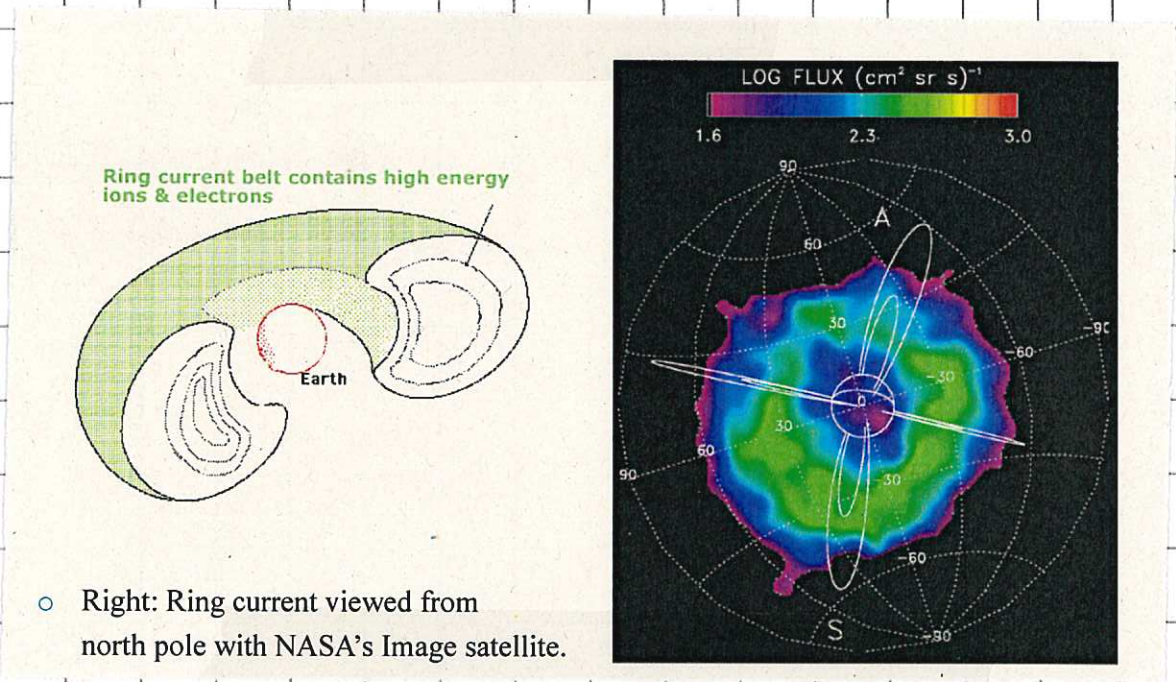
$$\begin{cases} B_r = \dfrac{\mu_0}{4\pi}\,\dfrac{2m}{r^3}\cos\theta \\[2ex] B_\theta = \dfrac{\mu_0}{4\pi}\,\dfrac{2m}{r^3}\sin\theta \end{cases} \qquad (61)$$

- OBSERVATION IN EQUATORIAL PLANE $B_r = 0$ AND $B_\theta = B_0/r^3$ $\Rightarrow$ A POSITIVE GRADIENT IN B_θ RADIALY DIRECTED OUTWARDS. THEREFORE THERE IS A GRAD-DRIFT $\perp$ TO $\vec{B}$ AND GRAD-B WHICH PRODUCES A RING CURRENT CIRCULATING ABOUT A PLANET.



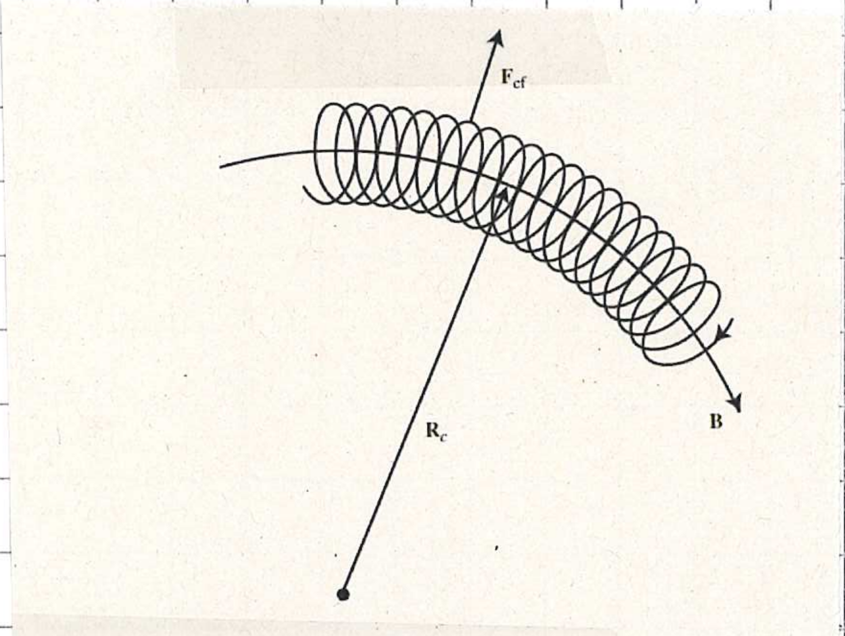
- Right: Ring current viewed from north pole with NASA's Image satellite.

- CURVED MAGNETIC DRIFT

WHEN A CHARGED PARTICLE MOVED ALONG CURVED MAGNETIC FIELD LINES, EXPERENCE CENTRIFUGAL FORCE PERPENDICULAR TO MAGNETIC FIELD. SEE FOLLOWING FIGURE.

- ASSUME A RADIUS OF CURVATURE $R_c \gg r_L$
THE OUTWARD CENTRIFUGAL FORCE IS

$$\bar{F}_{cf} = \frac{m v_{\parallel}^2}{R_c} \hat{r} \quad (62)$$



Curvature Drift

- THIS FORCE CAN BE DIRECTLY INSERTED INTO THE GENERAL FORM FOR GUIDING-CENTER DRIFT (SEE EQ. 48)

$$\bar{V}_f = \frac{1}{q} \frac{\bar{F} \times \bar{B}}{B^2} \Rightarrow$$

$$\bar{V}_R = \frac{m v_{\parallel}^2}{q R_c^2} \frac{\bar{R}_c \times \bar{B}}{B^2} \quad (63)$$

- THEREFORE, THE DRIFT IS INTO OR OUT OF THE PAGE DEPENDING ON SIGN OF q.

- OBSERVATION

THE TYPE OF FIELD CONFIGURATION STUDIED ABOVE HAVING CURVED BUT PARALLEL FIELD LINES, WILL NEVER OCCUR IN REALITY SINCE IT IMPLIES

$\bar{\nabla} \cdot \bar{B} \neq 0$. IN PRACTICE CURVED FIELD LINES WILL ALWAYS BE CONVERGING/DIVERGING. THUS A CURVATURE DRIFT WILL ALWAYS BE ACCOMPANIED BY A GRAD-B DRIFT. $\rightarrow$

$\bar{\nabla} |\bar{B}| \neq 0$ AND IN THE OPPOSITE DIRECTION TO R_c. THE GRAD-B AND THE CURVATURE DRIFTS ACT IN THE SAME DIRECTION.

- FLUID APPROACH TO PLASMA

  THE FLUID APPROACH DESCRIBES BULK PROPERTIES OF PLASMA. WE DO NOT ATTEMPT TO SOLVE UNIQUE TRAJECTORIES OF ALL PARTICLES IN A PLASMA.

  THIS SEMPLIFICATION WORKS VERY WELL FOR MAJORITY OF PLASMAS.

THE FLUID THEORY FOLLOWS DIRECTLY FROM MOMENTS OF THE BOLTZMAN EQ (NOT DERIVED HERE).

- EACH OF THE MOMENT OF THE BOLTZMANN EQUATION IS A TRANSPORT EQUATION DESCRIBING THE DYNAMICS OF A QUANTITY ASSOCIATED WITH A GIVEN POWER OF THE VELOCITY, $\bar{u}$.

$$\partial_t n + \bar{\nabla} \cdot (n\bar{u}) = 0$$

CONTINUITY OF MASS OR CHARGE TRANSPORT

(64)

DIVERG. OF PRESSURE TENSOR

$$mn\left[\partial_t \bar{u} + (\bar{u} \cdot \bar{\nabla})\bar{u}\right] = qn(\bar{E} + \bar{u} \times \bar{B}) - \bar{\nabla} \cdot \bar{\bar{P}} + P_i$$

MOMENTUM TRANSPORT

COLLISION TERM

$$\frac{\partial}{\partial t}\left[n\frac{1}{2}mu^2\right]+\bar{\nabla}\cdot\left[n\frac{1}{2}m\langle u^2\bar{u}\rangle\right]-nq\langle\bar{E}\cdot\bar{u}\rangle = \frac{m}{2}\int u^2\left(\frac{\partial f}{\partial t}\right)_{coll}du \qquad (64\text{-}b)$$

ENERGY TRANSPORT

THESE EQS CAN BE SEMPLIFIED BY CONSIDERING DIFFERENT TYPE OF PLASMA

- COLD-PLASMA NEGLETS THE THERMAL MOTIONS OF PARTICLES AND A SIMPLIFIED MOMENTUM TRANSFER EQ CAN BE ADOPTED
  $\Rightarrow$ SET THE KINETIC PRESSURE TENSOR TO ZERO, THE REMAINING VARIABLES ARE THEN n AND $\bar{u}$ DESCRIBED BY

$$\partial_t n+\bar{\nabla}\cdot(n\bar{u})=0$$

$$(65) \quad mn\left[\partial_t\bar{u}+(\bar{u}\cdot\bar{\nabla})\bar{u}\right]=qn(\bar{E}+\bar{u}\times\bar{B})+\bar{P}_{ij}$$

- THE COLLISION $\bar{P}_{ij}$ CAN BE APPROXIMATED BY AN EFFECTIVE COLLISION FREQUENCY
- ASSUMING THAT COLLISIONS CAUSE A RATE OF DECREASE IN MOMENTUM $\bar{P}_{ij}=-mn\nu_{eff}\bar{u}$
- WARM-PLASMA

AN ALTERNATIVE SET OF MACROSCOPIC EQS IS OBTAINED BY TRUNCATING THE ENERGY CONSERVATION EQ. LET'S START CONSIDERING THE PRESSURE TENSOR, AS REPORTED IN THE FOLLOWING,

$$\bar{\bar{P}} = \begin{vmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{vmatrix} \quad (66)$$

THE COMPONENTS REPRESENT THE TRANSPORT OF MOMENTUM. DIAGONAL ELEMENTS REPRESENT PRESSURE, WHILE OFF-DIAGONAL REPRESENT SHARING STRESSES.

- IN WARM PLASMA MODEL WE ONLY CONSIDER THE DIAGONAL PRESSURE ELEMENTS, SO

$$\bar{\nabla} \cdot \bar{\bar{P}} = \nabla p$$

THE PHYSICAL MEANING IS THAT THE VISCOUS FORCES ARE NEGLECTED. WE THEN HAVE

$$(67) \begin{cases} \partial_t n + \bar{\nabla} \cdot (n\bar{u}) = 0 \\ mn\left[\partial_t \bar{u} + (\bar{u} \cdot \bar{\nabla})\bar{u}\right] = qn(\bar{E} + \bar{u} \times \bar{B}) - \bar{\nabla} p + \bar{\bar{P}}_{ij} \end{cases}$$

- SINGLE-FLUID THEORY: MHD

UNDER CERTAIN CIRCUMSTANCES IS APPROPRIATE TO CONSIDER THE ENTIRE PLASMA AS A SINGLE FLUID, THAT MEANS WE DO NOT DIFFERENTIATE BETWEEN IONS AND ELECTRONS. THIS APPROACH IS CALLED MAGNETO HYDRO DYNAMICS (MHD).

- THIS APPROACH IS APPROPRIATE WHEN DEALING WITH SLOW VARYING CONDITIONS.
- MHD IS USEFUL WHEN PLASMA IS HIGHLY IONISED AND e^- AND i^+ ARE FORCED TO ACT AT

UNISON, EITHER BECAUSE OF FREQUENT COLLISIONS OR BY THE ACTION OF A STRONG EXTERNAL MAGNETIC FIELD.

- MHD – MASS AND CHARGE CONSERVATION

WE START BY COMBINING MULTIPLE-FLUID EQs INTO A SET OF EQs FOR A SINGLE FLUID. LET'S ASSUME A TWO SPECIES PLASMA OF i^+ AND e^- ($j = e^-$ OR i^+)

$$\partial_t n_j + \bar{\nabla}(n_j v_j) = 0 \quad (a)$$

(68)

(WE USE HERE $\bar{V} = \bar{u}$)

$$m_j n_j \left[\partial_t \bar{V}_j - (\bar{V}_j \cdot \bar{\nabla}) \bar{V}_j \right] = -\bar{\nabla} \cdot \bar{P}_j + q_j n_j (\bar{E} + \bar{V}_j \times \bar{B}) + + P_{ij} \quad (b)$$

FOR A FULLY IONIZED TWO-SPECIES PLASMA THE TOTAL MOMENTUM MUST BE CONSERVED

$$P_{ei} = -P_{ie} \quad (69)$$

- OBSERVATION AS $m_i \gg m_e$ THE TIME SCALE IN CONTINUITY AND MOMENTUM EQs FOR IONS AND ELECTRONS ARE DIFFERENT. THE CARATTERISTIC FREQUENCY OF PLASMA, SUCH AS PLASMA FREQUENCY OR CYCLOTRON FREQUENCY ARE MUCH LARGER FOR ELECTRONS

- SINGLE-FLUID EQ FOR FULLY IONIZED PLASMA.

WHEN PLASMA PHENOMENA ARE LARGE-SCALE ($L \gg \lambda_D$) AND HAVE RELATIVELY LOW FREQUENCY

$(\omega \ll \omega_p$ AND $\omega \ll \omega_c)$ THE PLASMA IS ON THE AVERAGE ELECTRICALLY NEUTRAL $(n_i \simeq n_e) \Rightarrow$ INDEPENDENT MOTION OF e^- AND i^+ CAN BE NEGLECTED $\Rightarrow$ THE PLASMA CAN BE TREATED AS A SINGLE CONDUCTING FLUID, WHOSE INERTIA IS PROVIDED BY THE MASS OF THE IONS. THE GOVERNING EQs ARE OBTAINED BY COMBINING THE 68 EQs, BY FOLLOWING THE STEPS:

1 - DEFINE THE MACROSCOPIC PARAMETERS OF THE PLASMA FLUID

70
$$\left\{\begin{array}{ll} \rho_m = n_e m_e + n_i m_i & \text{MASS DENSITY} \\ \bar{J} = n_e q_e \bar{V}_e + n_i q_i \bar{V}_i & \text{CURRENT DENSITY} \\ \bar{V} = \dfrac{n_e m_e \bar{V}_e + n_i m_i \bar{V}_i}{n_e m_e + n_i m_i} & \text{MASS VELOCITY} \\ \bar{\mathbb{P}} = \bar{\mathbb{P}}_e + \bar{\mathbb{P}}_i & \text{TOTAL PRESSURE TENSOR} \end{array}\right.$$

- MHD MASS AND CHARGE CONSERVATION

USING EQ. 68(a)

$$\partial_t n_j + \bar{\nabla} \cdot (n_j \bar{v}_j) = 0$$

BY MULTIPLYING BY q_i AND q_e AND ADD THE CONTINUITY EQS WE OBTAIN

$$\partial_t \rho + \bar{\nabla} \cdot \bar{J} = 0 \qquad \text{CHARGE CONSERV.}$$

WHERE $\bar{J}$ IS THE ELECTRIC CURRENT DENSITY

$$\bar{J} = n_e q_e \bar{V}_e + n_i q_i \bar{V}_i$$

AND THE ELECTRIC CHARGE DENSITY $\rho = n_e q_e + n_i q_i$.

BY MULTIPLYING EQ 68a BY m_i AND m_e WE OBTAIN

$$\partial_t \rho_m + \rho_m \bar{\bar{\nabla}} \cdot (\bar{V}) = 0 \quad \text{(MASS CON.)}$$

WHERE $\rho_m = n_e m_e + n_i m_i$ IS THE SINGLE FLUID MASS DENSITY AND $\bar{V}$ IS THE FLUID MASS VELOCITY,

$$\bar{V} = \frac{n_e m_e \bar{V}_e + n_i m_i \bar{V}_i}{n_e m_e + n_i m_i}$$

- MHD EQUATION OF MOTION

EQ. OF MOTION FOR BULK PLASMA CAN BE OBTAINED BY ADDING INDIVIDUAL MOMENTUM TRASPORT EQs (68b) FOR i^+ AND e^-:

$$(n_e m_e + n_i m_i) \partial_t \bar{V} = -\bar{\bar{\nabla}} \cdot (\mathbb{P}_e + \mathbb{P}_i) + (n_e q_e + n_i q_i)(\bar{E} + \bar{J} \times \bar{B}) \quad (71)$$

- NOTE THAT WE HAVE NEGLECTED $(\bar{V} \cdot \bar{\bar{\nabla}}) \bar{V}$ AS WE ARE DEALING WITH SMALL PERTURBATIONS FOR WHICH THE GRADIENTS ARE NEGLIGIBLE. FOR NEUTRAL PLASMA THE SECOND RIGHT TERM OF (71) IS ZERO $\Rightarrow$

$$\rho_m \partial_t \bar{V} = -\bar{\nabla} \cdot \bar{\mathbb{P}} + \bar{J} \times \bar{B} \quad (72)$$

THIS IS THE MOTION EQ. THAT FOR ISOTROPIC PLASMA BECOMES $\bar{\nabla} \cdot \bar{\mathbb{P}} = \nabla p$ WHERE p IS THE TOTAL PRESSURE $p = p_e + p_i \Rightarrow$

$$\rho_m \partial_t \bar{V} = -\nabla p + \bar{J} \times \bar{B} \quad (73)$$

- GENERALIZED OHM LAW

THE FINAL SINGLE FLUID EQ DESCRIBES THE VARIATION OF CURRENT DENSITY $\bar{J}$. LET'S CONSIDER THE MOMENTUM EQ 68 b

$$m_j n_j\left[\partial_t \bar{V}_j + (\bar{V}_j\cdot\bar{\nabla})V_j\right] = -\bar{\nabla}\cdot\bar{\bar{\Pi}}_j + q_j n_j\left(\bar{E}+\bar{V}_j\times\bar{B}\right) + P_{ij} \quad (74)$$

MULTIPLYING THE e^- BY q_e/m_e AND THE ION EQ BY q_i/m_i AND ADDING, WE OBTAIN

$$\partial_t \bar{J} = -\frac{q_e}{m_e}\bar{\nabla}\cdot\bar{\bar{\Pi}}_e - \frac{q_i}{m_i}\bar{\nabla}\cdot\bar{\bar{\Pi}}_i + \left(\frac{n_e q_e^2}{m_e}+\frac{n_i q_i^2}{m_i}\right)\bar{E} + \left(\frac{n_e q_e^2}{m_e}\bar{V}_e + \frac{n_i q_i}{m_i}\bar{V}_i\right)\times\bar{B} + \frac{q_e}{m_e}P_{ei} + \frac{q_i}{m_i}P_{ie} \quad (75)$$

FOR AN ELECTRICALLY NEUTRAL PLASMA $|q_e n_e| \simeq |q_i n_i|$ AND USING $\bar{J} = n_e q_e \bar{V}_e + n_i q_i \bar{V}_i$ AND $\bar{V} = \dfrac{n_e m_e \bar{V}_e + n_i m_i \bar{V}_i}{n_e m_e + n_i m_i}$ WE CAN WRITE

$$\partial_t \bar{J} = -\frac{q_e}{m_e}\bar{\nabla}\cdot\bar{\bar{\Pi}}_e - \frac{q_i}{m_i}\bar{\nabla}\cdot\bar{\bar{\Pi}}_i + \left(\frac{n_e q_e^2}{m_e}+\frac{n_i q_i^2}{m_i}\right)\left(\bar{E}+\bar{V}\times\bar{B}\right) + \left(\frac{q_e}{m_e}+\frac{q_i}{m_i}\right)\left(\bar{J}\times\bar{B}\right) + \left(\frac{q_e}{m_e}-\frac{q_i}{m_i}\right)P_{ei} \quad (76)$$

AS $m_e \ll m_i \Rightarrow q_e/m_e \gg q_i/m_i$ AND $n_e q_e^2/m_e \gg n_i q_i^2/m_i$

AND IN THERMAL EQUILIBRIUM THE KINETIC PRESSURES OF e^- AND i^+ ARE SIMILAR $\bar{\mathbb{P}}_e \simeq \bar{\mathbb{P}}_i$, WE GET

$$\partial_t \bar{J} = -\frac{q_e}{m_e}\bar{\nabla}\cdot\bar{\mathbb{P}}_e + \frac{n_e q_e^2}{m_e}\left(\bar{E}+\bar{V}\times\bar{B}\right) + \\ + \frac{q_e}{m_e}\left(\bar{J}\times\bar{B}\right) + \frac{q_e}{m_e} P_{ei}$$

THE COLLISIONAL TERM $P_{ei} = \xi\, q^2 n_e^2 (\bar{V}_i - \bar{V}_e)$ WHERE ξ HERE IS THE SPECIFIC RESISTIVITY, q^2 RELATES TO THE FACT THAT COLLISIONS RESULT FROM COULOMB FORCES BETWEEN IONS (q_i) AND ELECTRONS (q_e) AND TOTAL MOMENTUM TRANSFERRED TO ELECTRONS IN AN ELASTIC COLLISION WITH AN ION IS $\bar{V}_i - \bar{V}_e$. IN ADDITION CONSIDERING THAT $q_i = -q_e$ AND $n_e = n_i$ AND $\bar{J} = n_e q_e (\bar{V}_e - \bar{V}_i) \Rightarrow$

$$P_{ei} = -n_e q_e \xi \bar{J}$$

EQ. 76 BECOMES

$$\boxed{\partial_t \bar{J} = -\frac{q_e}{m_e}\bar{\nabla}\cdot\bar{\mathbb{P}}_e + \frac{n_e q_e^2}{m_e}\left(\bar{E}+\bar{V}\times\bar{B}\right) + \frac{q_e}{m_e}\left(\bar{J}\times\bar{B}\right) + \\ - \frac{n_e q_e^2}{m_e}\hat{\xi} \quad \text{WHERE } \hat{\xi} \text{ IS A TENSOR.}}$$

THIS IS THE GENERALIZED OHM LAW.