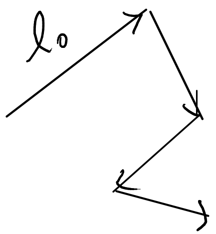
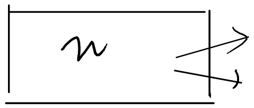
ES: stima di $\tau$ e $l_0$ nel rame $(T = 300\ K)$




$$\sigma = \frac{n e^2}{m_e} \tau \quad \Rightarrow \quad \tau = \frac{n e^2}{m_e \sigma}$$




Cu: $1 e^-$ di conduzione / atomo

densità di n. di atomi $\left[\frac{1}{m^3}\right]$

$$\rho_{Cu} = 8900 \ \frac{kg}{m^3} \qquad \left[\text{cf. } \rho_{H_2O} = 1000 \ \frac{kg}{m^3}\right] \quad \to \quad n = ?$$

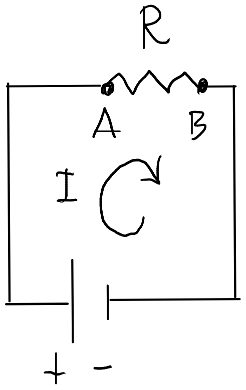
$$M_{Cu} = 63{,}5 \ \frac{g}{mol} = 63{,}5 \times 10^{-3} \ \frac{kg}{m^3}$$

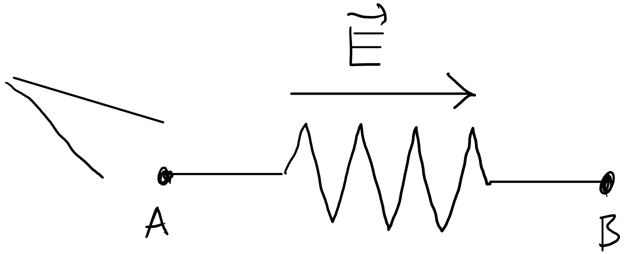
$$\frac{\rho_{Cu}}{M_{Cu}} \to \frac{mol}{m^3} \qquad n = N_A \frac{\rho_{Cu}}{M_{Cu}} \to \frac{1}{m^3} \quad \Rightarrow \quad \tau \approx 10^{-14}\ s$$

Calcolo alternativo per $v_d$: $J_e = e n v_d \Rightarrow v_d = \dfrac{J_e}{e n} = \ldots \approx 10^{-4} \ \dfrac{m}{s}$

$$J_e = \frac{1A}{10^{-6} m^2}$$

# EFFETTI TERMICI DELLA CONDUZIONE ELETTRICA




$V_A > V_B$ es: 12V

$V_B$ es: 0V

$$\vec{F} = -\vec{\nabla} E_p \rightarrow \vec{E} = -\vec{\nabla} V$$

$$E = -\frac{dV}{dx}$$

cost

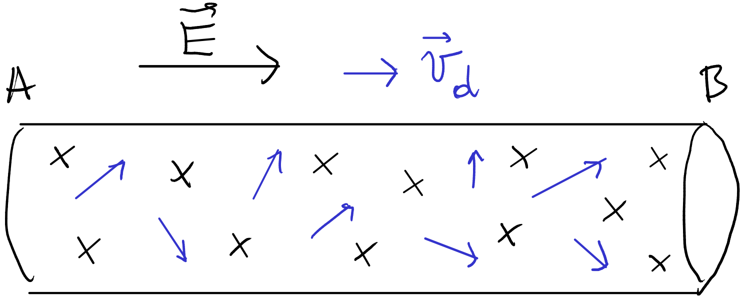
$$\Delta V = -E \Delta x$$

• $\delta q > 0$, $A \to B$, $V_B - V_A < 0$, $W = -\Delta E_p = -\delta q (V_B - V_A) > 0$

• $\delta q < 0$, $B \to A$, $V_A - V_B$, $W = -\Delta E_p = -\delta q (V_A - V_B) > 0$

$\Delta V = V_A - V_B$, $\delta q > 0 \Rightarrow \underline{W = \delta q \, \Delta V > 0}$

Corrente stazionaria, intervallo di tempo dt, trasferimento di carica $\delta q$




Sistema: {cariche, reticolo}
interagisce con l'ambiente esterno

Bilancio energetico

$$dE_c + dU = \delta W + \delta Q \Rightarrow \delta W = dU - \delta Q$$

$dE_c$: cariche, reticolo

$dE_c = 0$ , $E_c = 0$

$= 0$

$dU$: cariche, reticolo

$dU = C_v \, dT$

$\delta W$: $\delta q \, \Delta V$

$\delta Q$: calore scambiato dal sistema con l'ambiente

$dU$ ①, $\delta Q$ ②

Effetto Joule: potenza (energia per unità di tempo) fornita dal campo al sist.

$$P_e \equiv \frac{\delta W}{dt} = \frac{\delta q \, \Delta V}{dt} = I_e \, \Delta V \qquad (P = I \, \Delta V)$$

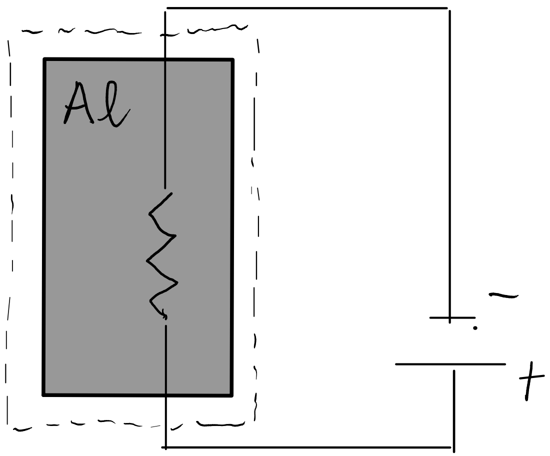
$$P_e \, dt = dU - \delta Q$$

Se conduttore ohmico, $\Delta V = I_e R \Rightarrow P_e = I_e^2 R = \frac{(\Delta V)^2}{R}$ $> 0$

Conduttore $T = \text{cost}$ (cariche, reticolo $\rightarrow T = \text{cost}$) $\rightarrow dT = 0 \Rightarrow P_e \, dt = -\delta Q$

calore ceduto all'esterno $\leftarrow \delta Q < 0$

Es.: capacità termica dell'alluminio




$m = 320\,g$

$I_e = 0.2\,A$

$\Delta t = 5\,min$

$\Delta V = 15{,}6\,V$

$\Delta T = 2{,}9\,°C \longrightarrow$ stessa per Al e resistenze

$C' = 1.5\,cal/K \rightarrow$ capacità termica resistenza

$\Rightarrow c_{Al} = ?\ [J/K/kg]$

Al + resist. isolato termicamente

$$\underbrace{\overset{=0}{dE_c} + dU}_{\text{variazioni del sistema}} = \underbrace{\delta W + \delta Q}_{\text{scambiate dal sistema con l'ambiente}}$$

$V = cost$ solidi incompressibili $\rightarrow$ lavoro meccanico $\overset{-P_{est}\,dV = 0}{=} 0$

$(\Rightarrow dU = \delta Q$ solo scambi termici$)$

Ipr. per {Al, resistenza} : $\Delta U = W + \overset{=0}{Q} = W = P_e \Delta t = I_e \Delta V \Delta t$

$$\Delta U = I_e \Delta V \Delta t$$

$U = U_{Al} + U_r$ additività

[ $V = \text{cost}$

$$dU = \frac{\partial U}{\partial T}\Big|_V dT + \cancel{\frac{\partial U}{\partial V}\Big|_T dV}^{=0}$$

$$\boxed{dU = C_V\, dT} \;\; ] \;\; = 0$$

$$\Delta U_{Al} + \Delta U_r = I_e \Delta V \Delta t$$

$$m_{Al}\, c_{Al}\, \Delta T + C\, \Delta T = I_e \Delta V \Delta t$$

per unità di massa

$$\Delta U = Q + W$$

$$Q_{Al} + \cancel{W_{Al}} + Q_r + \cancel{W_r} = I_e \Delta V \Delta t$$

$$m_{Al}\, c_{Al}\, \Delta T = I_e \Delta V \Delta t - C\, \Delta T$$

$$1.5\ \text{cal}/\text{K} = 1.5 \times 4.18\ \frac{\text{J}}{\text{cal}}\ \frac{\text{cal}}{\text{K}}$$

$$c_{Al} = \frac{I_e \Delta V \Delta t - C\, \Delta T}{m_{Al}\, \Delta T} = \frac{1}{m_{Al}}\left[\frac{I_e \Delta V \Delta t}{\Delta T} - C\right].$$

$$= \frac{1}{0.32\ \text{kg}}\left[\frac{0.2\,\text{A} \times 15.6\,\text{V} \times 60\,\text{s} \times 5}{2.9\ \text{K}} - 6.27\ \frac{\text{J}}{\text{K}}\right]$$

$$= 990\ \text{J}/\text{kg}/\text{K} = 0.99\ \text{J}/\text{g}/\text{K} < 4.18\ \text{J}/\text{g}/\text{K}$$

ES: analogia viscosa della conduzione elettrica

$q \longrightarrow \vec{E}$

$\longrightarrow \vec{v}$

modello: moto di una carica + attrito viscoso dipendente da $\vec{v}$

$\vec{F}_v = -K\vec{v}$

$\vec{F}_v$ $\quad$ $\vec{F}_e = q\vec{E}$

II Newton: $\Sigma \vec{F} = m\vec{a}$

$$m\frac{d\vec{v}}{dt} = q\vec{E} - K\vec{v}$$

1) stazionario; $\frac{d\vec{v}}{dt} = 0 \rightarrow \vec{v}$ limite? $\vec{v}_d = \frac{q}{K}\vec{E} \rightarrow \vec{J}_e = qn\vec{v}_d = \left(\frac{q^2 n}{k}\right)\vec{E}$ $\quad \sigma$

2) A $t = t_i$ spengo $\vec{E} \rightarrow \vec{v}(t) = ?$ $\vec{E} = E\vec{e}_x$, $\vec{v}_d = v_d\vec{e}_x \rightarrow m\frac{dv}{dt} = qE - kv$ 1d

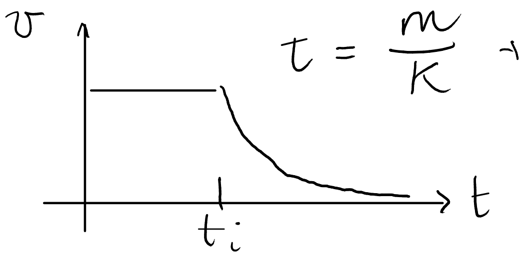
$m\frac{dv}{dt} = -kv$ $\quad \nearrow$ $A\exp\left(-\frac{k}{m}(t-t_i)\right)$

$$\boxed{\frac{dv}{dt} = -\frac{k}{m}v} \longrightarrow \frac{dv}{v} = -\frac{k}{m}dt \rightarrow \int_{v_i}^{v_f}\frac{dv}{v} = -\frac{k}{m}\int_{t_i}^{t_f}dt \rightarrow \ln\left(\frac{v_f}{v_i}\right) = -\frac{k}{m}(t_f - t_i)$$

$$v_f = v_i\exp\left(-\frac{k}{m}(t_f-t_i)\right) \rightarrow v(t) = v_i\exp\left(-\frac{k}{m}(t-t_i)\right)$$

$$= v_i\exp\left(-\frac{t-t_i}{\tau}\right)$$

$\tau = \frac{m}{k} \rightarrow k = \frac{m}{\tau}$



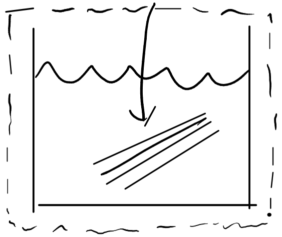
$$\sigma = \frac{q^2 n}{k} = \frac{q^2 n \tau}{m} \rightarrow \text{Drude !}$$

3) A $t = t'$, accendo $\vec{E} \rightarrow \vec{v}(t) = ?$ $\quad m \frac{dv}{dt} = qE - kv$

$v(t') = v' = 0$

$\underline{\tilde{v} = v + A}$ $\quad \frac{d\tilde{v}}{dt} = B\,\tilde{v}$

ES. colttura della pasta



$T_{ia} = 100°C$ $\qquad c_a = 4.18 \times 10^3\ J/kg/K$

$m_p = 500g$ $\qquad c_p = 3.5 \times 10^3\ J/kg/K$

$T_{ip} = 20°C$ $\qquad T_f = 95°C \rightarrow V_a = ? \rightsquigarrow m_a = ?$

1) isolato

Ipr sistema {acqua, pasta} : $\Delta U = 0$ $\qquad V = cost$

$\Delta U_a + \Delta U_p = 0 \rightarrow Q_a + Q_p = 0$

$m_a c_a (T_f - T_{ia}) + m_p c_p (T_f - T_{ip}) = 0$

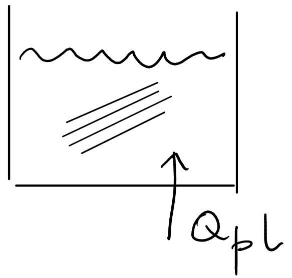
$$m_a = m_p \frac{c_p}{c_a} \frac{T_{ip} - T_f}{T_f - T_{ia}} = m_p \frac{c_p}{c_a} \frac{T_f - T_{ip}}{T_{ia} - T_f} = 0.5\,kg \frac{3.5 \times 10^3}{4.18 \times 10^3} \cdot \frac{75}{5} = 6.3\,kg$$

$V_a = 6.3\,l$ !!

2) placca di cottura $\qquad m_a = 1\,kg$

$$P_e = 2000\ W \rightarrow P_e \Delta t \cdot \underset{\text{efficacia}}{e} = c_a m_a (T_f - T_i) \rightarrow e = \frac{c_a m_a (T_f - T_i)}{P_e \Delta t}$$

$$e = \frac{\text{utile}}{\text{speso}} \qquad = 0.56$$




{acqua, pasta} non isolato

$$\Delta U_a + \Delta U_p = Q_{pl} \qquad Q_{pl} = P_e \overbrace{\Delta t}^{\approx 1\,min} \cdot e$$

$$m_a c_a (T_f - T_{ia}) + m_p c_p (T_f - T_{ip}) = P_e \Delta t \, e$$

$$m_a = m_p \frac{c_p}{c_a} \frac{T_f - T_{ip}}{T_{ia} - T_f} + \frac{P_e \Delta t\, e}{\underbrace{c_a (T_f - T_{ia})}_{<0}} \rightsquigarrow <0 \qquad = 6.3\,kg - \underbrace{\frac{2000 \times 60 \times 0.56}{4.18 \times 10^3 \times 5}}_{3,2}\,kg$$

$$= 3,1\,kg \rightarrow \underline{V_a = 3,1\,l}$$

- Conservazione energia $\quad E_{ci} + E_{pi} = E_{cf} + E_{pf} \ \cdots$

  ↑

  forze conservative

- legge adiabatiche $\quad PV^\gamma = \text{cost} \qquad P_i V_i^\gamma = P_f V_f^\gamma$

  ↑

  g.p., adiab., Q-S

$W = -P\Delta V \rightarrow W = -10^5\,\text{Pa}\,\Delta V$ $\qquad$ determina → simbolico

$P = 10^5\,\text{Pa}$ $\qquad$ calcola → numerico

- funzioni e variabili di stato $\qquad$ grandezze di trasformazione

  $P, T, V, U, S, \ldots$

  $\Delta P, \Delta T, \Delta V, \Delta U, \Delta S, \ldots \qquad W, Q$

  $dP, dT, dV, dU, dS, \ldots \qquad \delta W, \delta Q$

  ↓ $\qquad$ ↓

  differenziali esatti $\qquad$ differenziali (non esatti)

I pr.

$dE_c + dU = 0$

conservazione dell'energia

II pr.

$dS \geq 0$

senso trasformazioni dei corpi macro (freccia del tempo)

isolato

$dE_c + \boxed{dU = \delta W + \delta Q}$

$\downarrow$

$= 0$

$\boxed{dS = \dfrac{\delta Q}{T}}$

QS

non isolato

$dU = C_V dT$

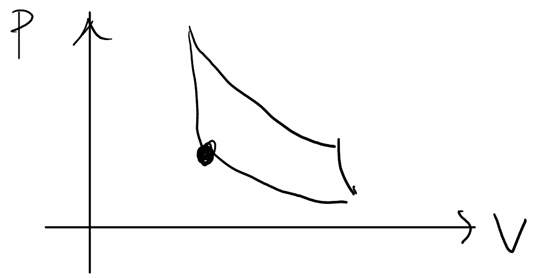
Eq. di stato: gas perfetto

- $PV = nRT = Nk_BT$
- $U = C_V T$ (mono: $U = \frac{3}{2} Nk_BT = \frac{3}{2} nRT$)

Solido armonico

- $V = \text{cost}$
- $U = C_V T = 3Nk_BT$
  $= 3nRT$

<u>Ciclo Carnot</u> : trasf. reversibili $\Delta S_u = 0$ (universo)




$\Delta S = 0$ perchè ciclo

del sistema

$W = \int_i^f \delta W \qquad Q = \int_i^f \delta Q^{=0}$

$W = -\int_i^f P\,dV = -P\Delta V$

<u>QS</u>

$dU = C_v\,dT$

$dU = \underbrace{\delta Q}_{=0} + \delta W = \delta W$

$\Delta U = \int_i^f dU = C_v \int_{T_i}^{T_f} dT = C_v(T_f - T_i)$

$\| W$

$\Delta U = \underbrace{W}_{=0} + Q = Q$

<u>n moli di g.p.</u> : trasf. <u>QS</u> $i \to f$

| | $W$ | $Q$ |
|---|---|---|
| isoterma | $-nRT\ln(V_f/V_i)$ | $nRT\ln(V_f/V_i)$ |
| isobara | $-P(V_f - V_i)$ | $C_p(T_f - T_i)$ |
| isocora | $\emptyset$ | $C_v(T_f - T_i)$ |
| adiabatica | $C_v(T_f - T_i)$ | $\emptyset$ |

$C_p = C_v + nR = \frac{3}{2}nR + nR = \frac{5}{2}nR$

$$W = \int_i^f \delta W \underset{\substack{\uparrow \\ QS}}{=} -\int_i^f P\,dV \underset{\substack{\uparrow \\ PV=nRT}}{=} -\int_{V_i}^{V_f} \frac{nRT}{V}\,dV \underset{\substack{\uparrow \\ T=cost}}{=} -nRT\int_{V_i}^{V_f}\frac{dV}{V} = -nRT \ln\frac{V_f}{V_i}$$

$$\underbrace{\Delta U}_{=0} = W + Q \quad \Rightarrow \quad Q = -W$$