$$m\frac{dV}{dt} = q\left(\bar{E} + \bar{V}\times\bar{B}\right).$$ THE V_z IS THEN

$$\frac{dV_z}{dt} = \frac{q}{m}E_z \Rightarrow V_z = \frac{qE_z}{m}t + V_{z0}.$$ THIS IS A STRAIGHT ACCELERATION ALONG $\bar{B}$, THE TRANSVERSE COMPONENTS ARE

$$\frac{dV_x}{dt} = \frac{q}{m}E_x \pm \omega_c V_y$$

(40)

$$\frac{dV_y}{dt} = 0 \pm \omega_c V_x$$

TAKING THE TIME DERIVATIVE

$$\ddot{V}_x = -\omega_c^2 V_x \;;\; \ddot{V}_y = \pm\omega_c\left(\frac{q}{m}E_x \pm \omega_c V_y\right)$$

(41)

$$= -\omega_c^2\left(\frac{E_x}{B} + V_y\right).$$

USING THE SAME PROCEDURE AS FOR EQS. (35) WE OBTAIN

$$V_x = V_\perp e^{i\omega_c t}$$

(42)

$$V_y = \pm V_\perp e^{i\omega_c t} - \frac{E_x}{B}$$

- OBSERVATION

LARMOR MOTION IS SIMILAR TO CASE WHEN $E=0$ BUT NOW THERE IS A SUPERIMPOSED DRIFT $\bar{V}_g$ OF THE GUIDING CENTER IN THE $-Y$ DIRECTION. TO OBTAIN A GENERAL FORMULA FOR $\bar{V}_g$ WE CAN SOLVE THE EQ. OF MOTION

$$m \frac{d\bar{V}}{dt} = q\left(\bar{E} + \bar{V} \times \bar{B}\right)$$

AS $m \frac{d\bar{V}}{dt}$ GIVES RISE TO CIRCULAR MOTION

(43) $\bar{E} + (\bar{V} \times \bar{B}) = 0$. TAKING THE CROSS PRODUCT OF

$$\bar{E} = -(\bar{V} \times \bar{B}) \quad (44)$$

WITH $\bar{B}$ WE OBTAIN

$$\bar{E} \times \bar{B} = \bar{B} \times (\bar{V} \times \bar{B}) = \bar{V} B^2 - \bar{B}(\bar{V} \cdot \bar{B})$$

THE TRANSVERSE COMPONENTS OF THIS EQ. ARE

$$\bar{V}_{\perp gc} = \bar{E} \times \bar{B} / B^2 \equiv \bar{V}_E \quad (45)$$

WHERE $\bar{V}_E$ IS THE ELECTRIC FIELD DRIFT OF THE GUIDING CENTER WHICH HAS THE MAGNITUDE

$$|V_E| = \frac{E\,(V/m)}{B\,(T)}\ m\,s^{-1} \quad (46)$$

- DRIFT IN UNIFORM AND NON-UNIFORM FIELDS

IN THE PREVIOU SECTION WE HAVE SOLVED THE EQS OF MOTION FOR CHARGED PARTICLES IN UNIFORM $\bar{E}$ AND $\bar{B}$ FIELDS, WE HAVE ALSO SHOWN THAT THE DRIFT OF THE GUIDING-CENTER IS

$$\bar{V}_E = \frac{\bar{E} \times \bar{B}}{B^2} \quad (47)$$

THIS CAN BE EXTENDED TO A FORM OF A GENERAL FORCE

$$\bar{V}_f = \frac{1}{q} \frac{\bar{F} \times \bar{B}}{B^2} \quad (48)$$

FOR EXAMPLE IN A GRAVITATIONAL FIELD

$$\bar{V}_g = \frac{m}{q}\,\frac{\bar{g}\times\bar{B}}{B^2} \qquad (49)$$

- OBSERVATION: SIMILAR TO THE $\bar{V}_E$ DRIFT, WHERE V_E IS $\perp$ TO BOTH FORCES IN THIS CASE PARTICLES OF OPPOSITE CHARGES DRIFT IN OPPOSITE DIRECTIONS.

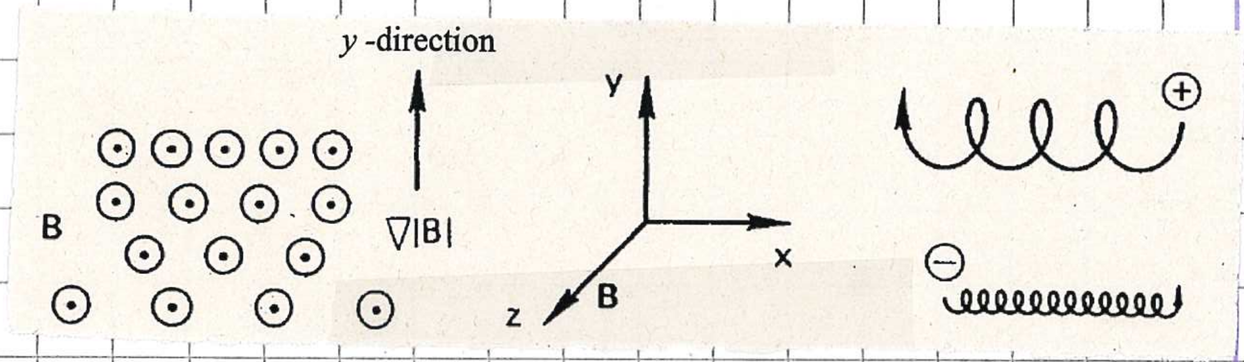
UNIFORM FIELDS PROVIDE VERY LIMITED DESCRIPTIONS FOR MANY PHENOMENA, SUCH AS PLANETARY FIELDS, CORONAL LOOPS, TOKAMACS, WHICH HAVE SPATIALLY AND TEMPORALLY VARYING FIELDS.

- DRIFT IN NON-UNIFORM MAGNETIC FIELDS

CHARGED PARTICLES DRIFT IN NON-HOMOGENEOUS FIELDS ARE CLASSIFIED IN SEVERAL WAYS. HERE WE CONSIDER ONLY TWO DRIFTS ASSOCIATED WITH SPATIALLY NON-UNIFORM $\bar{B}$: GRDIENT DRIFT AND CURVATURE DRIFT, BUT THERE ARE MANY OTHERS. VERY OFTEN THE INHOMOGENEITIES INTRODUCED END UP IN TOO COMPLICATED TRAJECTORIES TO BE OBTAINED FROM ANALYTICAL SOLUTIONS THEREFORE THE ORBIT THEORY APPROXIMATION IS USED. THIS MEANS ASSUMING THAT WITHIN ONE LARMOR ORBIT $\bar{B}$ IS $\sim$ UNIFORM, i.e. THE TYPICAL LENGTH SCALE, L, OVER WHICH $\bar{B}$ VARIES IS SUCH THAT $L \gg r_L \Rightarrow$ THE GYRO-ORBIT IS NEARLY A CIRCLE.

GRAD-$\bar{B}$ DRIFT.

WE ASSUME THE LINES OF FORCES ARE STREIGHT BUT THEIR DENSITY INCREASES IN THE $\hat{y}$ DIRECTION (SEE BELOW)



THE GRADENT IN $|\bar{B}|$ CAUSES THE LARMOR RADIUS ($r_L = mv/qB$) TO BE LARGER AT THE BOTTOM OF THE ORBIT THAN THE TOP WHICH LEADS TO A DRIFT. THIS SHOULD BE $\perp$ TO THE $\bar{\nabla}|\bar{B}|$ AND q^+ AND q^- DRIFT IN OPPOSITE DIRECTIONS.

LET'S CONSIDER SPATIALLY-VARYING MAGNETIC FIELD, $\bar{B} = (0, 0, B_z(y))$, $\bar{B}$ HAS ONLY $\hat{z}$ COMPONENT THE STRENGTH OF WHICH VARIES WITH y.

ASSUME $\bar{E} = 0 \Rightarrow \bar{F} = q(\bar{v} \times \bar{B})$, WHICH COMPONETS ARE

$$(50) \quad \begin{cases} F_x = q(v_y B_z) \\ F_y = -q(v_x B_z) \\ F_z = 0 \end{cases}$$

NOW THE GRADIENT OF B_z IS

$$\frac{dB_z}{dy} \sim \frac{B_z}{L} \ll \frac{B_z}{r_L} \Rightarrow r_L \frac{dB_z}{dy} \ll B_z \quad (51)$$

THIS MEANS THAT THE $|\bar{B}|$ CAN BE EXPANDED IN TAYLOR SERIES FOR DISTANCES $y < r_L$

$$B_z(y) = B_0 + y\frac{dB_z}{dy} + \dots \quad (52)$$

THUS THE (50) BECOME

$$(53)\quad \begin{cases} F_x = qV_y\left(B_0 + y\dfrac{dB_z}{dy}\right) \\ F_y = -qV_x\left(B_0 + y\dfrac{dB_z}{dy}\right) \end{cases}$$

THEREFORE, CHARGED PARTICLES IN A $\bar{B}$ FIELD TRAVELLING ABOUT A GUIDE CENTER (0,0) HAVE A HELICAL TRAJECTORY

$$(54)\quad \begin{cases} x = r_L \sin(\omega_c t) \\ y = \pm r_L \cos(\omega_c t) \end{cases} \Rightarrow$$

THE VELOCITIES CAN BE WRITTEN IN A SIMILAR FOR

$$(55)\quad \begin{cases} V_x = V_\perp \cos(\omega_c t) \\ V_y = \pm V_\perp \sin(\omega_c t) \end{cases}$$

BY SUBSTITUTING 55 INTO 53 GIVES

$$(56)\quad \begin{cases} F_x = -qV_\perp \sin(\omega_c t)\left[B_0 \pm r_L \cos(\omega_c t)\dfrac{dB_z}{dy}\right] \\ F_y = -qV_\perp \cos(\omega_c t)\left[B_0 \pm r_L \cos(\omega_c t)\dfrac{dB_z}{dy}\right] \end{cases}$$

SINCE WE ARE ONLY INTERESTED IN THE GUIDING CENTRE MOTION WE AVERAGE THE FORCE

OVER A GYROPERIOD $\Rightarrow$ IN THE $\hat{x}$

$$\langle F_x \rangle = -qV_\perp \left[B_0 \langle \sin(\omega_c t) \rangle \pm r_L \langle \sin(\omega_c t) \cos(\omega_c t) \rangle \frac{dB_z}{dy} \right]$$

BUT $\langle \sin(\omega_c t) \rangle = 0$ AND $\langle \sin(\omega_c t) \cos(\omega_c t) \rangle = 0$

$$\Downarrow$$

$$\langle F_x \rangle = 0 \quad (57)$$

IN THE $\hat{y}$ DIRECTION

$$\langle F_y \rangle = -qV_\perp \left[B_0 \langle \cos(\omega_c t) \rangle \pm r_L \langle \cos^2(\omega_c t) \rangle \frac{dB_z}{dy} \right]$$

$$= \mp \frac{qV_\perp r_L}{2} \frac{dB_z}{dy} \quad (58)$$

WHERE $\langle \cos(\omega_c t) \rangle = 0$ AND $\langle \cos^2(\omega_c t) \rangle = \frac{1}{2}$

IN GENERAL, DRIFT OF GUIDING-CENTER IS

$$\bar{V}_f = \frac{1}{q} \frac{\bar{F} \times \bar{B}}{B^2}$$

THEREFORE USING EQ 58 WE OBTAIN

$$\bar{V}_{\nabla B} = \frac{1}{q} \frac{\langle F_y \rangle \hat{y} \times B_z \hat{z}}{B_z^2} = \mp \frac{V_\perp r_L}{2B_z} \frac{dB_z}{dy} \hat{x}$$

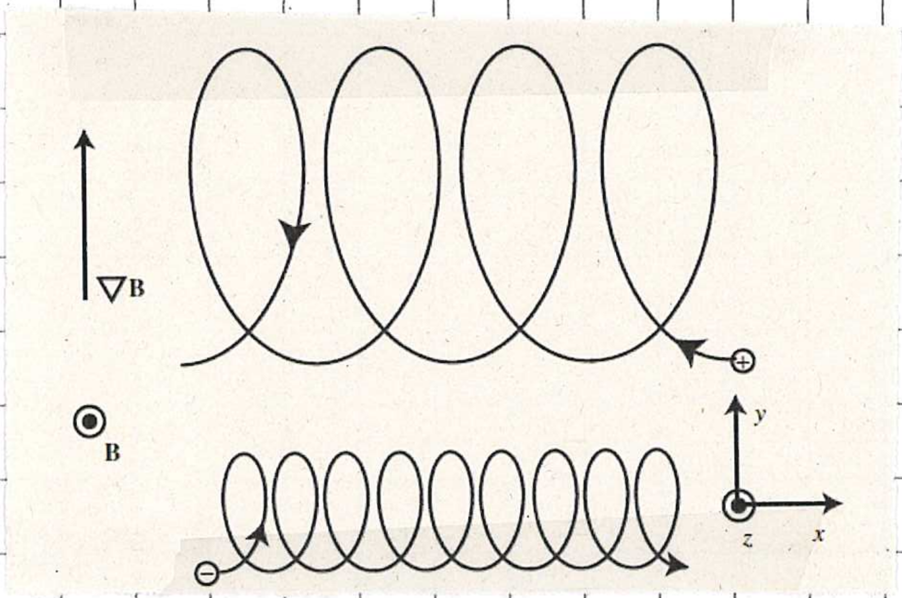
$\Rightarrow$ + CHARGES DRIFT TO $-\hat{x}$ AND VICEVERSA.

IN 3D THE RESULT CAN BE GENERALIZED TO

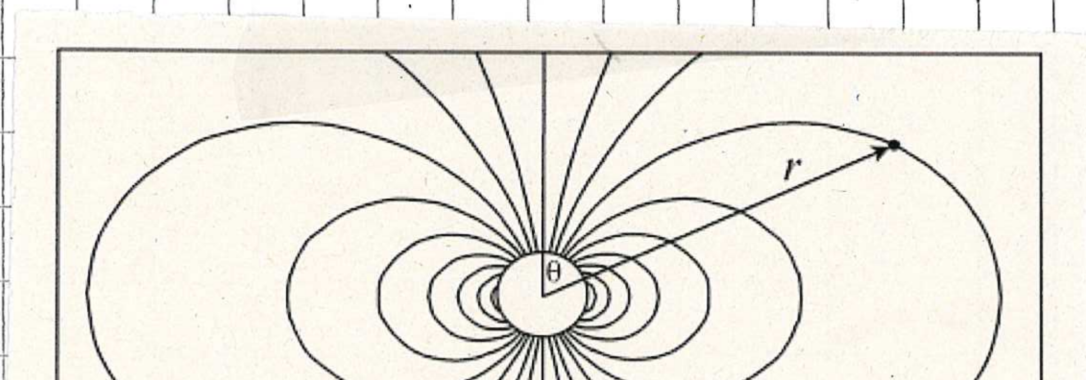
$$\boxed{\bar{V}_{\nabla B} = \pm \frac{1}{2} V_\perp r_L \frac{\bar{B} \times \bar{\nabla}|B|}{B^2}} \Leftarrow \text{GRAD-B DRIFT}$$

OBSERVATION THE GRAD-B DRIFT IN OPPOSITE DIRECTIONS FOR e^- AND i^+ GENERATES A CURRENT DENSITY $\perp$ TO $\bar{B}$

THE FIGURE BELOW SHOWS A PARTICLE DRIFT DUE TO A $\bar{B}$-FIELD GRADIENT WHERE $\bar{B}(y) = \hat{z} B_z$



OBSERVATION THE GRADIENT DRIFT IS RESPONSIBLE FOR CURRENT IN INNER PARTS OF PLANETARY MAGNETOSPHERES

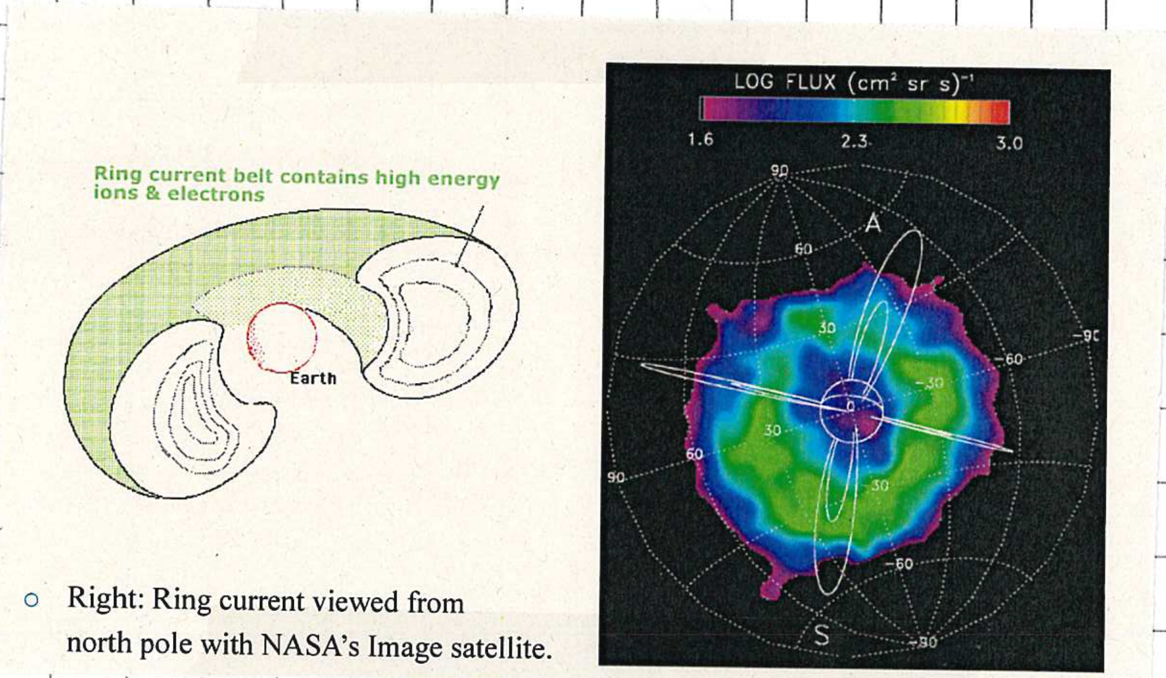


APPROXIMATE FIELD IS A DIPOLE

$$B_r = \frac{\mu_0}{4\pi}\frac{2m}{r^3}\cos\theta$$

$$B_\theta = \frac{\mu_0}{4\pi}\frac{2m}{r^3}\sin\theta$$

- OBSERVATION IN EQUATORIAL PLANE $B_r = 0$ AND $B_\theta = B_0/r^3 \Rightarrow$ A POSITIVE GRADIENT IN B_θ RADIALLY DIRECTED OUTWARDS. THEREFORE THERE IS A GRAD-DRIFT $\perp$ TO $\overline{B}$ AND GRAD-B WHICH PRODUCES A RING CURRENT CIRCULATING ABOUT A PLANET.



- Right: Ring current viewed from north pole with NASA's Image satellite.