b. Use the Lee-Kosterlitz analysis at the specific heat peak to determine if there is a phase transition.

Project 17.27. Ground state energy of the Ising spin glass

from Gould-Tobochnick

A spin glass is a magnetic system with frozen-in disorder. An example of such a system is the Ising model with the exchange constant J_{ij} between nearest neighbor spins randomly chosen to be ± 1. The disorder is said to be "frozen-in" because the set of interactions $\{J_{ij}\}$ does not change with time. Because the spins cannot arrange themselves so that every pair of spins is in its lowest energy state, the system exhibits frustration similar to the antiferromagnetic Ising model on a triangular lattice (see Problem 17.15). Is there a phase transition in the spin glass model, and if so, what is its nature? The answers to these questions are very difficult to obtain by doing simulations. One of the difficulties is that we need to do not only an average over the possible configurations of spins for a given set of $\{J_{ij}\}$, but we also need to average over different realizations of the interactions. Another difficulty is that there are many local minima in the energy (free energy at finite temperature) as a function of the configurations of spins, and it is very difficult to find the global minimum. As a result, Monte Carlo simulations typically become stuck in these local minima or metastable states. Detailed finite size scaling analyses of simulations indicate that there might be a transition in three dimensions. It is generally accepted that the transition in two dimensions is at zero temperature. In the following, we will look at some of the properties of an Ising spin glass on a square lattice at low temperatures.

a. Write a program to apply simulated annealing to an Ising spin glass using the Metropolis algorithm with the temperature fixed at each stage of the annealing schedule (see Problem 17.22a). Search for the lowest energy configuration for a fixed set of $\{J_{ij}\}$. Use at least one other annealing schedule for the same $\{J_{ij}\}$ and compare your results. Then find the ground state energy for at least ten other sets of $\{J_{ij}\}$. Use lattice sizes of $L = 5$ and $L = 10$. Discuss the nature of the ground states you are able to find. Is there much variation in the ground state energy E_0 from one set of $\{J_{ij}\}$ to another? Theoretical calculations give an average over realizations of $\overline{E_0}/N \approx -1.4$. If you have sufficient computer resources, repeat your computations for the three-dimensional spin glass.

optional

b. Modify your program to do simulated annealing using the demon algorithm (see Problem 17.22b). How do your results compare to those that you found in part (a)?

Project 17.28. Zero temperature dynamics of the Ising model We have seen that various kinetic growth models (Section 14.3) and reaction-diffusion models (Section 12.4) lead to interesting and nontrivial behavior. Similar behavior can be seen in the zero temperature dynamics of the Ising model. Consider the one-dimensional Ising model with $J > 0$ and periodic boundary conditions. The initial orientation of the spins is chosen at random. We update the configurations by choosing a spin at random and computing the change in energy ΔE. If $\Delta E < 0$, then flip the spin; else if $\Delta E = 0$, flip the spin with 50% probability. The spin is not flipped if $\Delta E > 0$. This type of Monte Carlo update is known as Glauber dynamics. How does this algorithm differ from the Metropolis algorithm at $T = 0$?

The quantity of interest is $f(t)$, the fraction of spins that flip for the first time at time t. As usual, the time is measured in terms of Monte Carlo steps per spin. Published results (Derrida,