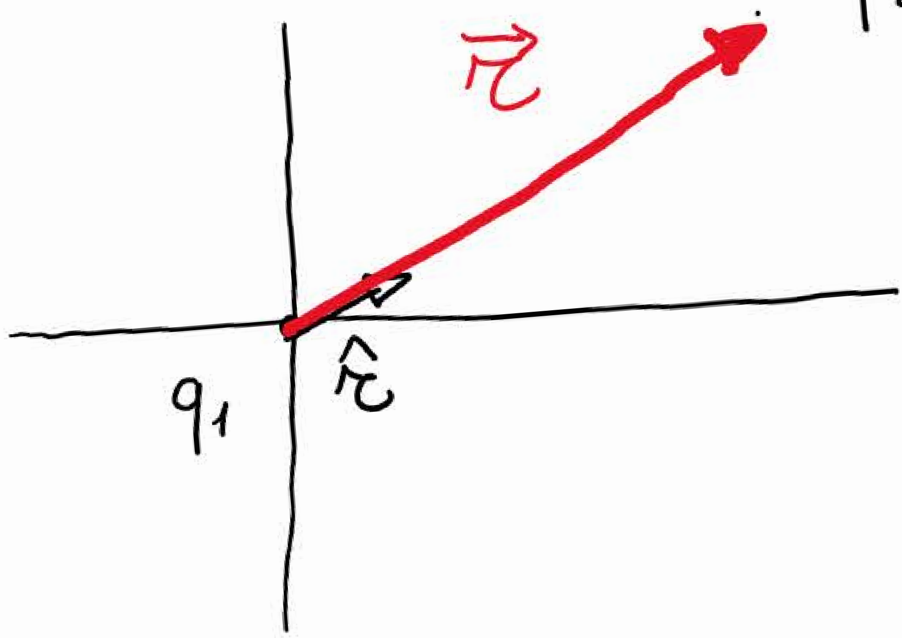
ELETTROSTATICA - FORZA DI COULOMB E CAMPO ELETTRICO

$$e = 1{,}602 \cdot 10^{-19}\ C$$



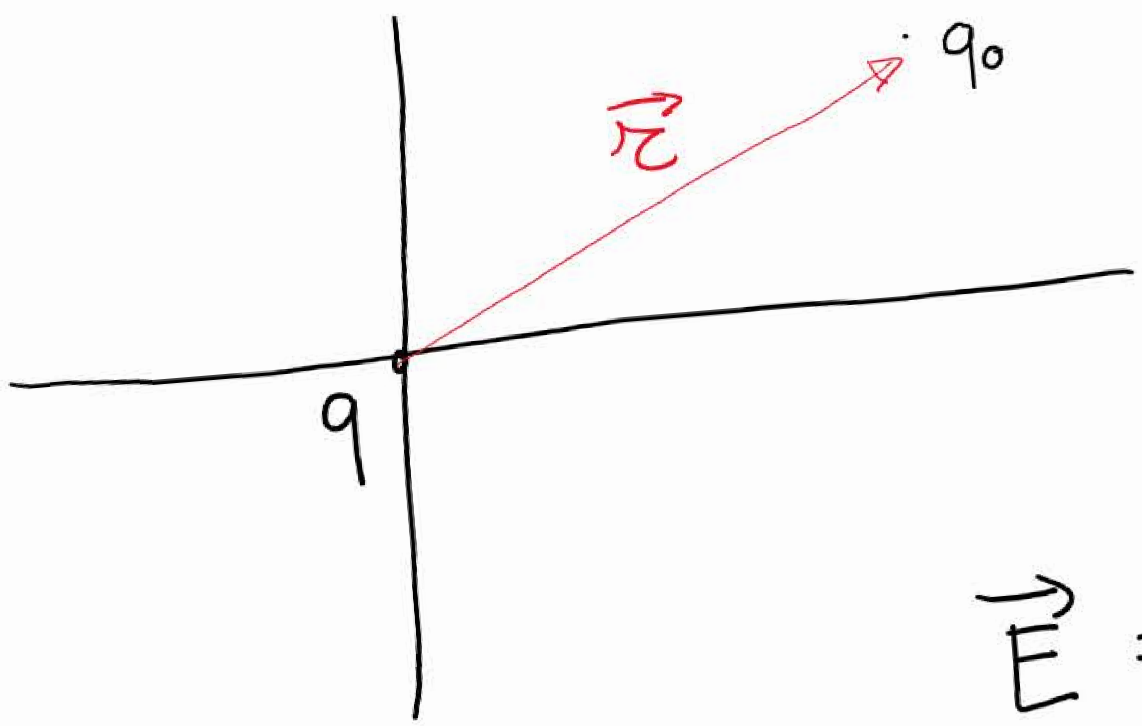
$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$$

$$r = |\vec{r}|$$

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{C^2}{m^2 N}$$

$$\varepsilon_r \geq 1$$

$\varepsilon_r = 1$ nel vuoto



$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{q q_0}{r^2} \hat{r}$$

$$\vec{E} = \frac{\vec{F}_e}{q_0} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{q}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{q}{r^2} \hat{r}}$$

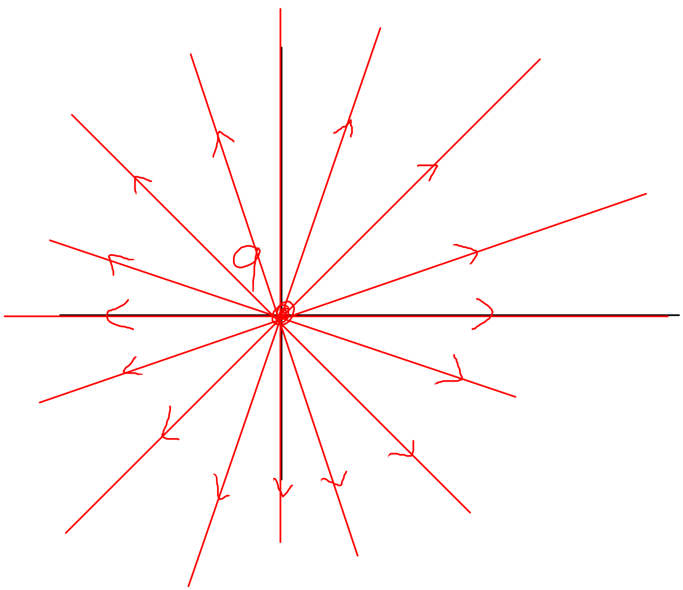
CAMPO ELETTRICO

$$\vec{E} = \frac{\vec{F}_e}{q_0}$$ dove q_0 è una carica di prova

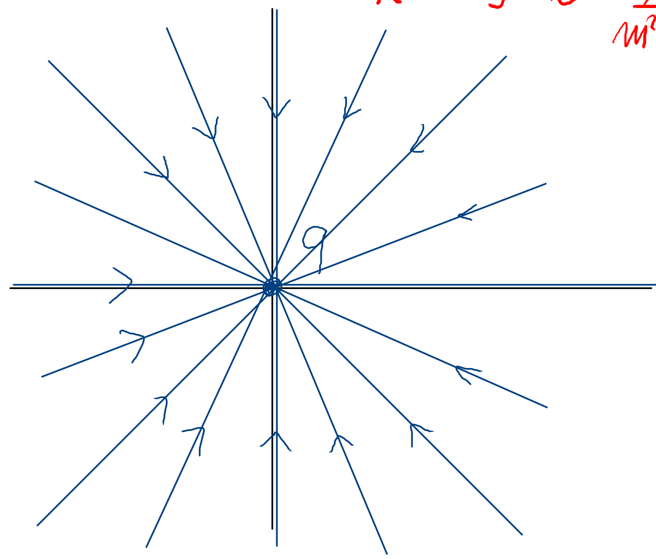
Per carica q puntiforme in O: $\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r^2}\hat{r}$

$8.854 \cdot 10^{-12}\ \frac{C^2}{m^2 N}$

$K = 9 \cdot 10^9\ \frac{N}{m^2 C^2}$



$q > 0$



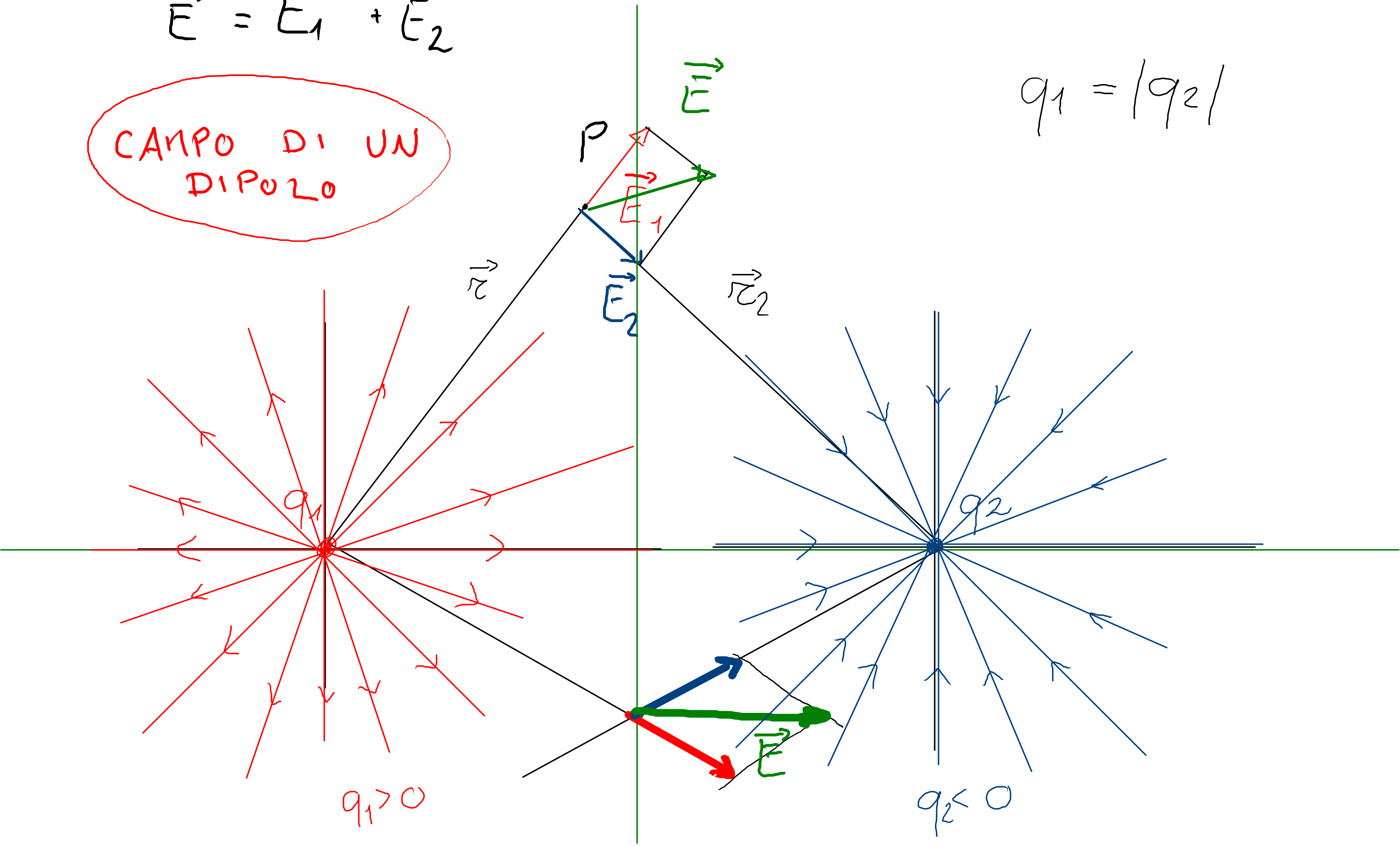
$q < 0$

PRINCIPIO DI SOVRAPPOSIZIONE

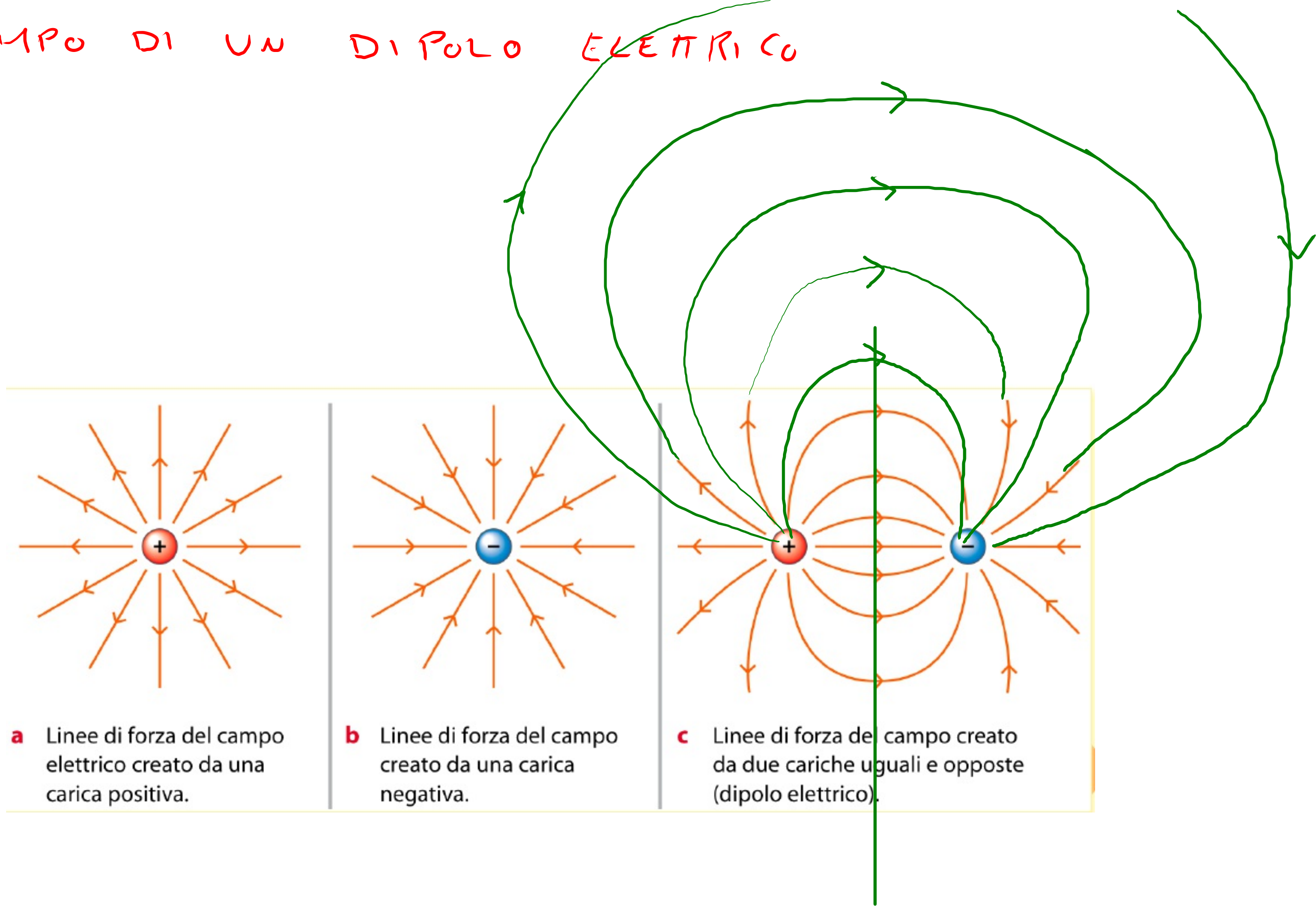
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

CAMPO DI UN DIPOLO

$q_1 = |q_2|$



CAMPO DI UN DIPOLO ELETTRICO

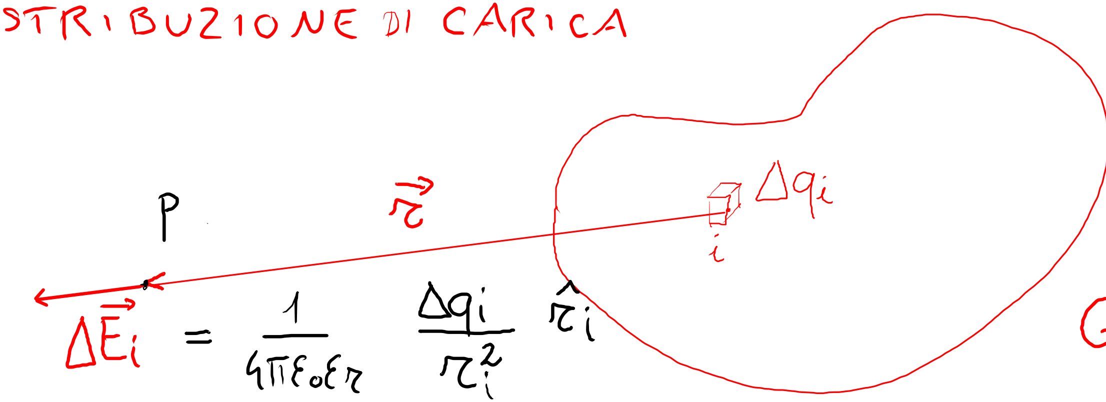


a Linee di forza del campo elettrico creato da una carica positiva.

b Linee di forza del campo creato da una carica negativa.

c Linee di forza del campo creato da due cariche uguali e opposte (dipolo elettrico).

CAMPO ELETTRICO PER UNA GENERICA DISTRIBUZIONE DI CARICA



Q

$$\Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{\Delta q_i}{r_i^2}\,\hat{r}_i$$

$$Q = \sum_i \Delta q_i$$

$$\vec{E} \cong \sum_i \Delta\vec{E}_i = \sum_i \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{\Delta q_i}{r_i^2}\,\hat{r}_i$$

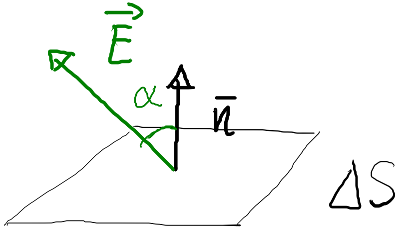
$$\vec{E} = \lim_{\Delta q_i \to 0} \sum_i \Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\int_V \frac{dq}{r^2}\,\hat{r}$$

se Q è unif. distrib in V: $\rho = \dfrac{Q}{V}$ $\qquad \displaystyle\int_V \frac{\rho\, dV}{r^2}\,\hat{r}$

Q " in S: $\sigma = \dfrac{Q}{S}$ $\qquad \displaystyle\int_S \frac{\sigma\, dS}{r^2}\,\hat{r}$

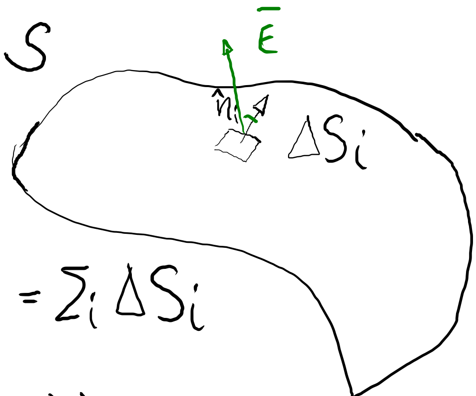
Q " in ℓ: $\lambda = \dfrac{Q}{\ell}$ $\qquad \displaystyle\int_\ell \frac{\lambda\, d\ell}{r^2}\,\hat{r}$

FLUSSO ϕ DI $\vec{E}$ ATTRAVERSO S



$$\Delta\phi(\vec{E}) = \vec{E}\cdot\underbrace{\hat{n}\cdot\Delta S}_{\Delta\vec{S}}$$

$$\Delta\phi(\vec{E}) = \vec{E}\cdot\Delta\vec{S} = E\cdot\Delta S\cdot\cos\alpha$$



$$\phi(E) \cong \sum_i \Delta\phi_i(E)$$

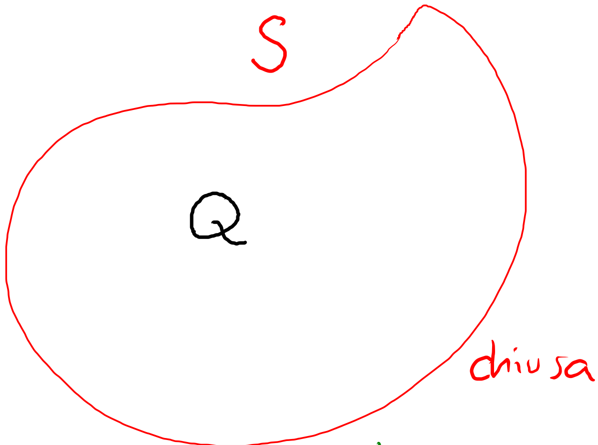
$$S = \sum_i \Delta S_i$$

$$\phi(\vec{E}) = \lim_{\Delta S_i\to 0} \sum_i \Delta\phi_i(E)$$

$$\phi(\vec{E}) = \lim_{\Delta S_i\to 0} E_i\,\Delta S_i\cos\alpha_i = \int_S \vec{E}\cdot\underbrace{\hat{n}\,dS}_{d\vec{S}} = \int_S \vec{E}\cdot d\vec{S}$$

TEOREMA DI GAUSS

(flusso di $\vec{E}$ attraverso S chiusa)



$$\phi(\vec{E}) = \oint_S \vec{E} \cdot d\vec{S}$$

$$\boxed{\phi(\vec{E}) = \frac{Q}{\varepsilon_0 \varepsilon_r}}$$

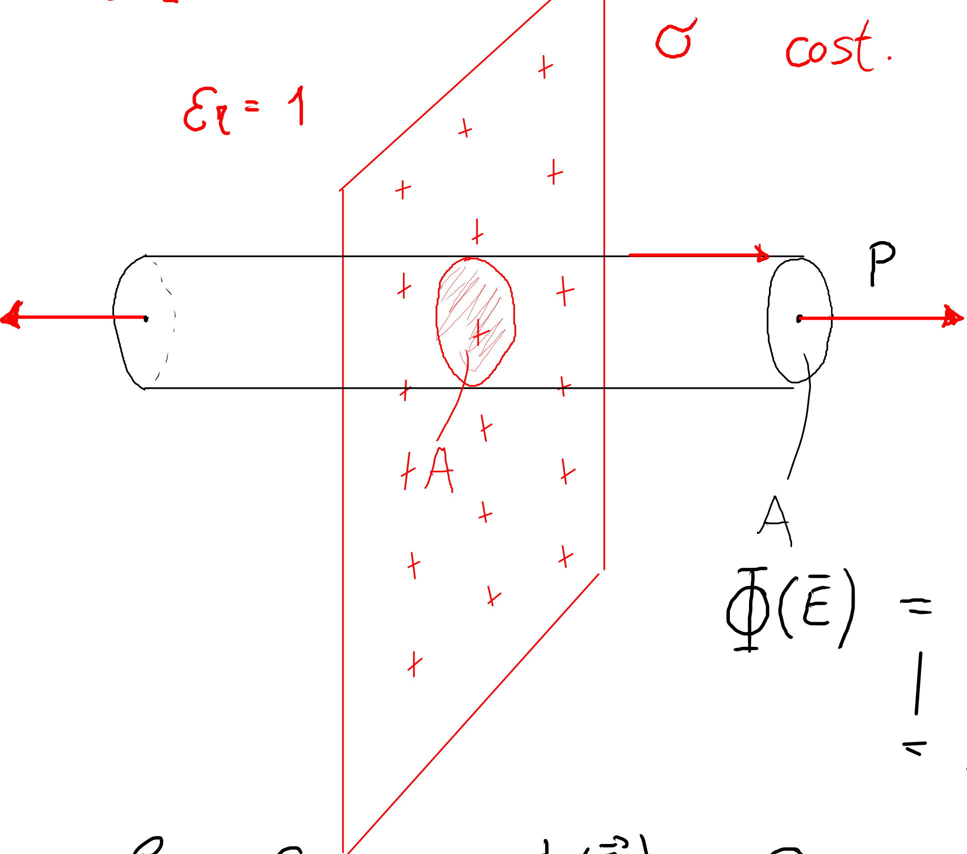
Q è la carica totale racchiusa in S

Se $Q = 0$, $\phi(\vec{E}) = 0$

FOGLIO INFINITO DI CARICHE

σ cost.

$\varepsilon_r = 1$



$$\vec{E}(P) = \,?$$

$$Q = \sigma A$$

$$\Phi(\vec{E}) = \Phi(\vec{E})_{BASI} + \cancel{\Phi(\vec{E})_{SUP.\ LAT.}} = 0$$

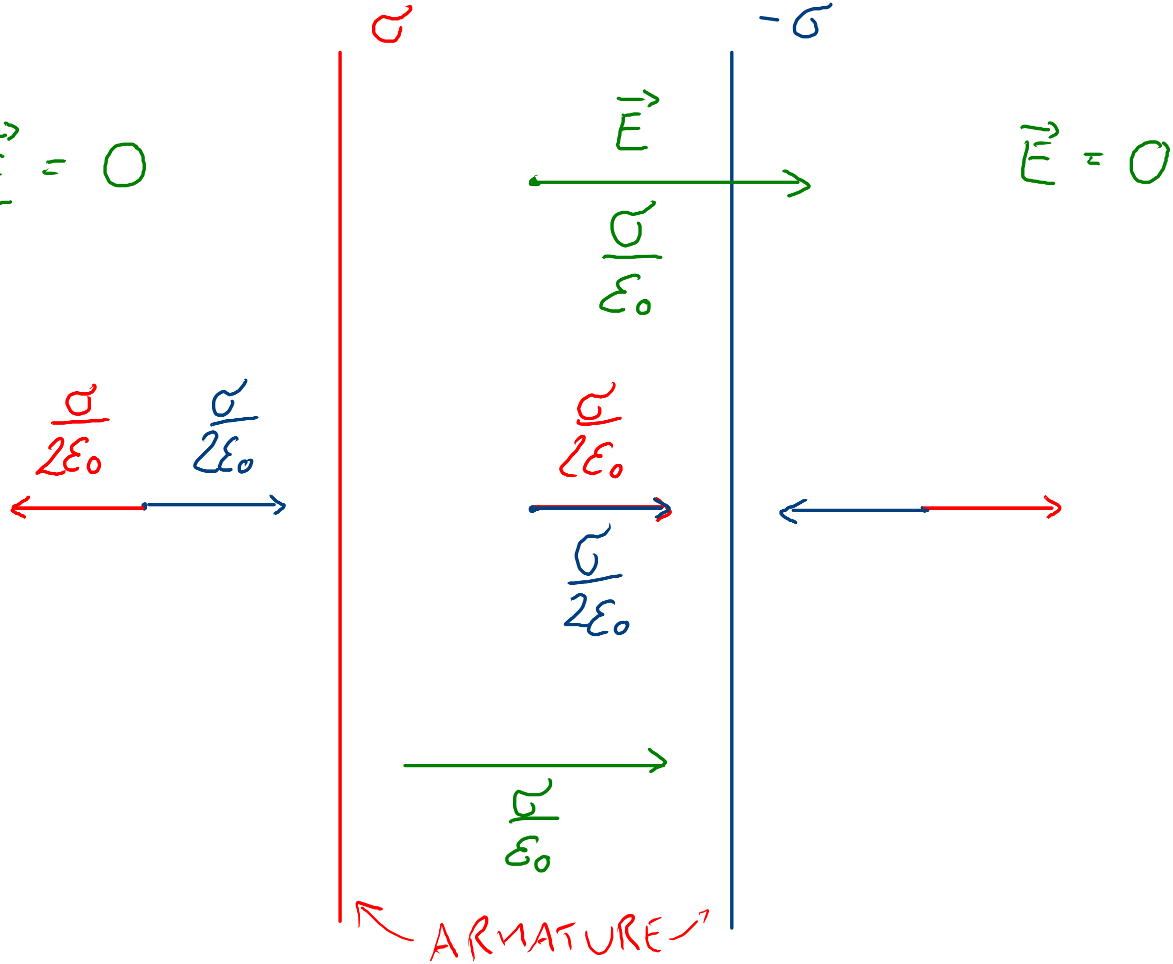
$$= 2|\vec{E}(P)| \cdot A$$

Per Gauss $\Phi(\vec{E}) = \dfrac{Q}{\varepsilon_0}$

$$2|\vec{E}(P)| \cdot \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{|\vec{E}(P)| = \frac{\sigma}{2\varepsilon_0}}$$

DOPPIO FOGLIO INFINITO DI CARICHE → CONDENSATORE



Riprendiamo il discorso nel condensatore a facce piane parallele

ENERGIA POTENZIALE ELETTRICA

$$\mathcal{L} = -\Delta U = U_A - U_B$$

$$\mathcal{L} = \int_A^B \vec{F}_e \cdot d\vec{s} = \int_A^B q_0 \vec{E} \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{F}_e = q_0 \vec{E}$$

$$\vec{E} = \frac{\vec{F}_e}{q_0}$$

$$U_A - U_B = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

$$\Delta U = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

POTENZIALE ELETTRICO

$$V = \frac{U}{q_0} \qquad \Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = V_B - V_A$$

$$[V] = \frac{J}{C} = \text{Volt} = V$$

Potenziale elettrico in un punto P

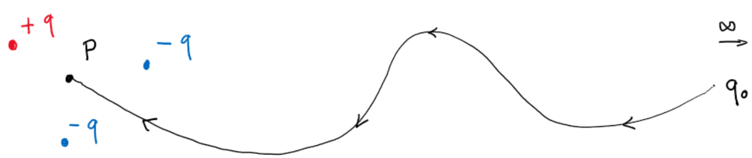
$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = V_B - V_A$$

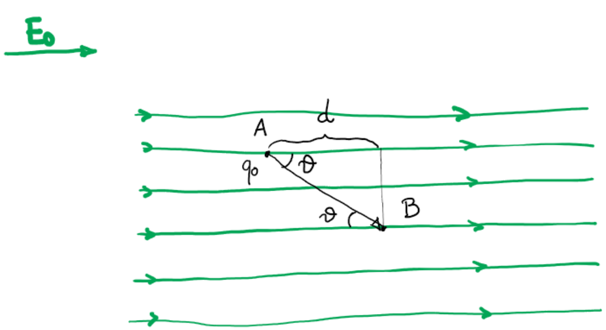
$B \to P$
$A \to \infty$
$V_\infty = 0$

$$V_P - \underbrace{\cancel{V_\infty}}_{=0} = -\int_\infty^P \vec{E} \cdot d\vec{s} = \int_P^\infty \vec{E} \cdot d\vec{s}$$

$$V_P = \int_P^\infty \vec{E} \cdot d\vec{s} = -\int_\infty^P \vec{E} \cdot d\vec{s}$$

Il potenziale V_P è il lavoro per unità di carica fatto CONTRO le forze elettriche per portare una carica q_0 dall'∞ a P.





$$\mathcal{L}_{AB} = q_0 \vec{E} \cdot \vec{AB} = q_0 |\vec{E}| \cdot |\vec{AB}| \cos\vartheta = q_0 E d \; *$$

$$\Delta U = U_B - U_A = -\mathcal{L} = -q_0 E d$$

$$\Delta V = \frac{\Delta U}{q_0} = \underbrace{-Ed = V_B - V_A}$$

$$V_A - V_B = Ed \; *$$

$$\mathcal{L} = q_0 (V_A - V_B)$$

$$\boxed{[E] = \frac{N}{C} = \frac{V}{m}} = \frac{N \cdot \cancel{m}}{C \, \cancel{m}}$$

$$\Delta V = -\int_A^B \bar{E} \cdot d\bar{s}$$

$$dV = -\vec{E} \cdot d\vec{s} \xrightarrow[\text{asse } x]{1D} dV = -E_x \cdot dx$$

$$\frac{dV}{dx} = -E_x$$

$$E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad E_z = -\frac{dV}{dz}$$

$$\xleftarrow{3D}$$

$$\vec{E} = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k}$$

$$\boxed{\vec{E} = -\overline{grad}\, V = -\bar{\nabla} V}$$

Potenziale generato da una carica puntiforme q posta in O

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q}{r^2} \hat{r}$$

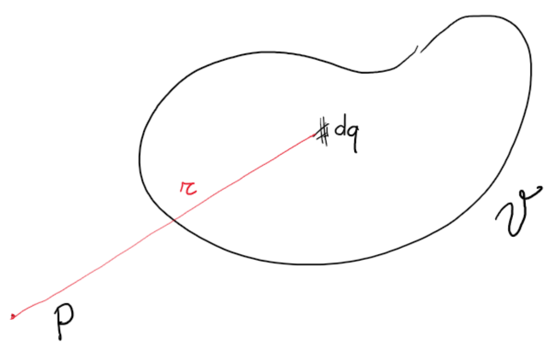
$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q}{r}$$ ← dimostrazione a pag. 63bis degli appunti

$$dV = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{dq}{r}$$ (per una carica infinitesima)

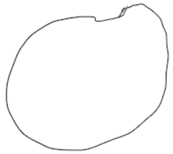
Potenziale generato da una distribuzione di carica qualsiasi

$$V(P) = \int_{\mathcal{V}} \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{dq}{r}$$

$$\boxed{V(P) = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \int_{\mathcal{V}} \frac{dq}{r}}$$

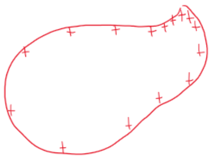


I CONDUTTORI (METALLICI)



CONDUTTORE NEUTRO $Q = 0$

$V = 0$



CONDUTTORE CARICO $Q \neq 0$

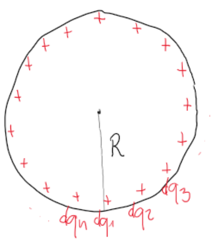
$V \neq 0$

CAPACITÀ

$\boxed{\frac{Q}{V} = C}$ capacità del conduttore $\frac{C}{V} = \text{Farad}$

$\boxed{Q = C \cdot V}$ $\frac{C}{V} = F$

CAPACITÀ DI UNA SFERA CONDUTTRICE



Q $C = \frac{Q}{V}$

V è lo stesso su tutta la sfera

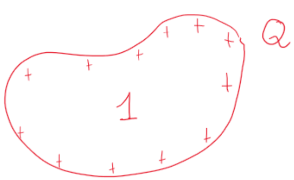
$Q = \sum_1^n dq_i$

$dV_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{dq_i}{r_i}$ con $r_i = R$ per tutte le cariche

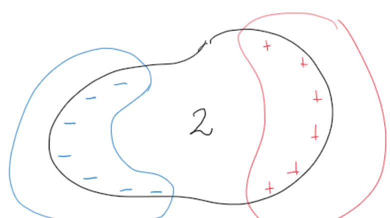
$V = \sum_1^n dV_i = \sum_1^n \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{dq_i}{R} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{1}{R} \sum_1^n dq_i = \frac{Q}{4\pi\varepsilon_0\varepsilon_r R}$

$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0\varepsilon_r R}} = 4\pi\varepsilon_0\varepsilon_r R$

CONDENSATORE



conduttore carico

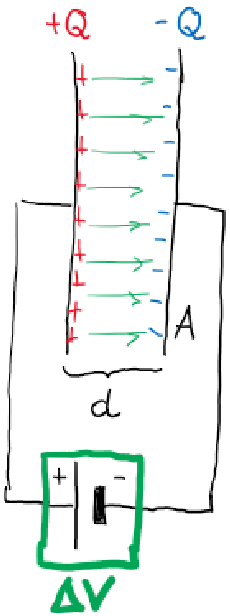


conduttore neutro

le cariche negative ⊖ abbassano il potenziale in 1 più di quanto lo alzino le cariche positive ⊕

$\Rightarrow$ la capacità di 1 è aumentata!

CONDENSATORE A FACCE PIANE & PARALLELE



1) $E = \frac{\sigma}{\varepsilon_0 (\varepsilon_r)}$

2) $\sigma = \frac{Q}{A}$

$C = \frac{Q}{\Delta V}$

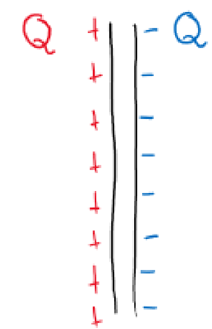
3) $E = \frac{\Delta V}{d}$

$$= \frac{\sigma A}{\Delta V} \underset{(1)}{=} \frac{E \varepsilon_0 A}{\Delta V} \underset{(3)}{=} \frac{\cancel{\Delta V} \varepsilon_0 A}{d \cancel{\Delta V}} = \varepsilon_0 \frac{A}{d}$$

(2)

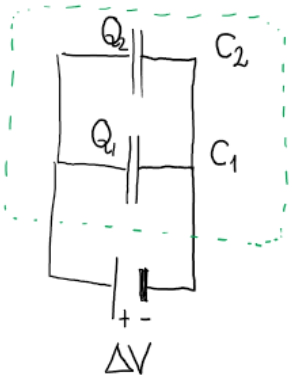
$$\boxed{C = \varepsilon_0 (\varepsilon_r) \frac{A}{d}}$$

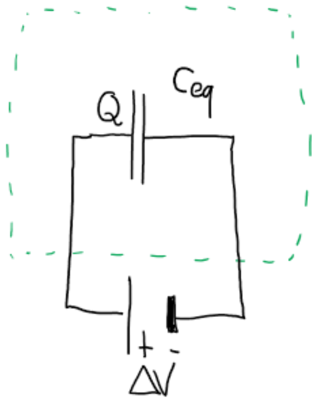
$C = \frac{A}{d} \varepsilon_0 \varepsilon_r$



$\sigma = \frac{Q}{A}$

CONDENSATORI IN PARALLELO





$$Q_1 = C_1 \Delta V_1$$
$$Q_2 = C_2 \Delta V_2$$

$$Q = C_{eq} \Delta V$$

$$\boxed{C_{eq} = C_1 + C_2}$$

$$\Delta V_1 = \Delta V_2 = \Delta V$$

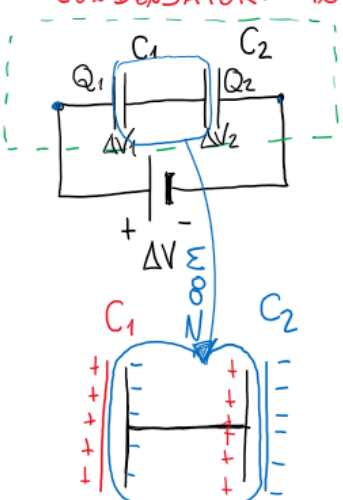
$$Q = Q_1 + Q_2$$

$$C_{eq} \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2$$

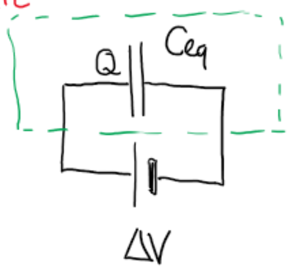
$$C_{eq} \cancel{\Delta V} = C_1 \cancel{\Delta V} + C_2 \cancel{\Delta V}$$

$$C_{eq} = C_1 + C_2$$

CONDENSATORI IN SERIE



$$Q_1 = Q_2$$



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$Q = Q_1 = Q_2$$

$$C_1 = \frac{Q_1}{\Delta V_1} \qquad C_2 = \frac{Q_2}{\Delta V_2} \qquad C_{eq} = \frac{Q}{\Delta V}$$

$$\Delta V_1 = \frac{Q_1}{C_1} \qquad \Delta V_2 = \frac{Q_2}{C_2} \qquad \Delta V = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{\cancel{Q}}{C_{eq}} = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$