simeq 1\,\mu m$.