From:
An Introduction to Computer Simulation Methods Third Edition (revised)
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http://www.opensourcephysics.org/items/detail.cfm?ID=7375 chap 12

or Edition II chap 13

We also can consider *continuum* percolation models. For example, we can place disks at random into a two-dimensional box. Two disks are in the same cluster if they touch or overlap. A typical continuum (off-lattice) percolation configuration is depicted in Fig. 13.7. One quantity of interest is the quantity ϕ, the fraction of the area (volume in three dimensions) in the system that is covered by disks. In the limit of an infinite size box, it can be shown that

$$\phi = 1 - e^{-\rho\pi r^2}, \tag{13.1}$$

where ρ is the number of disks per unit area, and r is the radius of a disk (see Xia and Thorpe). Equation (13.1) is significantly inaccurate for small boxes because disks located near the edge of the box might have a significant fraction of their area located outside of the box. `Program site` can be modified to simulate continuum percolation. Instead of placing the disks on regular lattice sites, place them at random within a square box of area L^2. The relevant parameter is the density ρ, the number of disks per unit area, instead of the probability p. Because the disks overlap, it is convenient to replace the `BOX SHOW` statement in `Program site` with

```
BOX SHOW occup$ at x(i)-0.5,y(i)-0.5 using "or"
```

where the arrays `x(i)` and `y(i)` are used to store the disk positions of disk `i`. It also is a good idea to set the background color to red (not black or white).

Problem 13.4. Continuum percolation

a. For site percolation, we can define ϕ as the area fraction covered by the disks that are placed on the sites as in `Program site`. Convince yourself that $\phi_c = (\pi/4)p_c$ (for disks of unit diameter and unit lattice spacing). It is easy to do a Monte Carlo calculation of the area covered by the disks to confirm this result. (Choose points at random in the box and calculate the fraction of points within any disk.)

b. Modify `Program site` to simulate continuum percolation as discussed in the text. Estimate the value of the percolation threshold ρ_c. Given this value of ρ_c, use a Monte Carlo method to estimate the corresponding area fraction ϕ_c, and compare the value of ϕ_c for site and continuum percolation. Explain why you might expect ϕ_c to be bigger for continuum percolation than for site percolation. Compare your direct Monte Carlo estimate of ϕ_c with the indirect value of ϕ_c obtained from (13.1) using the value of ρ_c. Explain any discrepancy.

c. Consider the simple model of the cookie problem discussed in Section 13.1. Write a program that places disks at random into a square box and chooses their diameter randomly between 0 and 1. Estimate the value of ρ_c at which a spanning cluster first appears. How is the value of ρ_c changed from your estimate found in part (b)? Is your value for ϕ_c more or less than what was found in part (b)?

d. A more realistic model of the cookie problem is to place disks with unit diameter at random into a square box with the constraint that the disks do not overlap. Continue to add disks until the probability of placing an additional disk becomes less than 1%, i.e., when one hundred successive attempts at adding a disk are not successful. Then increase the diameters of all the disks at a constant rate (in analogy to the baking of the cookies) until a spanning cluster is attained. How does ϕ_c for this model compare with ϕ_c found in part (c)?

e. A continuum model that is applicable to random porous media is known as the *Swiss cheese* model. In this model the relevant quantity (the cheese) is the space between the disks. For the Swiss cheese model in two dimensions, the cheese area fraction at the percolation threshold, $\tilde{\phi}_c$, is given by $\tilde{\phi}_c = 1 - \phi_c$, where ϕ_c is the disk area fraction at the threshold of the disks. Do you think such a relation holds in three dimensions (see Project 13.15)? Imagine that the disks are conductors and that the cheese is an insulator and let $\sigma(\phi)$ denote the conductivity of this system. Alternatively, we can imagine that the cheese is a conductor and the disks are insulators and define a conductivity $\sigma(\tilde{\phi})$. Do you think that $\sigma(\phi) = \sigma(\tilde{\phi})$ when $\phi = \tilde{\phi}$? This question is investigated in Project 13.15.