ESERCIZI TUTORATO - 14/05/2021

ELETTROSTATICA E CAMPO ELETTRICO (PARTE 1)

#0) RISCALDAMENTO (II PROVETTA ANNO SCORSO)

UN DIPOLO ELETTRICO È COSTITUITO DA 2 CARICHE UGUALI (CON $q = 1.5$ nC) E DI SEGNO OPPOSTO SEPARATE DA UNA DISTANZA $2a$ ($a = 2$ cm). CONSIDERIAMO IL RIFERIMENTO COME IN FIGURA →
IL PUNTO P SI TROVA A DISTANZA $y = 3,0$ cm DALL' ORIGINE LUNGO L' ASSE y, MENTRE R SI TROVA A $x = 5,0$ cm DALL' ORIGINE, LUNGO L'ASSE x
CALCOLARE:
a). V_0, E_0 (POTENZIALE ELETTRICO E CAMPO ELETTRICO IN O)
b). V_P, E_P (" " E " " IN P)
c). V_R, E_R (" " E " " IN R)

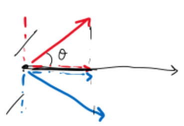
a) $V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi a}\frac{-q}{a} = 0$

$q \; \cdot \longrightarrow \; \rightarrow\cdot \; -q$

$$\vec{E}_0 = \hat{\imath}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)$$
$$= \hat{\imath}\,\frac{2q}{a^2}\frac{1}{4\pi\varepsilon_0} = 8{,}99\times10^9\,\frac{N\cancel{m^2}}{\cancel{C^2}}\,\frac{2\;1{,}5\times10^{-9}\cancel{C}}{(2\times10^{-2}\cancel{m})^2}\,\hat{\imath}$$
$$= 6{,}75\times10^4\,\frac{N}{C}\,\hat{\imath}$$

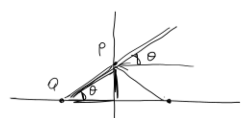
b) $V_P = 0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d} - \frac{q}{d}\right) = 0$

$$\vec{E}_P = \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a^2+y^2} + \frac{q}{a^2+y^2}\right)\cos\theta$$

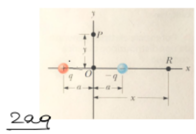


$\cos\theta = \frac{a}{\sqrt{a^2+y^2}}$

$$\vec{E}_P = \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{a^2+y^2}\right)\cos\theta$$
$$= \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\,\frac{2qa}{(a^2+y^2)^{3/2}}$$



$$= \hat{\imath}\;8{,}99\times10^9\,\frac{Nm^2}{C^2}\,\frac{2\;1{,}5\times10^{-9}C\;\;2{,}0\times10^{-2}m}{((2\times10^{-2}m)^2+(3\times10^{-2}m)^2)^{3/2}}$$
$$= 1{,}15\times10^4\,\frac{N}{C}\,\hat{\imath}$$



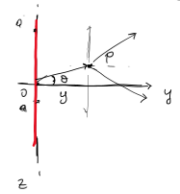
c) $$V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x+a} - \frac{q}{x-a}\right)$$
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\cancel{x}-a-\cancel{x}-a}{(x+a)(x-a)}\right) = -\frac{2aq}{4\pi\varepsilon_0(x^2-a^2)}$$
$$= -8{,}99\times10^9\,\frac{N\cancel{m^2}}{\cancel{C^2}}\,\frac{2\;1{,}5\times10^{-9}\cancel{C}\;\;2\cdot10^{-2}m}{(5^2-2^2)\,10^{-4}\cancel{m^2}}$$
$$= -2{,}57\times10^2\,V$$

$$\vec{E}_R = \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(x+a)^2} - \frac{q}{(x-a)^2}\right)$$
$$= \hat{\imath}\,\frac{q}{4\pi\varepsilon_0}\left(\frac{(x-a)^2-(x+a)^2}{(x-a)^2(x+a)^2}\right) =$$
$$= \hat{\imath}\;1{,}5\times10^{-9}C\;\;8{,}99\times10^9\,\frac{Nm^2}{C^2}\left(\frac{1}{7\times10^{-4}} - \frac{1}{3\times10^{-4}}\right)\frac{1}{m^2}$$
$$= -1{,}22\times10^4\,\frac{N}{C}\,\hat{\imath}$$

#1) CAMPI ELETTRICI E FORME GEOMETRICHE:

CALCOLARE IL CAMPO ELETTRICO GENERATO DALLE SEGUENTI DIST. DI CARICA:

- FILO CARICO (INFINITO)
- SFERA CARICA (INTERNO ED ESTERNO)
- GUSCIO SFERICO CARICO (INTERNO ED ESTERNO)



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\,\hat{r}$$

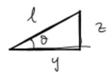
$\lambda \Rightarrow \quad dq = \lambda ds = \lambda dz$

$$\vec{E}(\vec{r}) = \int d\vec{E} = \int d\vec{E}_y + \cancel{d\vec{E}_z}$$

DA QUALCHE PARTE

$z = y\,tg\theta \rightarrow dz = \frac{y}{\cos^2\theta}d\theta$

$$= \frac{1}{4\pi\varepsilon_0}\int\frac{dq\cos\theta}{(\sqrt{y^2+z^2})^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{dz\cos\theta\,\lambda}{y^2+z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\int_0^{\infty}\frac{2\cos\theta\,\lambda dz}{y^2+z^2}$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{\cancel{0}}^{\cancel{\infty}}\frac{2\cos\theta}{y^{\cancel{2}}+y^2tg^2\theta}\frac{\cancel{y}}{\cos^2\theta}d\theta$$



$y = l\cos\theta$, $z = l\sin\theta$ $\}$ $\frac{z}{y} = tg\theta$

$dz = y\,d(tg\theta) = \frac{y}{\cos^2\theta}d\theta$

$\uparrow \int_0^{\pi/2}$

$$= \frac{2\lambda}{y\,4\pi\varepsilon_0}\int_0^{\pi/2}d\theta\,\frac{\cos\theta}{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta}$$

(NON FATELA VEDERE AI MATEMATICI!)

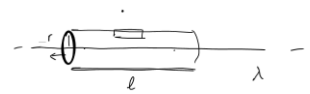
↳ $\frac{dz}{d\theta} = \frac{y\,d(tg\theta)}{d\theta} = y\,\frac{1}{\cos^2\theta}$

MOLTIPLICO PER $d\theta$ E TROVO:

$dz = \frac{y}{\cos^2\theta}d\theta$ (QUELLO CHE VOLEVO PRIMA)

$$= \frac{2\lambda}{4\pi y\varepsilon_0}\int_0^{\pi/2}d\theta\,\frac{\cos\theta}{(\cos^2\theta+\sin^2\theta)} \quad = 1$$
$$= \frac{2\lambda}{4\pi y\varepsilon_0}\int_0^{\pi/2}d\theta\cos\theta$$
$$= \frac{\lambda}{2\pi\varepsilon_0 y}$$

$\oint\vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$



$\frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

$\oint\vec{E}\cdot d\vec{s} = E_r\,2\pi r l$

$E_r\,2\pi r\cancel{l} = \frac{\lambda\cancel{l}}{\varepsilon_0}$

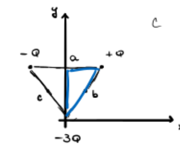
↳ $E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$

#2) IL TRIANGOLO NO ...

CONSIDERIAMO TRE CARICHE PUNTIFORMI $+Q, -Q$ E $-3Q$ POSTE AI VERTICI DI UN TRIANGOLO EQUILATERO CON $L = 12$ cm (COME IN FIGURA). CALCOLARE:

a) IL VALORE DEL POTENZIALE ELETTROSTATICO V_a V_b E V_c NEI PUNTI MEDI DEI 3 LATI

. IL VETTORE CAMPO ELETTRICO $\vec{E}_a$ NEL PUNTO a



a) $h = L\frac{\sqrt{3}}{2} \longrightarrow$ PITAGORA SU $(-3Q)\,Q\,a$ △

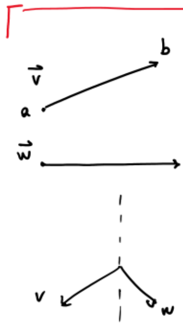
$$\rightarrow V_a = \frac{1}{2\cancel{4}\pi\varepsilon_0}\frac{-3Q}{\frac{L\sqrt{3}}{\cancel{2}}} = -\frac{3}{2}\frac{1}{\pi\varepsilon_0}\frac{Q}{L\sqrt{3}} = -8{,}72\times10^5\,V$$

$$V_b = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{L/2} - \frac{Q}{L\sqrt{3}/2} - \frac{3Q}{L/2}\right) =$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\left(2 - \frac{2}{\sqrt{3}} - 2\cdot3\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\left(2 - 6 - \frac{2\sqrt{3}}{3}\right) = -1{,}3\times10^6\,V$$

$$(k =)\,\underbrace{8{,}99\times10^9\,\frac{N\cancel{m^2}}{\cancel{C^2}}\,\frac{2{,}8\times10^{-6}\cancel{C}}{1{,}2\times10^{-4}\cancel{m}}}_{\text{COSO}} \rightarrow [\text{COSO}] = V$$

$$V_c = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\left(-2-6+\frac{2\sqrt{3}}{3}\right) = 1{,}72\times10^6\,V$$

VETTORE CAMPO ELETTRICO $\vec{E}_a$



$\vec{v}+\vec{w}$, $\vec{v}-\vec{w}$ $\}$ $\vec{z}$ (VETTORE) $\qquad \| 2\hat{\imath} + \cancel{\hat{\jmath}} - 3 \|$

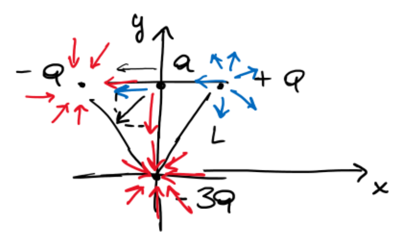
$a\vec{v}$

$\vec{w}\cdot\vec{v} = \vec{v}\cdot\vec{w} \rightarrow |\vec{v}|\,|\vec{w}|\cos(\alpha)$ → RITORNA SCALARE

$\vec{v}\times\vec{w}$ ↳ [RITORNA UN VETTORE!]

$\hat{\imath}\;\hat{\jmath}\;\hat{k} \rightarrow$ VERSORI (VETTORI CON MODULO 1)

$$\vec{E}(a) = \vec{E}_Q + \vec{E}_{-Q} + \vec{E}_{-3Q}$$



$\vec{E}_x$ $\begin{cases} \vec{E}_Q = -\hat{\imath}\,\frac{1}{\cancel{4}\pi\varepsilon_0}\frac{Q}{\frac{L^2}{\cancel{4}}} \\ \vec{E}_{-Q} = -\hat{\imath}\,\frac{1}{\cancel{4}\pi\varepsilon_0}\frac{Q}{\frac{L^2}{\cancel{4}}} \end{cases}$

$h = \frac{L\sqrt{3}}{2}$

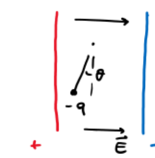
$$\vec{E}_y \rightarrow \vec{E}_{-3Q} = -\hat{\jmath}\,\frac{1}{\cancel{4}\pi\varepsilon_0}\frac{\cancel{3}Q}{\frac{L^2\cancel{3}}{\cancel{4}}} = -\hat{\jmath}\,\frac{1}{\pi\varepsilon_0}\frac{Q}{L^2}$$

$\hat{\imath}$ = VERSORE

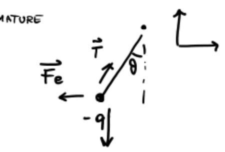
$$\vec{E}(a) = -\hat{\imath}\,\frac{1}{\pi\varepsilon_0}\frac{Q}{L^2} - \hat{\imath}\,\frac{1}{\pi\varepsilon_0}\frac{Q}{L^2} - \hat{\jmath}\,\frac{1}{\pi\varepsilon_0}\frac{Q}{L^2}$$
$$= \frac{1}{\pi\varepsilon_0}\frac{Q}{L^2}\left(-\hat{\imath}-\hat{\imath}-\hat{\jmath}\right)$$
$$= -\frac{1}{\pi\varepsilon_0}\frac{Q}{L^2}\left(2\hat{\imath}+\hat{\jmath}\right)$$

#3) ARMATURE E CONDENSATORI

IL CAMPO ELETTRICO TRA LE ARMATURE DI UN CONDENSATORE A FACCE PIANE PARALLELE È ORIZZONTALE, UNIFORME E HA INTENSITÀ E. UN PICCOLO OGGETTO DI MASSA $m = 0{,}025$ kg E CARICA $-3{,}10\ \mu C$ È IN QUIETE, SOSPESO AD UN FILO SITUATO TRA LE 2 ARMATURE. IL FILO FORMA UN ANGOLO DI $10{,}5°$ CON LA VERTICALE.



a) LA TENSIONE DEL FILO
b) L' INTENSITÀ DEL CAMPO ELETTRICO
c) LA DENSITÀ SUPERFICIALE DI CARICA
d) L' INTENSITÀ DEL CAMPO ELETTRICO NEL CASO IN CUI TRA LE ARMATURE VENGA POSTO UN MATERIALE ISOLANTE CON $\varepsilon_r = 2{,}45$



$\begin{cases} x: & T_x = |q|E \\ y: & T_y = mg \end{cases} \qquad T_x = T\sin\theta, \quad T_y = T\cos\theta$

$\begin{cases} x: & T\sin\theta = |q|E \;(*) \\ y: & T\cos\theta = mg \end{cases} \Rightarrow T = \frac{mg}{\cos\theta} = \frac{0{,}025\,kg\;9{,}81\,m/s^2}{\cos(10{,}5°)} = 0{,}249\,N$

$qE = T\sin\theta = \frac{mg}{\cos\theta}\sin\theta$

b) $$E = \frac{mg\,tg\theta}{q} = \frac{0{,}025\,kg\;9{,}81\,m/s^2\;tg(10{,}5°)}{+3{,}10\times10^{-6}\,C} = -1{,}4\times10^4\,N/C$$

c) $$E = \frac{\sigma}{\varepsilon_0} \longrightarrow \sigma = \varepsilon_0|E| = 8{,}85\times10^{-12}\,\frac{C^2}{Nm^2}\;1{,}46\times10^4\,\frac{N}{C} = 1{,}29\times10^{-7}\,C/m^2$$

(*) $\begin{cases} x: & T_x - |qE| = 0 \\ y: & \end{cases}$

d) $\varepsilon_0 \rightarrow \varepsilon = \varepsilon_0\varepsilon_r$

$$E' = \frac{E}{\varepsilon_r} = 5{,}96\times10^3\,\frac{N}{C}$$