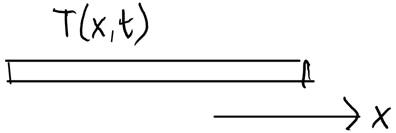
Eq. del calore: $\frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial x^2}$



Stato stazionario: $\underline{T(x)}$ $\quad \frac{\partial T}{\partial t} = 0$

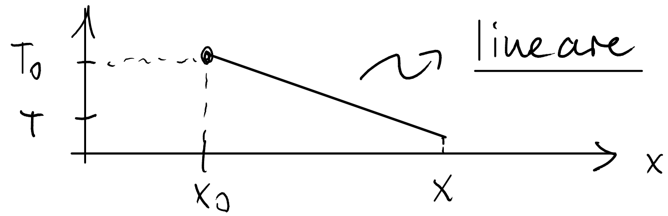
$\frac{\partial^2 T}{\partial x^2} = 0$ (stazionario) $\rightarrow \sum \vec{F} = \vec{0} \Rightarrow \frac{d^2 \vec{r}}{dt^2} = 0$

$\frac{d^2 T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = A \Rightarrow \int_{x_i}^{x_f} \frac{dT}{dx}\, dx = \int_{x_i}^{x_f} A\, dx \Rightarrow T_f - T_i = A(x_f - x_i)$

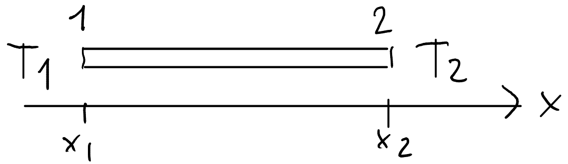
$\Rightarrow \boxed{T = A(x - x_0) + T_0}$

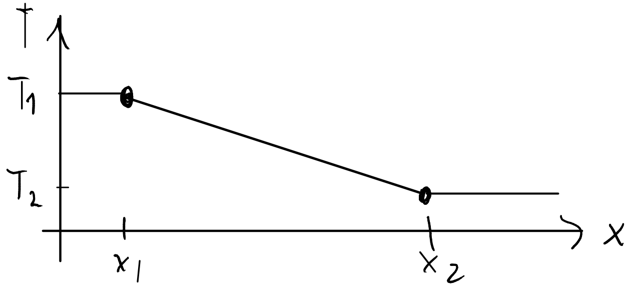
Legge di Fourier:

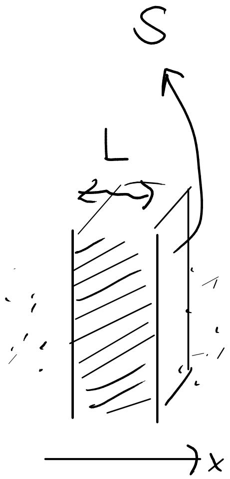
$J_u = -\lambda \frac{dT}{dx} \qquad A = cost \Longrightarrow J_u = cost$



lineare







$L = x_2 - x_1$

$T_2 = A(x_2 - x_1) + T_1 \qquad A = -\frac{J_u}{\lambda}$

$T_2 = -\frac{J_u}{\lambda} L + T_1 \qquad I_u = J_u \cdot S$

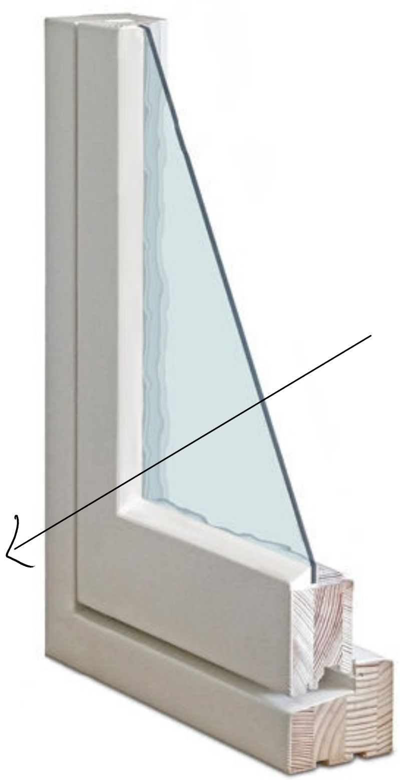
$T_1 - T_2 = \frac{J_u}{\lambda} L = \boxed{\frac{L}{\lambda S}} I_u$

$\Delta T \sim I_u$ $\qquad R \equiv \dfrac{L}{\lambda S}$ resistenza termica $\qquad \boxed{\Delta T = R I_u}$

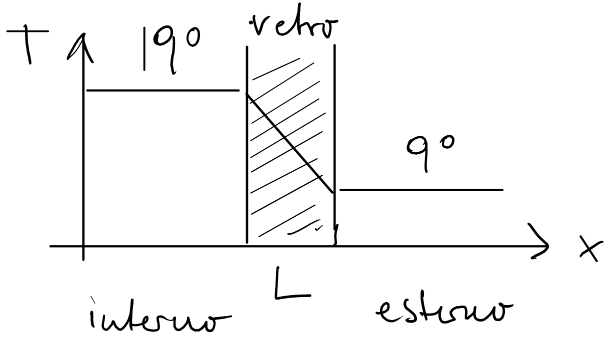
SI: $\lambda \rightarrow \dfrac{W}{K \cdot m}$ $\qquad R \rightarrow \dfrac{m}{W\, m^2} K \cdot m = \dfrac{K}{W}$

$R \sim L$; $R \sim \dfrac{1}{\lambda}$; $R \sim \dfrac{1}{S}$

corrente: $I \equiv I_u \rightarrow \Delta T = R I = I R \qquad (\Delta V = I R)$



Es.: vetro semplice $\quad L = 5\,mm$; $S = 5\,m^2$; $\lambda = 1 \dfrac{W}{K \cdot m}$



stazionario

$\Delta T = R I$

$$R = \frac{\cancel{5} \times 10^{-3}\, m}{\cancel{5}\, m^2 \times 1 \frac{W}{K \cdot m}} = 10^{-3} \frac{K}{W}$$

$$I = \frac{\Delta T}{R} = \frac{10\, K}{10^{-3} \frac{K}{W}} = 10^4\, W = 10\, kW$$

$\Delta t = 1g \rightarrow Q = ? \rightarrow J$; kWh

$Q = I \cdot \Delta t$

$= 10^4 W \cdot 3600 \cdot 24\, s$

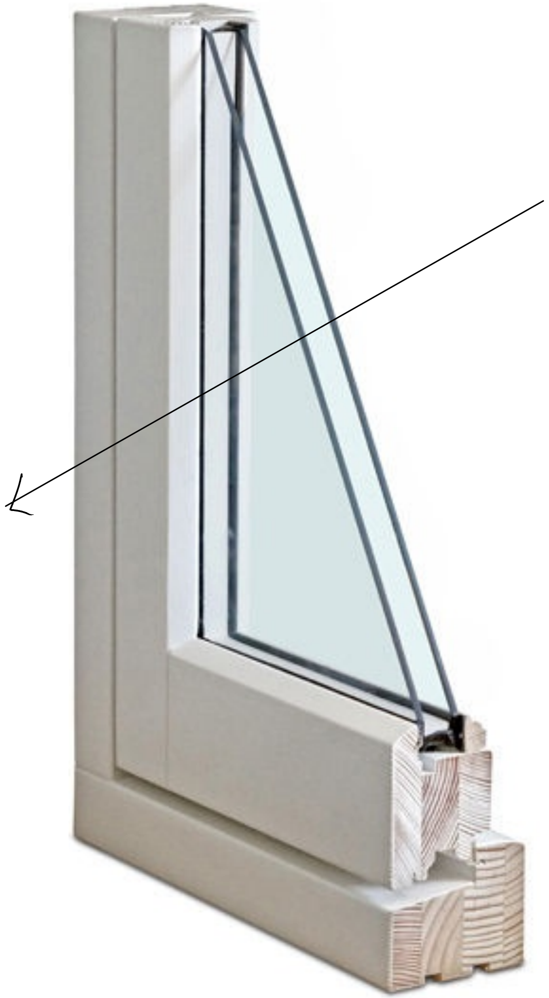
$= 8{,}64 \times 10^8\, J$

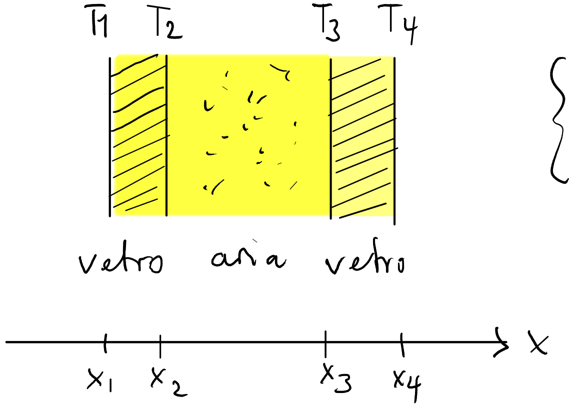
$= 10\, kW \cdot 24\, h = 240\, kWh$

Elettricità:

$0{,}05$ €/kWh $\rightarrow 12$ €/g

Resistenze termiche in serie





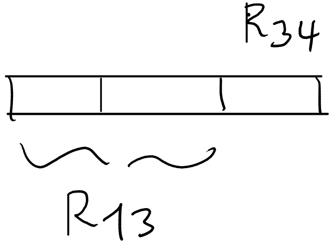
Stato stazionario : $I = cost$

$$\begin{cases} T_1 - T_2 = R_{12} I_{12} = R_{12} I & \text{vetro} \\ T_2 - T_3 = R_{23} I_{23} = R_{23} I & \text{aria} \end{cases}$$

$$T_1 - T_3 = (R_{12} + R_{23}) I$$

$$\Delta T = R \quad I$$

$R_{13} = R_{12} + R_{23} \rightarrow$ Resistenze in serie si sommano



$$R_{14} = R_{tot} = R_{13} + R_{34} = R_{12} + R_{23} + R_{34}$$

$$\boxed{R_{tot} = R_1 + R_2 + \dots + R_n}$$

Es: doppio vetro, $L = 5\,mm$, $S = 5\,m^2$, $x_3 - x_2 = 10\,mm$

$R_{12} = R_{34} = 10^{-3} \frac{K}{W}$ vetro

$\lambda_{aria} = 24 \times 10^{-3} \frac{W}{K\,m}$

$$R_{23} = \frac{L}{\lambda S} = \frac{0.01\,m}{5\,m^2 \times 24 \times 10^{-3} \frac{W}{K\,m}} = 8{,}3 \times 10^{-2} \frac{K}{W} = 83\,R_{12}$$

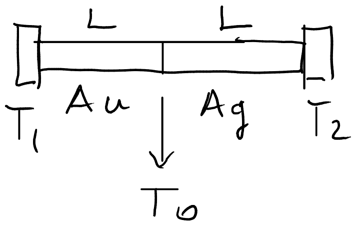
$$R_{tot} = R_{12} + R_{34} + R_{23} = 0{,}2 \times 10^{-2}\ \frac{K}{W} + 8{,}3 \times 10^{-2}\ \frac{K}{W} = 8{,}5 \times 10^{-2}\ \frac{K}{W}$$

$$\Delta T = R_{tot} I \Rightarrow I = \frac{\Delta T}{R_{tot}} = \frac{1}{85} \boxed{\frac{\Delta T}{R_{12}}} = \frac{10^4\ W}{85} \approx 120\ W \Rightarrow 0{,}14\ €/g$$

$$R_{tot} = 85\, R_{12}$$

singolo vetro

ES:



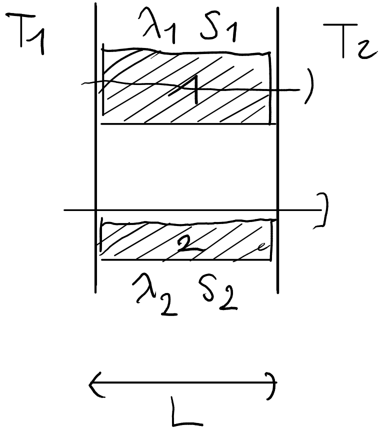
$T_1 = 80°$

$T_2 = 30°$

$T_0 = ?$

stazionario

Resistenze termiche in parallelo



$$R_1 = \frac{L}{\lambda_1 S_1}$$

$$R_2 = \frac{L}{\lambda_2 S_2}$$

$$\Delta T = T_1 - T_2$$

Corrente termica totale

$$I = I_1 + I_2 = \frac{\Delta T}{R_1} + \frac{\Delta T}{R_2} = \Delta T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{\cancel{\Delta T}}{R_{tot}} = \cancel{\Delta T}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow \frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

resistenze termiche in parallelo: si sommano i reciproci!

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_M}$$

$$R_{tot} = \frac{R_1 R_2}{R_1 + R_2} \qquad R \equiv R_1 = R_2 \Rightarrow R_{tot} = \frac{R}{2}$$

$$\Delta T = RI \rightarrow I = \frac{\Delta T}{R}$$