$$f(x) = \begin{cases} \displaystyle\int_x^{x+\operatorname{arctg} x} \frac{1}{g(t)}\, dt & x > 0 \\[2ex] \displaystyle\int_0^x \frac{t}{(t-1)(t-2)^2}\, dt & x \le 0 \end{cases}$$

$g(t)$ inversa di

$$\left.\begin{array}{l} x \longrightarrow x + 2x^3 + 4x^7 \\ x^{\frac{1}{3}} + 2x^3 + 4x^7 \end{array}\right\rangle h(x)$$

$$\begin{tikzcd} h: \mathbb{R} \arrow[r] & \mathbb{R} \arrow[l, bend right, "g"'] \end{tikzcd} \qquad \lim_{x \to +\infty} h(x) = +\infty$$

$$\Longrightarrow \quad \lim_{t \to +\infty} g(t) = +\infty$$

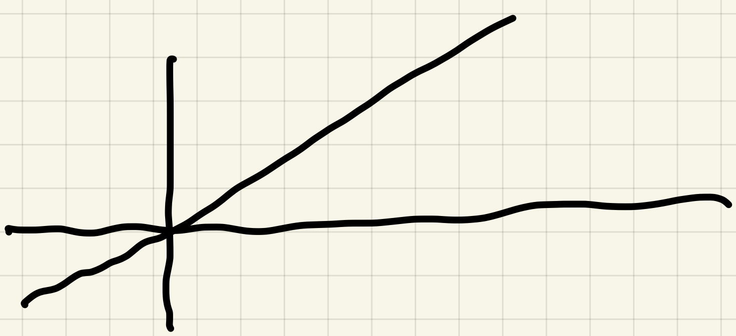
$g(t)$ non c'entra nulla $\dfrac{1}{h(t)}$

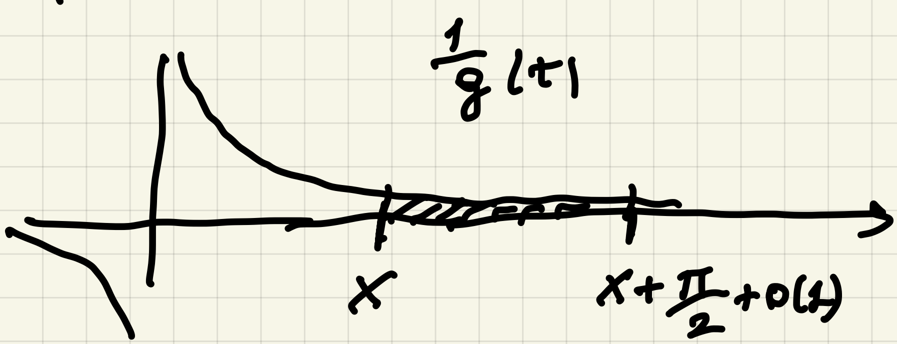
$$f(x) = \int_x^{x+\operatorname{arctg} x} \frac{1}{g(t)}\, dt$$

$$\lim_{x \to +\infty} f(x) = \,?$$

$$\int_x^{x+\operatorname{arctg} x} \frac{1}{g(t)}\, dt$$

$$= \int_x^{x+\frac{\pi}{2}+o(1)} \frac{1}{g(t)}\, dt \xrightarrow{x\to+\infty} 0$$





$$h(x) = x + 2x^3 + 4x^7$$

per $x > 0$

$h(0)=0$
$g(0)=0$

$$\lim_{t\to+\infty} g(t) = +\infty$$

$$\Downarrow$$

$$\lim_{t\to+\infty} \frac{1}{g(t)} = 0$$

$g \in C^0(\mathbb{R})$

$\frac{1}{g} \in C^0(\mathbb{R}\setminus\{0\})$

$h(x) = y$

$g(y) = x$

$x = y = 0$

$h(0) = 0$

$g(0) = 0$

$$0 < \int_x^{x+\frac{\pi}{2}+o(1)} \frac{1}{g(t)}\,dt = \left(\cancel{x}+\frac{\pi}{2}+o(1)-\cancel{x}\right)\frac{1}{g(c_x)}$$

dove $\quad x \le c_x \le x+\frac{\pi}{2}+o(1)$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\int_x^{x+\frac{\pi}{2}+o(1)} \frac{1}{g(t)}\,dt = \lim_{x\to+\infty}\left(\frac{\pi}{2}+o(1)\right)\frac{1}{g(c_x)}$$

$$= \frac{\pi}{2}\lim_{x\to+\infty}\frac{1}{g(c_x)}$$

$$= \frac{\pi}{2}\lim_{c\to+\infty}\frac{1}{g(c)} = 0$$

$$\lim_{t\to+\infty} g(t) = +\infty$$

$$h(x) = x + 2x^3 + 4x^7$$

$$x^{\frac{1}{3}} + 2x^3 + 4x^7$$

$$f(x) = \begin{cases} \displaystyle\int_x^{x+\operatorname{arctg} x} \frac{1}{g(t)}\,dt & x > 0 \\ \displaystyle\int_0^x \frac{t}{(t-1)(t-2)^2}\,dt & x \leq 0 \end{cases}$$

$g(t)$ inversa di

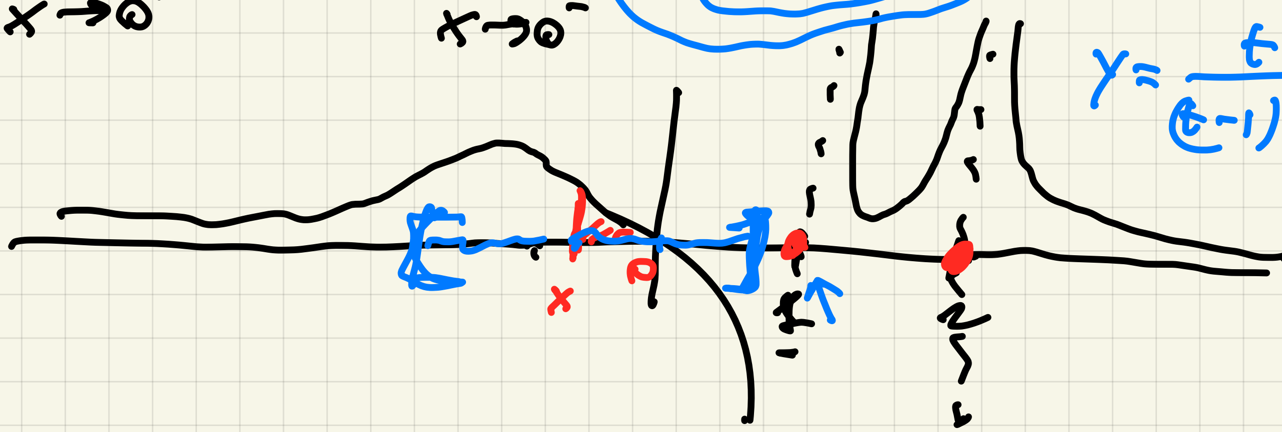
$$x \to x + 2x^3 + 4x^7 \quad \Big\} \; h(x)$$

$$x^{\frac{1}{3}} + 2x^3 + 4x^7$$

$f \in C^\infty(\mathbb{R} \setminus \{0\})$

$$\lim_{x\to 0^-} f(x) = \lim_{x \to 0^-} \int_0^x \frac{t}{(t-1)(t-2)^2}\,dt = \int_0^0 \frac{t}{(t-1)(t-1)^2}\,dt = 0 = f(0)$$

$$y = \frac{t}{(t-1)(t-2)^2}$$



$$\lim_{x\to 0^-} f(x) = f(0) = 0$$

$$\lim_{x \to 0^+} f(x) \neq f(0)$$

$$f(x)=\begin{cases} \displaystyle\int_x^{x+\operatorname{arctg} x} \frac{1}{g(t)}\,dt & x>0 \\ \displaystyle\int_0^x \frac{t}{(t-1)(t-2)^2}\,dt & x\le 0 \end{cases}$$

$g(t)$ inversa di

$$x \longrightarrow x+2x^3+4x^7 \longleftarrow h(x)$$

$$x^{\frac{1}{3}}+2x^3+4x^7$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \int_x^{x+\operatorname{arctg}(x)} \frac{1}{g(t)}\,dt$$

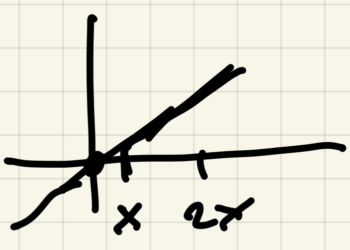
$$\operatorname{arctg}(x) = x+o(x)$$

$$\int_x^{2x+o(x)} \frac{1}{g(t)}\,dt$$

$g(t)$ inversa di

$x+2x^3+4x^7$

$$g(t)=g(0)+g'(0)t+o(t)$$
$$=0+g'(0)t+o(t)$$



$$g'(0)=\frac{1}{(x+2x^3+4x^7)'(0)}=1$$

$$g'(y_0)=\frac{1}{h'(x_0)} \qquad x_0=y_0=0$$

$$g(t)=t+o(t)=$$
$$g(t)=t\,(1+o(1))$$

$$\int_x^{2x+o(x)} \frac{1}{g(t)}\,dt = \int_x^{2x+o(x)} \frac{1}{t}\,\frac{1}{(1+o(1))}$$

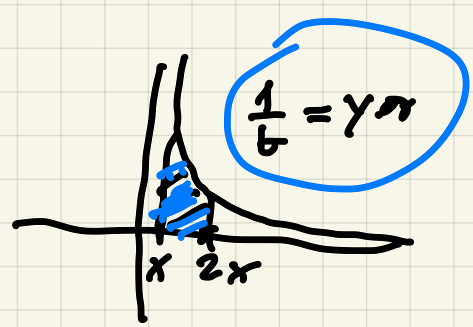
$$g(t)=t\,(1+o(1)) \qquad = \int_x^{2x+o(x)} \frac{1}{t}\,(1+o(1))\,dt$$

$$= \underbrace{\int_x^{2x+o(x)} \frac{1}{t}\,dt}_{\xrightarrow{x\to 0^+} \lg 2} + \underbrace{\int_x^{2x+o(x)} \frac{o(1)}{t}\,dt}_{\xrightarrow{x\to 0^+} 0}$$

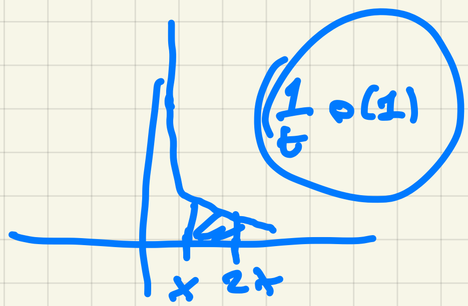
$$\int_x^{2x+o(x)} \frac{1}{t}\,dt = \lg(2x+o(x)) - \lg(x)$$

$$= \lg \frac{2x+o(x)}{x} = \lg(2+o(1)) \xrightarrow{x\to 0^+} \lg 2$$

$$\int_x^{2x+o(x)} \frac{o(1)}{t}\,dt =$$



$$= (x+o(x)) \left.\frac{o(1)}{t}\right|_{t=c_x}$$



$$x \le c_x \le 2x+o(x)$$

$$\left| \int_x^{2x+o(x)} \frac{o(1)}{t}\,dt \right| = \left| (x+o(x)) \left.\frac{o(1)}{t}\right|_{t=c_x} \right|$$

$$x \le c_x \le 2x+o(x)$$

$$\le (x+o(x)) \cdot \frac{|o(1)|}{c_x} =$$

$$= \frac{x+o(x)}{c_x} |o(1)|$$

$$= \frac{x}{c_x}(1+o(1)) \; |o(1)|$$

$$\le \cancel{\frac{x}{x}} (1+o(1)) \; |o(1)|_{c_x} \xrightarrow{x\to 0}$$

$$\lim_{x\to 0} o(1)|_{c_x} = \lim_{c\to 0} o(1) = 0$$

$$\lim_{x\to 0^+} f(x) = \lg(2) \neq 0 = f(0)$$

f non e' continua in 0.

$h(x) = x^{\frac{1}{3}} + 2x^3 + 4x^7$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \int_x^{x+\arctan(x)} \frac{1}{g(t)}\,dt$$

Per $h(x) = x^{\frac{1}{3}} + 2x^3 + 4x^7$ allora

vicino a 0 $\quad h(x) = x^{\frac{1}{3}}\left(1 + o(x^2)\right)$

La funzione inversa di $h(x) = x^{\frac{1}{3}}$?

$g(y) = y^3 \qquad g(h(x)) = x$

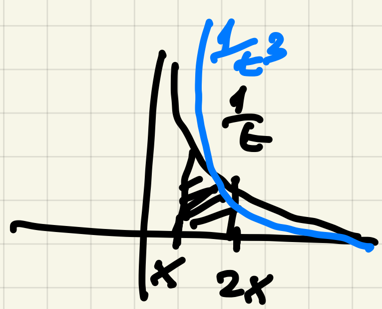
$g(h(x)) = h^3(x) = \left(x^{\frac{1}{3}}\right)^3 = x$

$h(g(x)) = x$

Nel nostro caso $\quad g(t) = t^3\left(1 + o(1)\right)$

$$\int_x^{2x+o(x)} \frac{1}{g(t)}\,dt = \int_x^{2x+o(x)} \frac{1}{t^3}\,\frac{1}{(1+o(1))}\,dt =$$

$$= \int_x^{2x+o(x)} \frac{1}{t^3}\,\overbrace{(1+o(1))}^{>\frac{1}{2}}\,dt$$



$$\geq \frac{1}{2}\int_x^{2x+o(x)} \frac{1}{t^3}\,dt \xrightarrow{x\to 0^+} +\infty$$

$$\int_{x}^{2x+o(x)} \frac{1}{t^3}\,dt = \left.\frac{1}{-2}\,\frac{1}{t^2}\right]_x^{2x+o(x)} =$$

$$= \frac{1}{2}\,\frac{1}{x^2} - \frac{1}{2}\,\frac{1}{(2x+o(x))^2} =$$

$$= \frac{1}{2}\,\frac{1}{x^2} - \frac{1}{2}\,\frac{1}{\left(2x\,(1+o(1))\right)^2} =$$

$$= \frac{1}{2}\,\frac{1}{x^2} - \frac{1}{8}\,\frac{1}{x^2}\,\frac{1}{(1+o(1))^2} \sim 1+o(1)$$

$$= \frac{1}{2}\,\frac{1}{x^2} - \frac{1}{8}\,\frac{1}{x^2}\,(1+o(1))$$

$$= \frac{1}{2x^2}\left(1 - \frac{1+o(1)}{4}\right) =$$

$$= \frac{1}{2x^2}\left(\frac{3}{4} + o(1)\right) \xrightarrow{x\to 0^+} +\infty$$

$$\int_1^{+\infty} \frac{\cos x}{[x]}\,dx$$

$$\boxed{\int_1^{+\infty} \frac{\cos x}{x}\,dx \quad \text{convergente.}}$$

$$\frac{\cos x}{[x]} = \frac{\cos x}{x} + \boxed{\cos x\left(\frac{1}{[x]} - \frac{1}{x}\right)}$$

integrabile non non integrabile ass.

$$\cos x\left(\frac{1}{[x]} - \frac{1}{x}\right) = \cos x \, \frac{x-[x]}{[x]\,x} =$$

$$= \cos x \, \frac{1}{x} \, \frac{1}{x} \, \frac{1}{1+o(1)} \, (x-[x])$$

$$[x] = x + [x] - x = x\left(1 + \frac{[x]-x}{x}\right) =$$

$$-1 < [x]-x \le 0 \qquad [x] \le x < [x]+1$$

$$[x] = x\,(1+o(1))$$

$$\cos x \left(\frac{1}{[x]} - \frac{1}{x} \right) = \cos x \, \frac{x - [x]}{[x]\, x} =$$

$$= \cos x \, \frac{1}{x} \, \frac{1}{x} \, \frac{1}{1+o(1)} \, (x - [x])$$

$$= \left| \cos x \, \frac{1}{x^2} \, (1+o(1)) \, \overbrace{(x-[x])}^{\leq 1} \right|$$

$$\leq \frac{1}{x^2} \, (1+o(1))$$