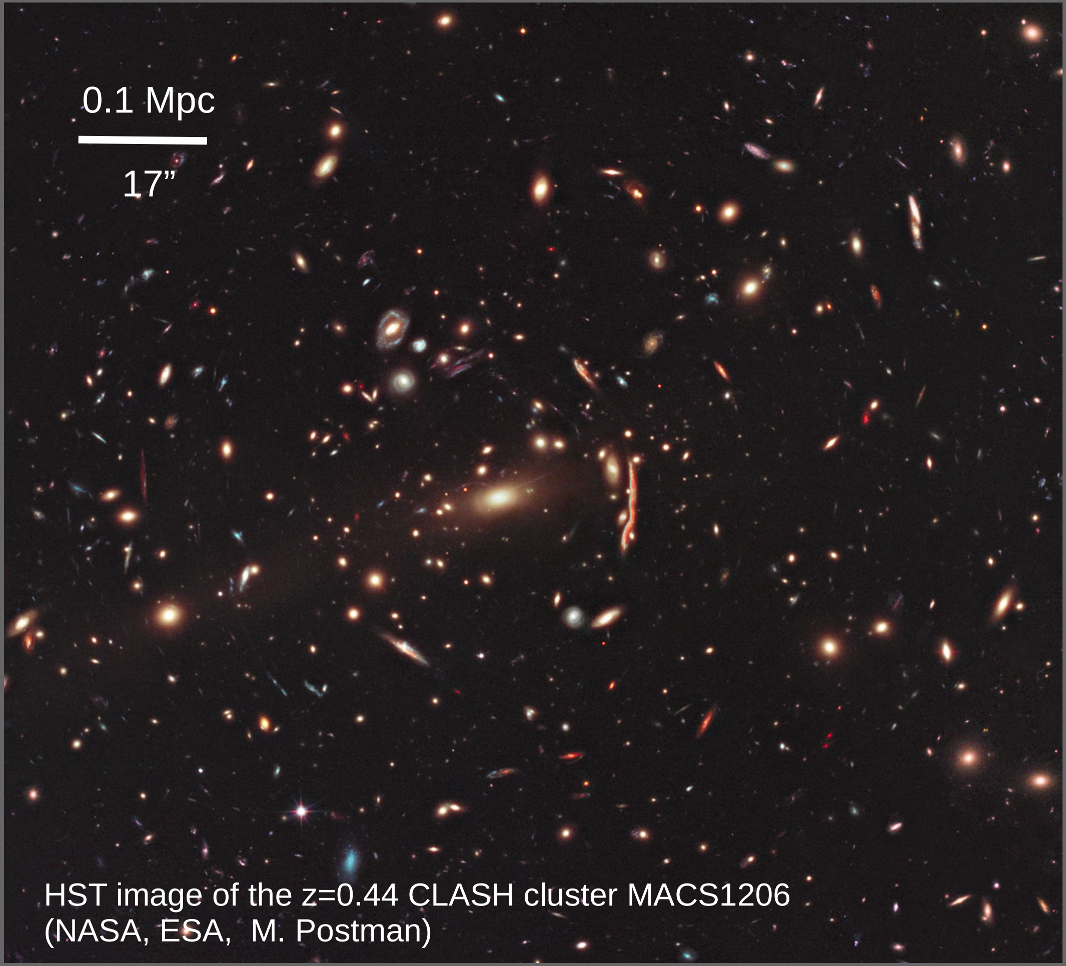
Dynamical mass measurements of clusters of galaxies



HST image of the z=0.44 CLASH cluster MACS1206
(NASA, ESA, M. Postman)

Andrea Biviano
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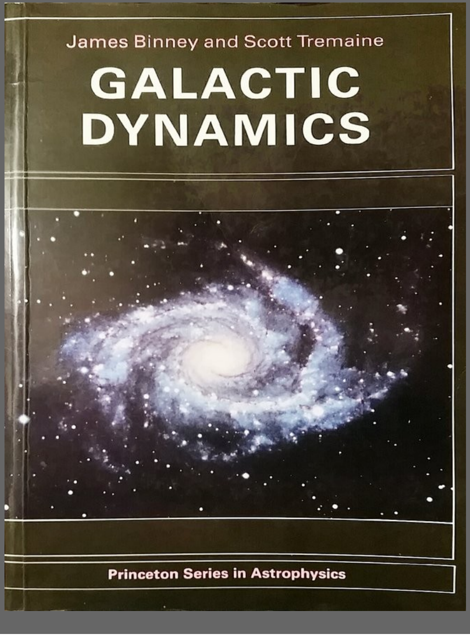
Dynamical mass measurements of clusters of galaxies

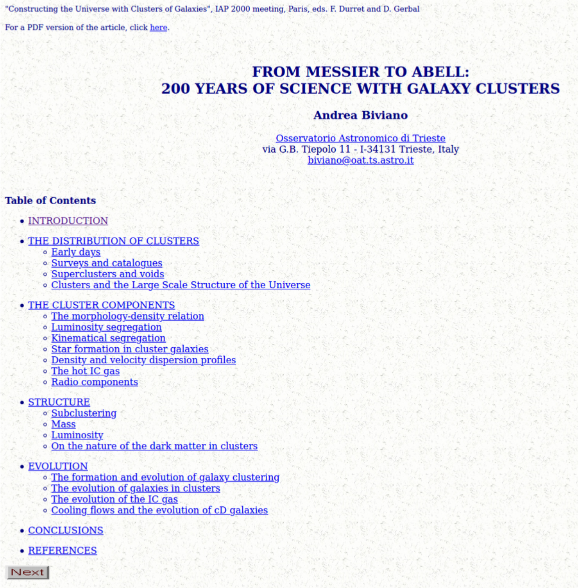
Main textbook:

Binney & Tremaine 1987,
Chapters 4.1, 4.2, 4.3



Additional readings:

AB 00,
http://ned.ipac.caltech.edu/level5/Biviano2/frames.html
(review on clusters of galaxies)



"Constructing the Universe with Clusters of Galaxies", IAP 2000 meeting, Paris, eds. F. Durret and D. Gerbal

For a PDF version of the article, click here.

FROM MESSIER TO ABELL:
200 YEARS OF SCIENCE WITH GALAXY CLUSTERS

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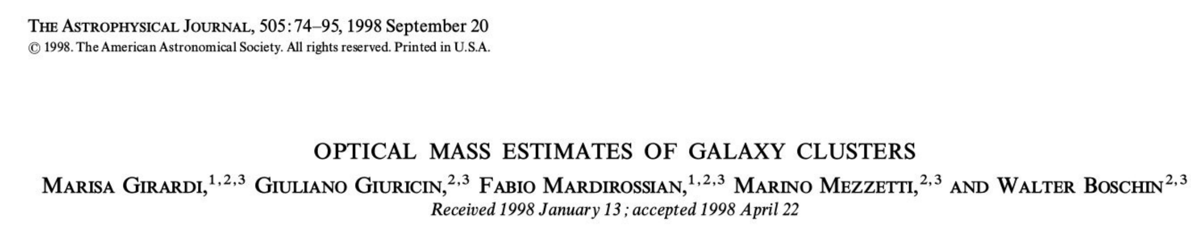
Girardi et al. 98,
The Astrophysical Journal,
505, 74 *(virial theorem)*



The Astrophysical Journal, 505: 74–95, 1998 September 20
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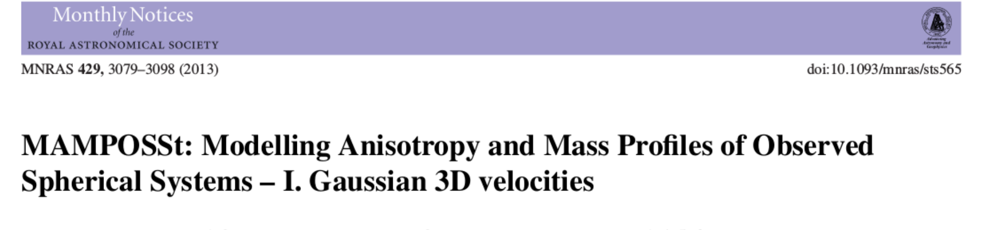
OPTICAL MASS ESTIMATES OF GALAXY CLUSTERS

Marisa Girardi,[1,2,3] Giuliano Giuricin,[2,3] Fabio Mardirossian,[1,2,3] Marino Mezzetti,[2,3] and Walter Boschin[2,3]
Received 1998 January 13 ; accepted 1998 April 22

Mamon, AB, Boué 13,
Monthly Notices of the Royal Astronomical Society,
429, 3079 *(MAMPOSSt)*



Monthly Notices of the Royal Astronomical Society

MNRAS **429**, 3079–3098 (2013) doi:10.1093/mnras/sts565

MAMPOSSt: Modelling Anisotropy and Mass Profiles of Observed Spherical Systems – I. Gaussian 3D velocities

Gary A. Mamon,[1,2★] Andrea Biviano[3] and Gwenaël Boué[1,4,5,6]

Where are the galaxies?

The Milky Way,
our own galaxy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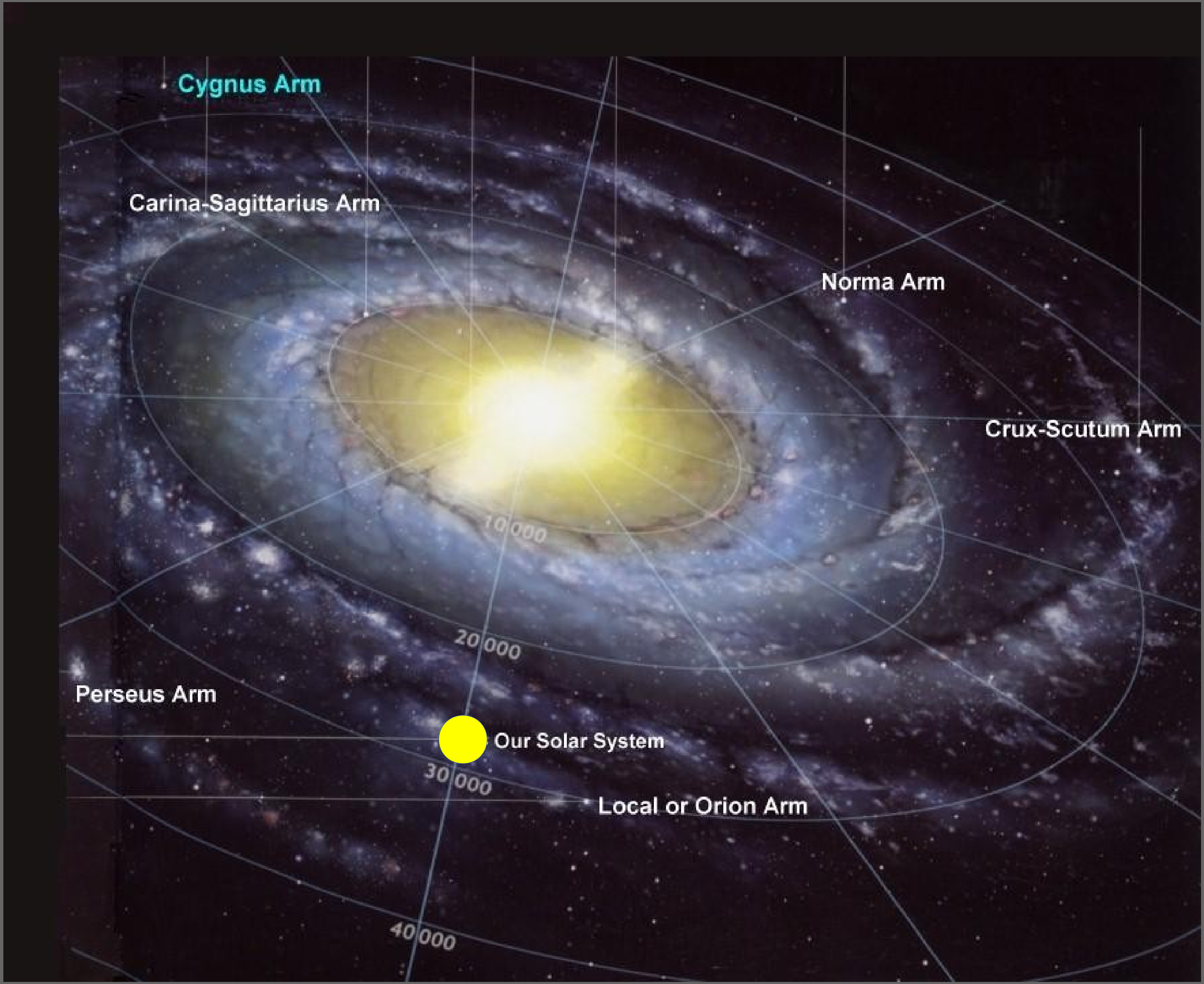
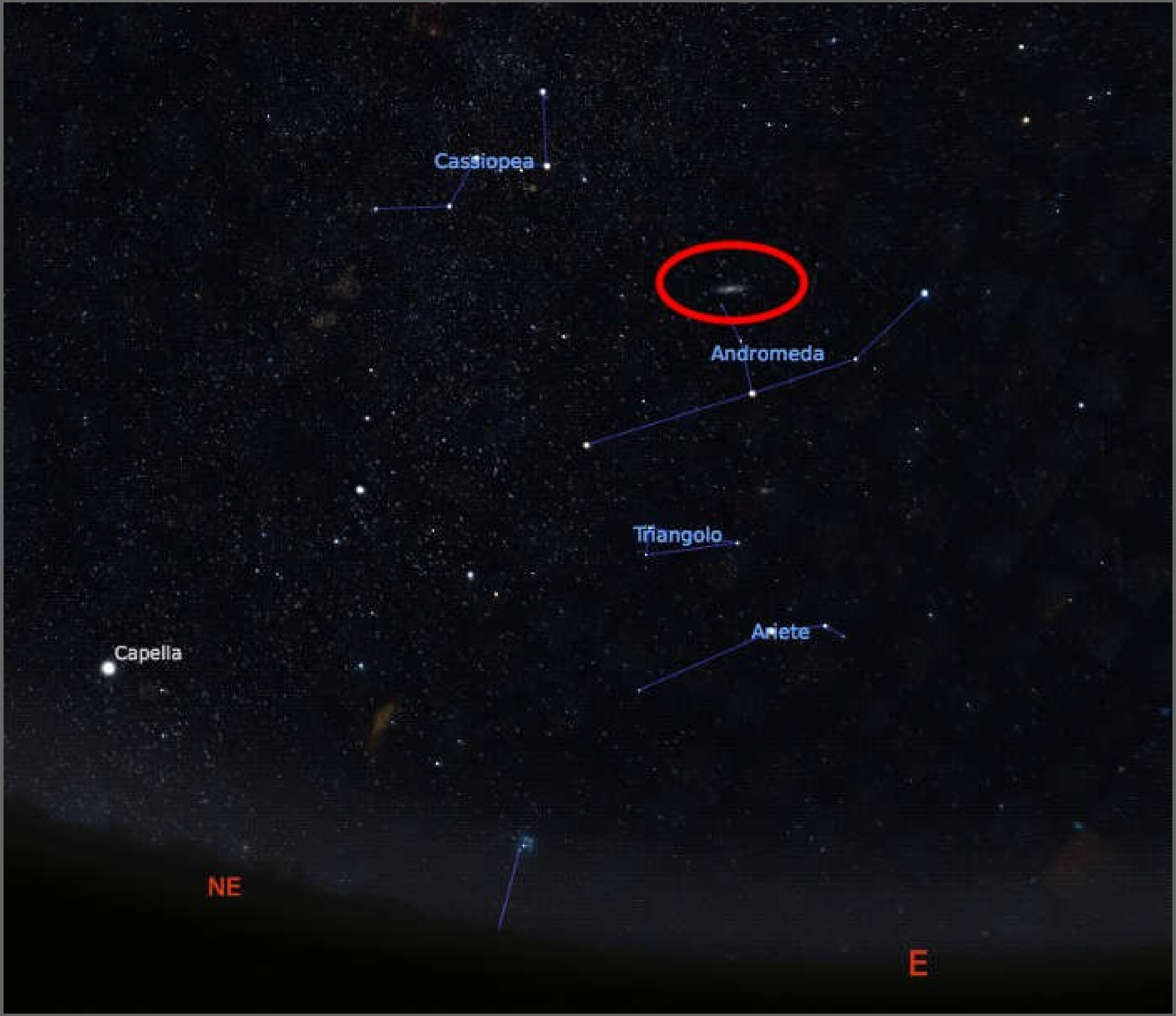
Where are the galaxies?

The Milky Way, our own galaxy



Where are the galaxies?

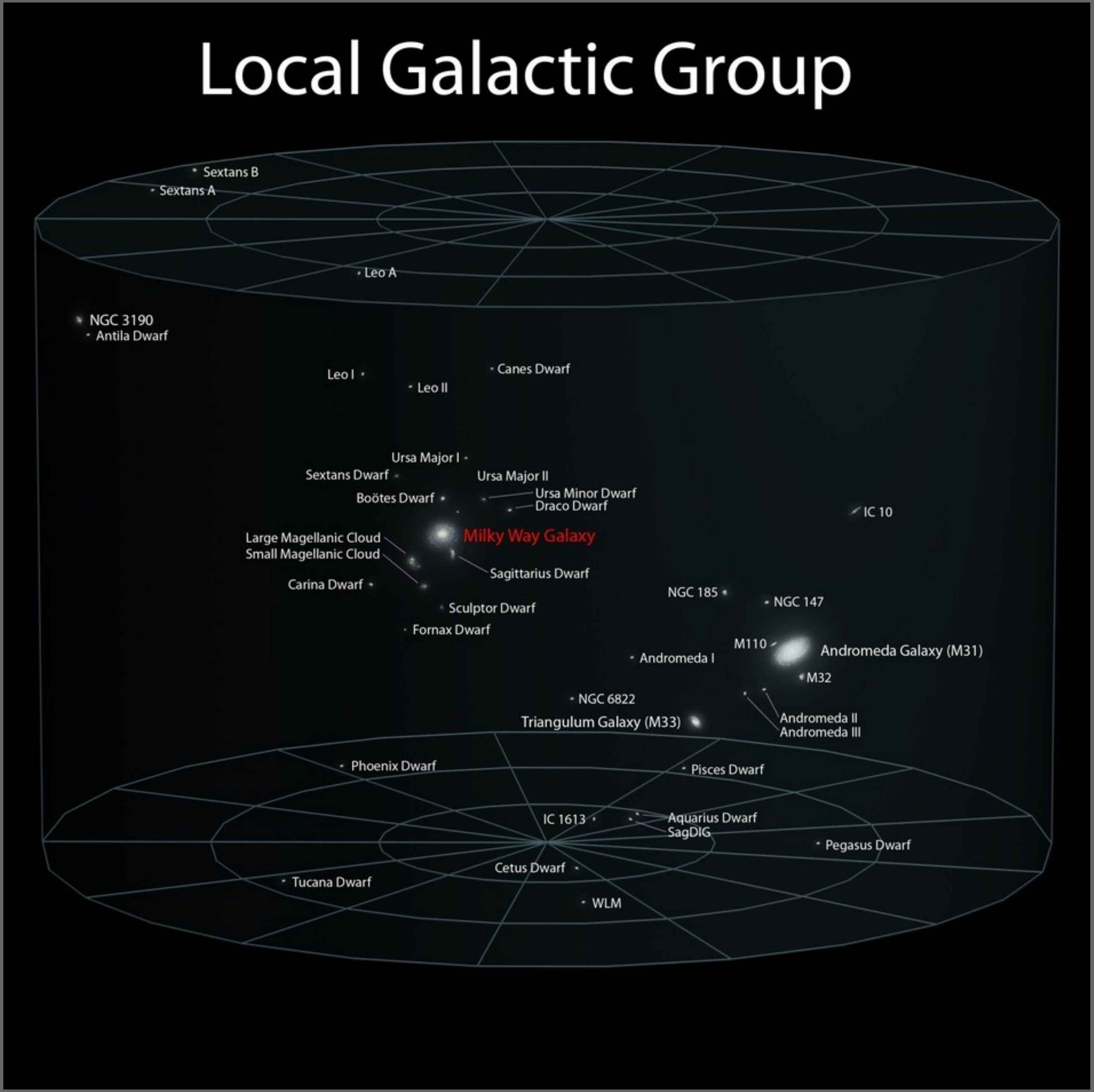
Where are the galaxies?



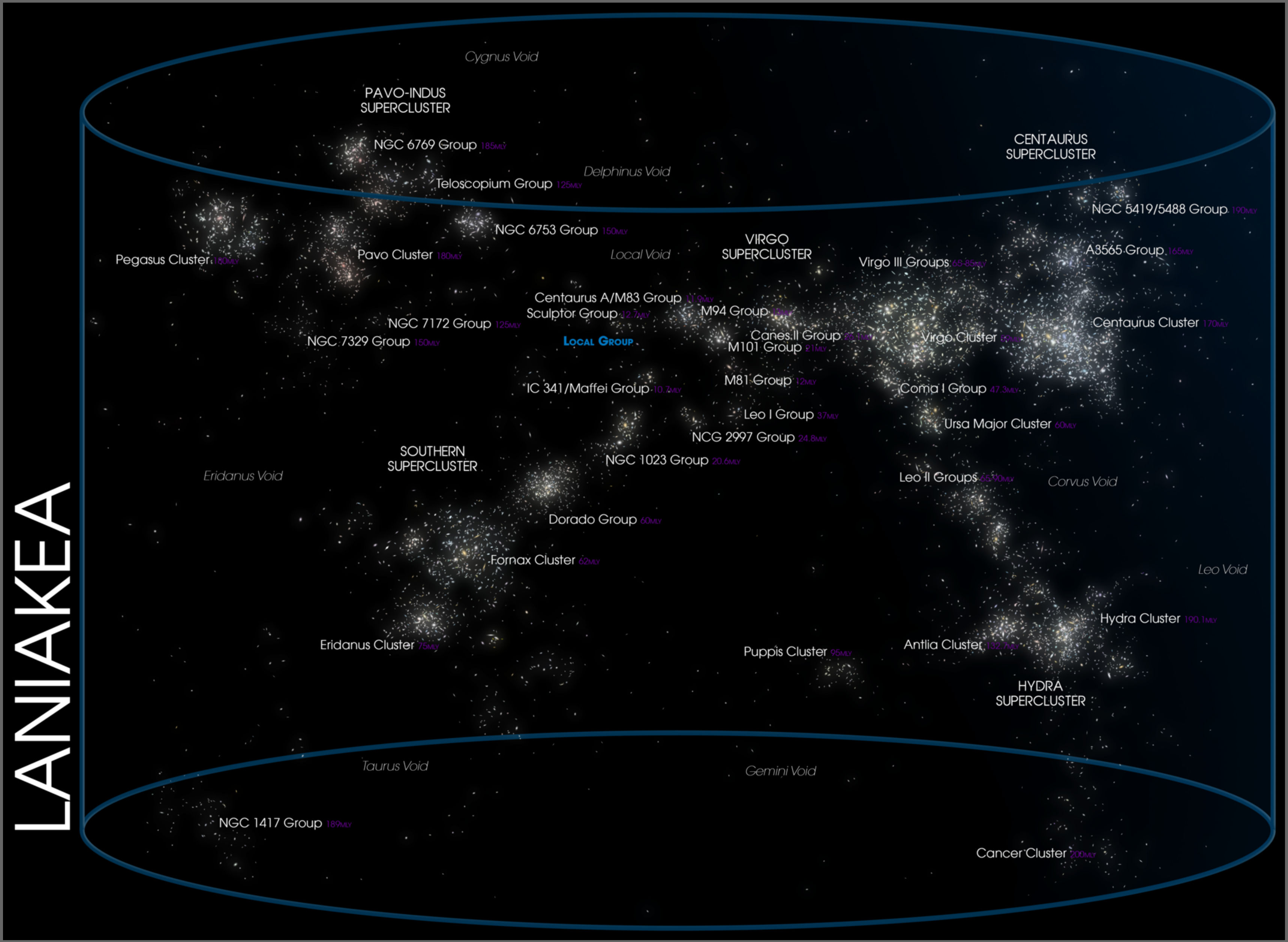
Where are the galaxies?

Andromeda and its two satellite galaxies

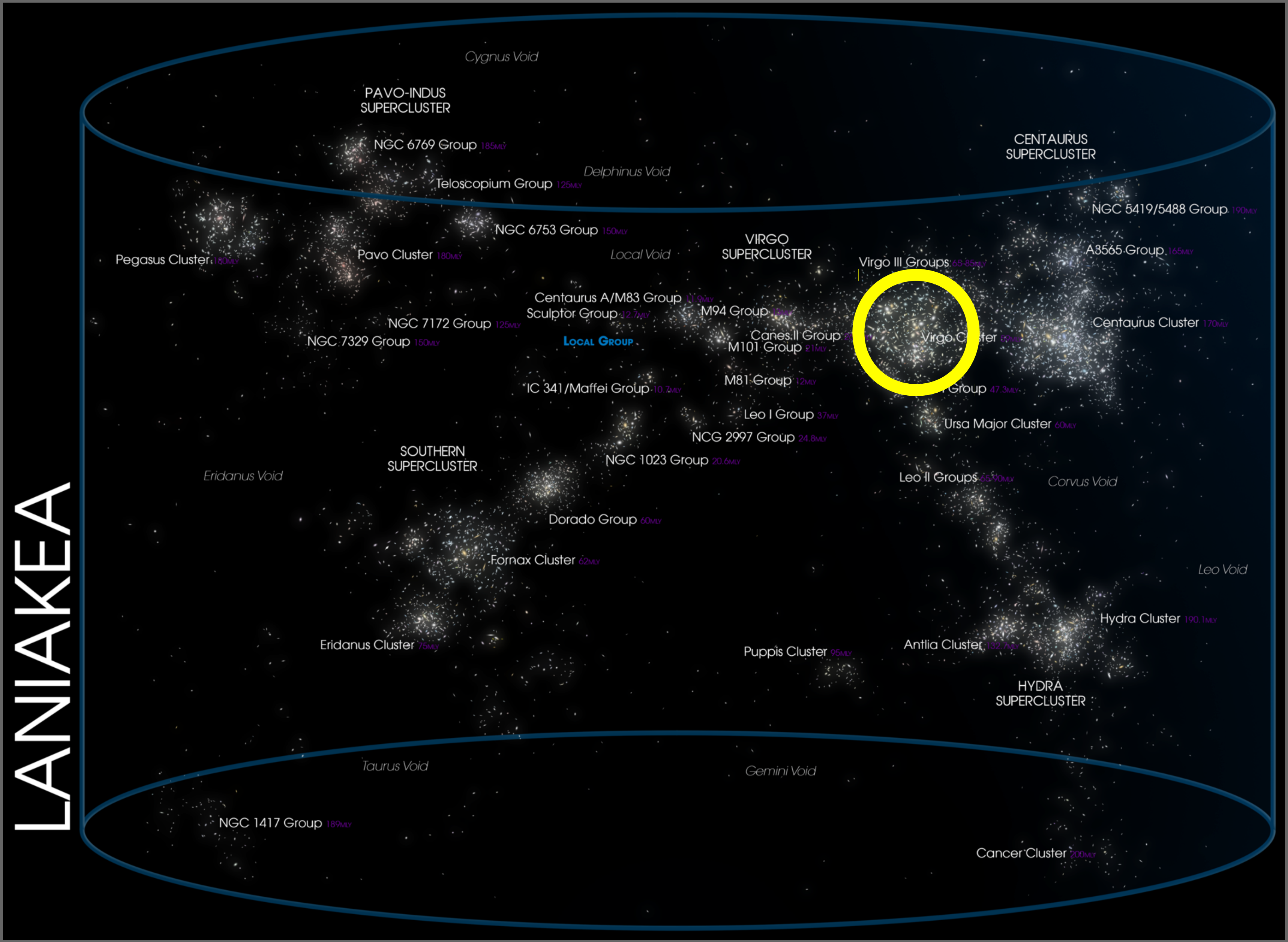
Where are the galaxies?



1/3 galaxies are in groups and clusters



1/3 galaxies are in groups and clusters



The Virgo cluster of galaxies

Rogelio Bernal Andreo
DeepSkyColors.com

The properties of clusters of galaxies

Each cluster contain from a few galaxies (“group of galaxies”) to several thousands

1/3 all galaxies belong to clusters (or groups, mostly)

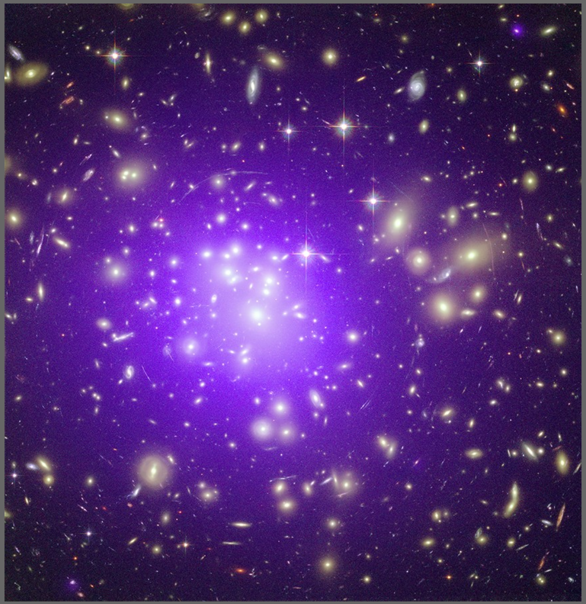
Most of the cluster baryonic mass is not in galaxies but in a diffuse intra-cluster plasma ($10^{7\text{-}8}$ K), that shines in X-ray by thermal bremsstrahlung

Most of the (gravitationally detected) mass is in an undetected form (Dark Matter)

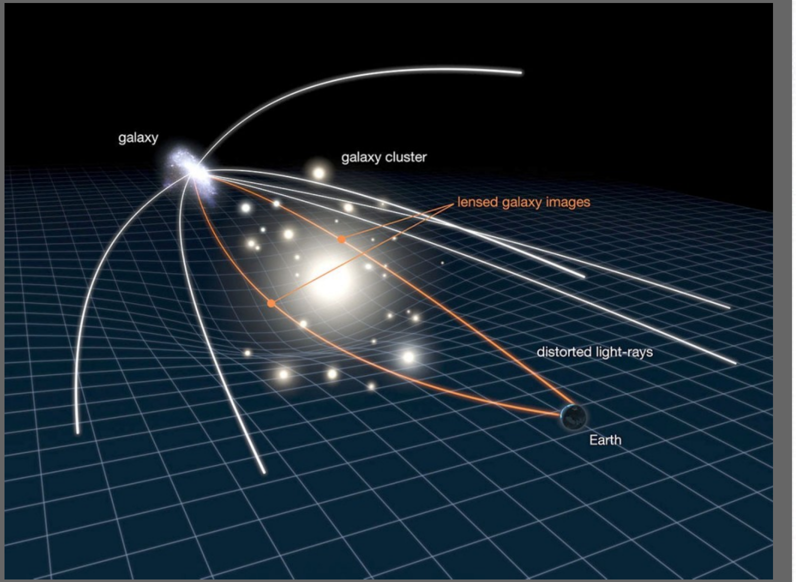
Typical size: $2\ 10^6$ pc (1 pc = $3\ 10^{16}$ m); typical mass: $10^{14\text{-}15}$ $M_\odot$ (1 $M_\odot$ = $2\ 10^{30}$ Kg)

Clusters are the largest gravitationally bound systems in the Universe. For this reason, they are the latest cosmological structures to form (in a cosmological model where hierarchical gravitational assembly is the dominant process)

How to determine cluster masses and cluster mass profiles?

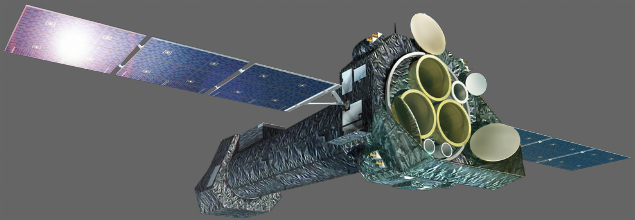


X-ray observations: assuming the intra-cluster, X-ray emitting gas is in hydrostatic equilibrium



Optical observations: using the deflected and amplified light from background galaxies due to the gravitational lensing effect

Optical observations: using the spatial and velocity distributions of cluster galaxies

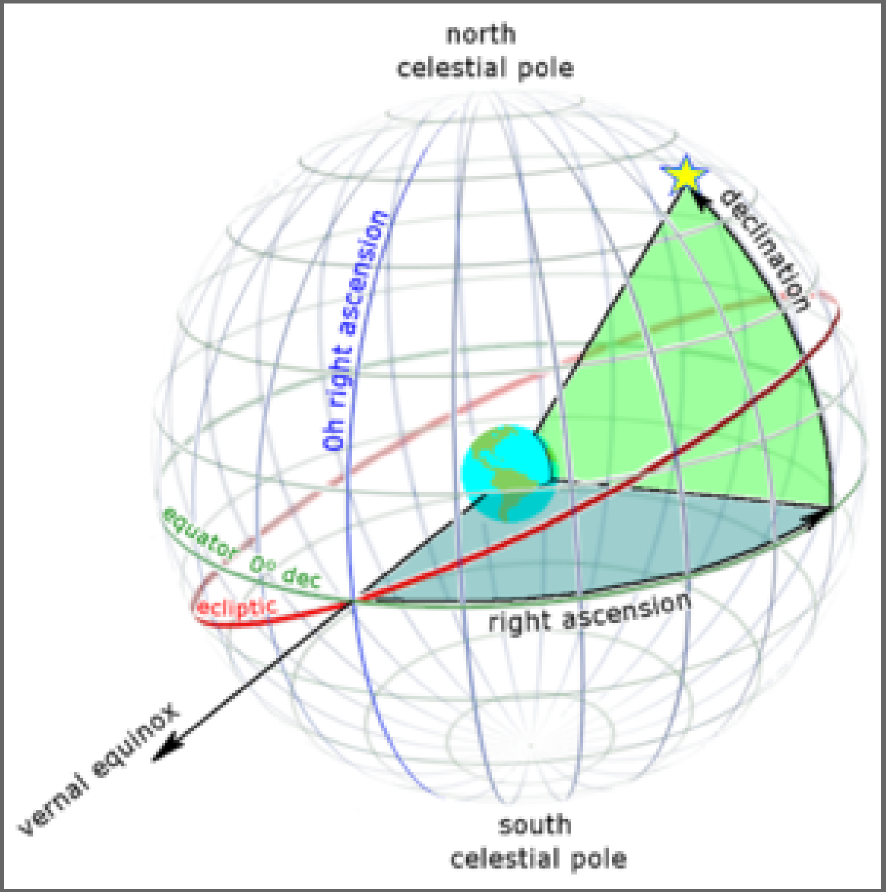


XMM-Newton space telescope

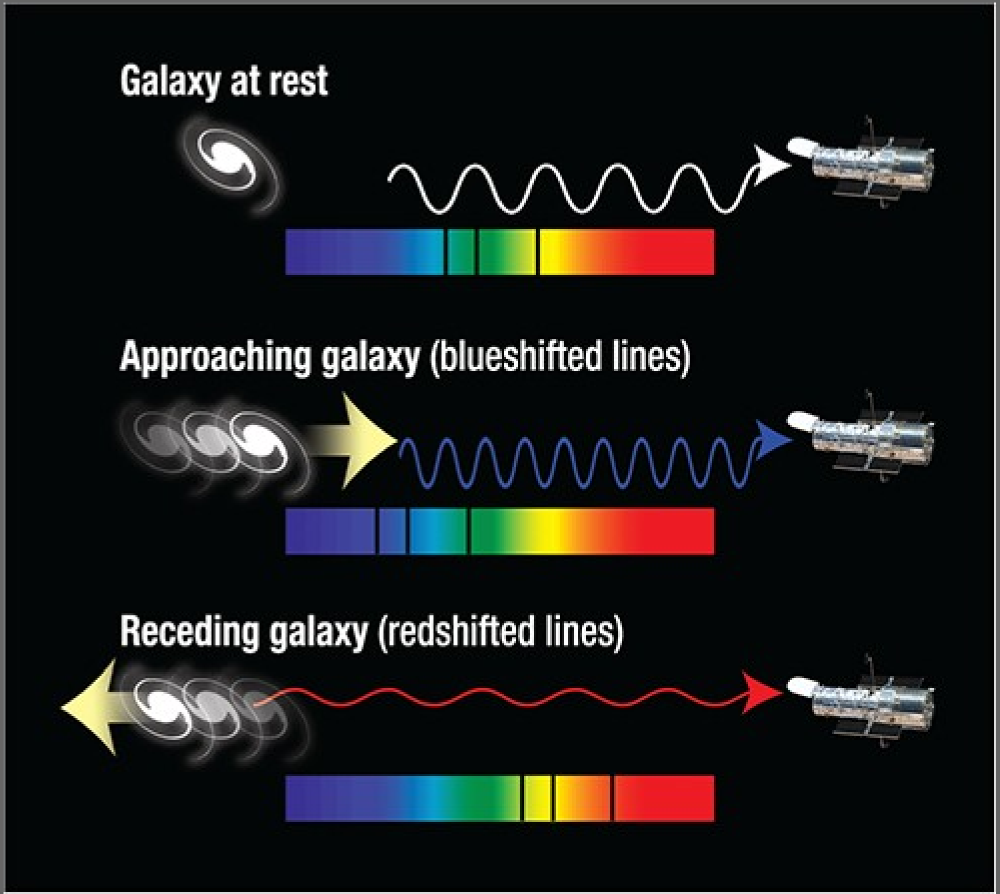
Hubble space telescope

Very Large Telescope

The Observables

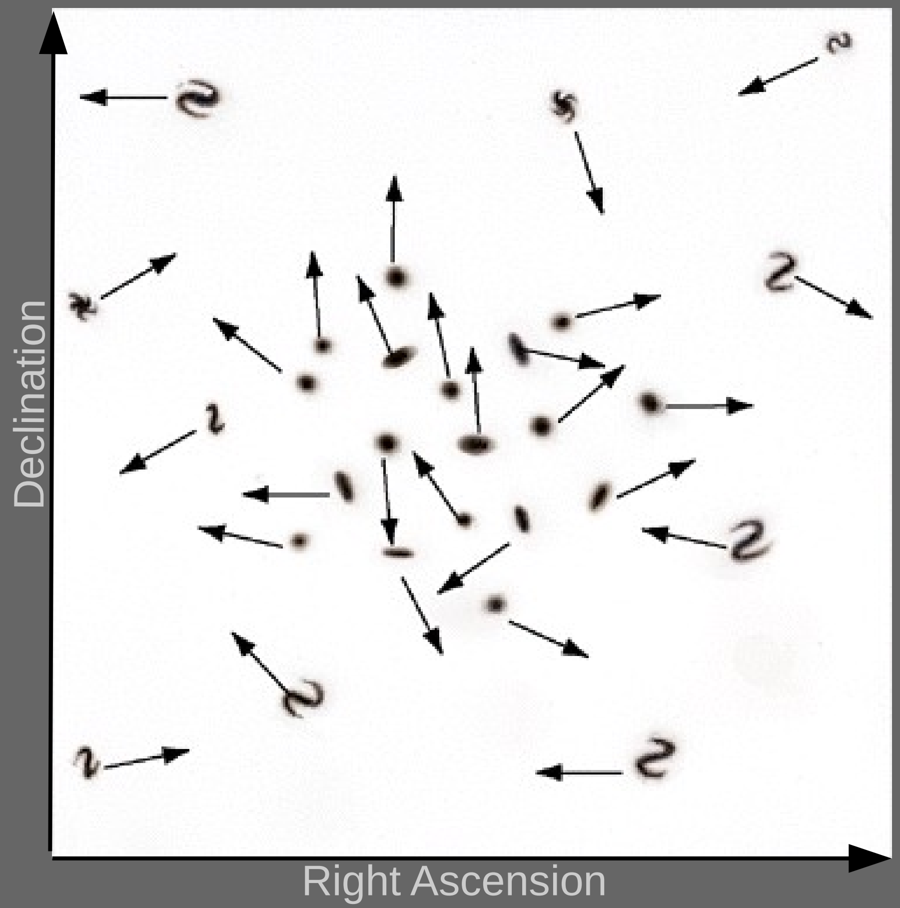


Right Ascension and Declination:
define the projected position of a
galaxy on the sky



Redshift: defines the velocity of a galaxy with respect to us (along the line of sight direction). It is the sum of the cosmological term (due to the expansion of the Universe) and the proper motion of the galaxy inside the cluster it belongs to

The Observables



Need the spatial and velocity distributions of the tracers of the cluster gravitational potential:

1) **positions** of cluster galaxies, RA & Dec
(from imaging observations)

- need to define a cluster center:
the Brightest Cluster Galaxy,
the peak of the X-ray emission from the intra-cluster medium,
the density peak of the galaxy distribution

2) **redshifts** of cluster galaxies, z,
(from spectroscopic observations)

- need to define a cluster mean redshift:
the Brightest Cluster Galaxy redshift,
the mean (median) of the cluster galaxy redshifts

Given a cluster center, RA & Dec → **projected distances from cluster center, R**

Given a cluster <z>, z → **rest-frame velocities, v_{rf}** $\equiv c\,(z-\langle z\rangle)/(1+\langle z\rangle)$
(removing the cosmological component of z, the kinematic component is left)

The Equations

System of collision-less galaxies moving in a smooth gravitational potential Φ(**x**,t)

The system state can be fully described by the phase-space density,
f(**x**,**v**,*t*) d^3**x** d^3**v**, i.e. the number of galaxies in a volume d^3**x** and in velocity range d^3**v**

Define the phase-space 6-dimensional vector: **w** ≡ (**x**,**v**)

If there are no collisions, the orbits of the galaxies in the system define a smooth incompressible flow in phase-space,
so that *f*(**w**,*t*) satisfies a continuity equation:

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{6} \frac{\partial f \dot{w}_i}{\partial w_i} = 0$$

rate of increasing number of galaxies inside a phase-space volume element

flow rate of galaxies out of the volume element

If **x** and **v** are independent coordinates of the phase space, and the gradient of the potential does not depend on velocities, we obtain the **collision-less Boltzmann (or Vlasov) equation:**

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{6} \dot{w}_i \frac{\partial f}{\partial w_i} = 0 \quad \longleftrightarrow \quad \frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) = 0$$

The Equations

Integrating the Vlasov equation, we obtain the **Jeans equations:**

$$\nu \frac{\partial \overline{v_j}}{\partial t} + \nu \overline{v_i} \frac{\partial \overline{v_j}}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \frac{\partial (\nu \sigma_{ij}^2)}{\partial x_i}$$

spatial density of galaxies

mean galaxy velocity

i, j = 1,2,3 coordinates

velocity dispersion tensor *(can be diagonalized because it is symmetric):*

$$\sigma_{ij}^2 \equiv \overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$$

If the system is spherically symmetric, we can re-write these equations in spherical coordinates r, θ, Φ, for the special case in which the system is **in steady state, does not contract or expand, and does not rotate**:

$$\frac{d(\nu \sigma_r^2)}{dr} + \frac{\nu}{r}\left[2\sigma_r^2 - (\sigma_\theta^2 + \sigma_\phi^2)\right] = -\nu \frac{d\Phi}{dr}$$

If there is no preference for one of the two tangential directions: $\sigma_\theta^2 = \sigma_\phi^2$
the Jeans equation becomes:

$$\frac{d(\nu \sigma_r^2)}{dr} + 2\beta \frac{\nu \sigma_r^2}{r} = -\nu \frac{d\Phi}{dr}$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

Velocity anisotropy

The Jeans equation

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\Phi}{dr}$$

dynamical
pressure
gradient

gravitational
potential
gradient

James Jeans

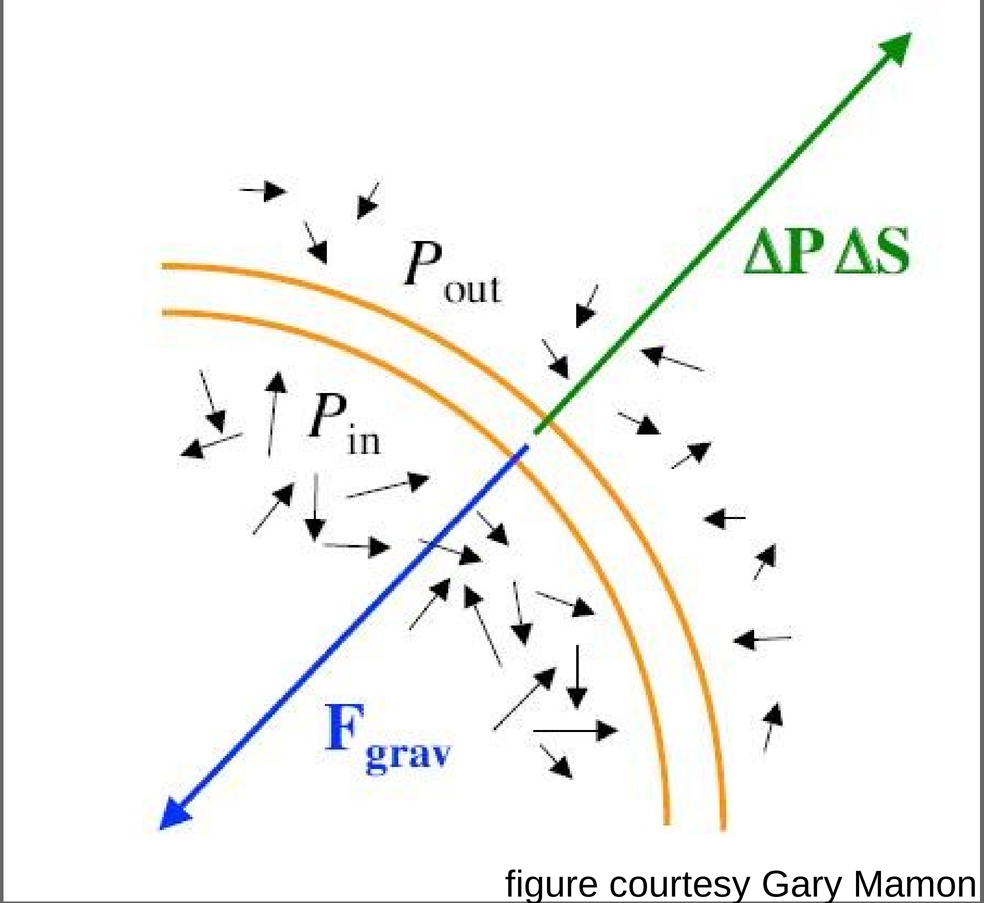


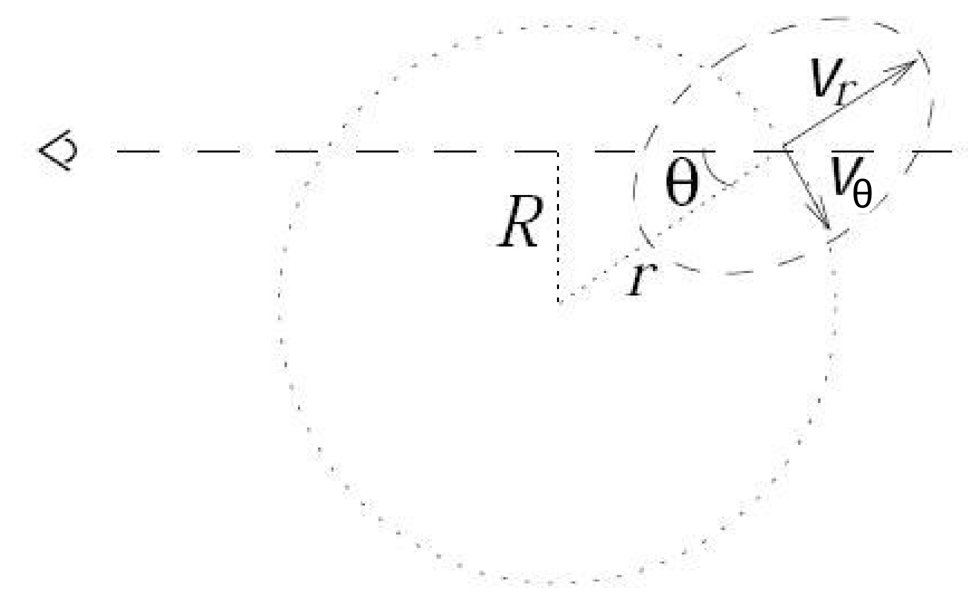
figure courtesy Gary Mamon

The Jeans equation

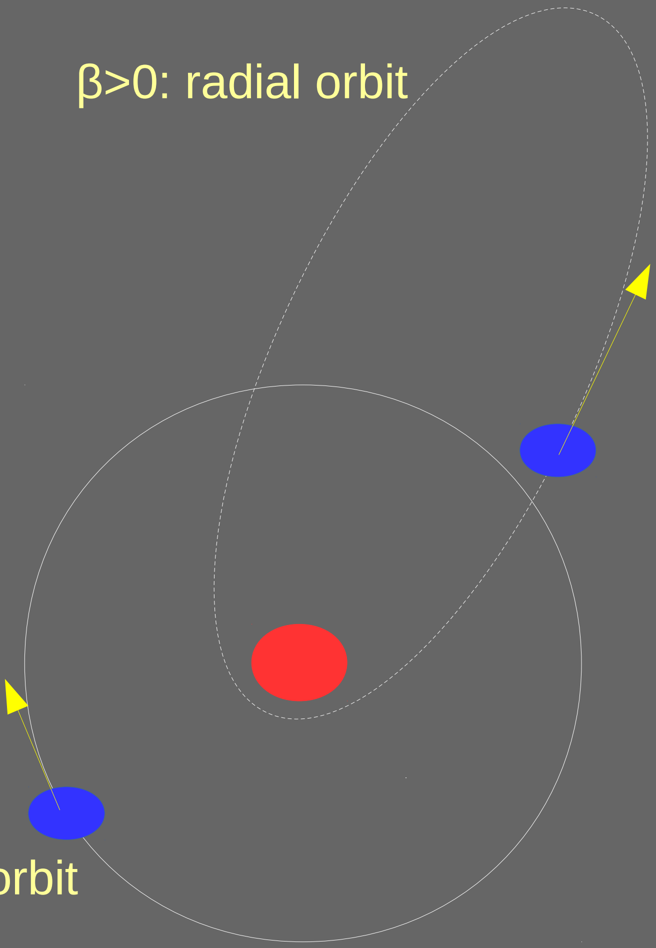
$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\Phi}{dr}$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

elongation of the velocity ellipsoid
→orbits of cluster galaxies



β>0: radial orbit



β<0: tangential orbit

The Equations

Given that $\frac{d\Phi}{dr} = \frac{GM(r)}{r^2}$, from the Jeans equation it is possible to derive **the system mass profile, M(r)**:

$$G\boxed{M(r)} = -r\sigma_r^2 \left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta \right)$$

Integrating the Jeans equation, and assuming the system is in steady state, we obtain the **scalar virial theorem:**

$$2\boxed{K} + \boxed{W} = 0$$

system's total kinetic energy

system's total potential energy

From the virial theorem it is possible to derive **the system total mass, M**:

$$G\boxed{M} = \frac{\sigma_{\mathrm{tot}}^2}{< b(r) r^{-1} >}$$

b(r) accounts for the possibility that the distribution of galaxies and the distribution of mass be different

Why do we care about cluster masses?


Fritz Zwicky

In 1933 Zwicky applied the virial theorem to the Coma cluster of galaxies, and found out that the combined mass of all galaxies, as inferred by their emitting light, cannot bind the cluster gravitationally. It is the discovery of

Dark Matter

De-projecting the equations

The Jeans equation and the virial theorem have been expressed in terms of the **6** full phase-space coordinates **w.**

However **we have access only to 3 coordinates**, two spatial coordinates + the velocity along the line-of-sight (from the redshift).

We need to de-project the two equations. For the **virial theorem:**

$$GM = \frac{\sigma^2_{\rm tot}}{< b(r) r^{-1} >} \Longrightarrow \boxed{GM = \frac{3\pi\sigma^2_{\rm los}}{< R_{ij}^{-1} >}}$$

los=
line-of-sight

i,i = galaxy id. number

If we observe the **entire** (spherically symmetric) system, so that:

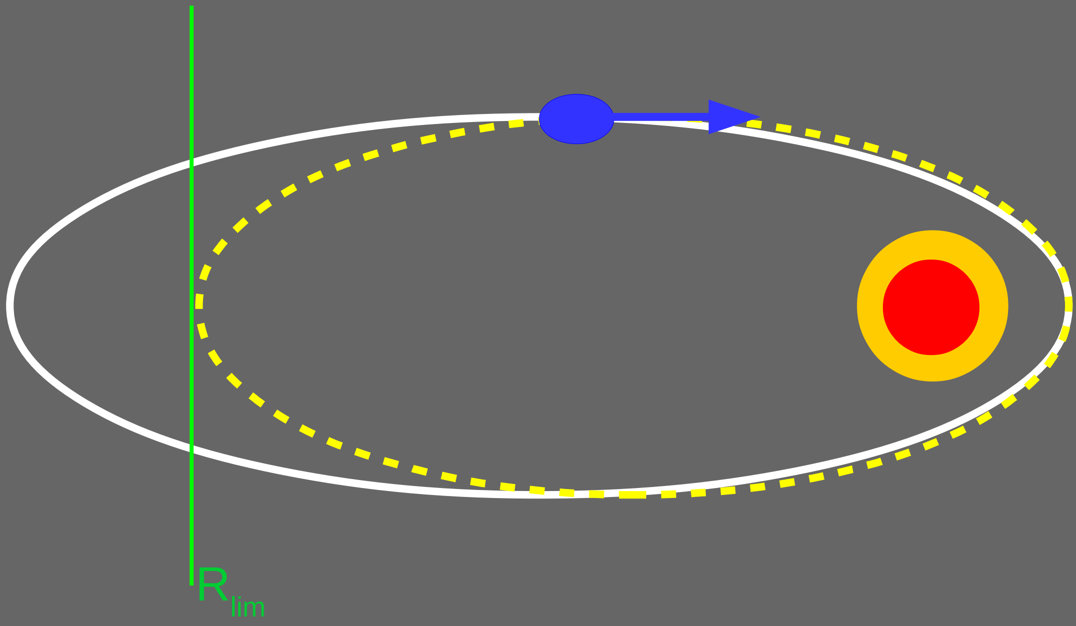
- $\sigma^2_{\rm tot} = 3\,\sigma^2_{\rm los}$ (since the velocity dispersion tensor has 3 components), independent of the shape of galaxy orbits (tangential vs. radial)
- $< b(r)\, r^{-1} > = < R_{ij}^{-1} > / \pi$,
  if we assume $b(r) \equiv 1$, the "light traces mass" hypothesis

De-projecting the equations

In case we do not observe the entire system, the projected virial theorem needs to be corrected for the **surface pressure** term *(see The & White 1986)*:

$$GM = \frac{3\pi\sigma_{\rm los}^2}{< R_{ij}^{-1} >} - \boxed{S[M(r), \beta(r), R_{\rm lim}]}$$

where S is a function of the limiting radius of observation $R_{\rm lim}$, of the galaxy orbital distribution within the cluster, β(r), but also (unfortunately) of the mass distribution itself, M(r)



Blue ellipse: orbiting galaxy

Solid line: true orbit;

Dashed line: inferred orbit due to observational limitation;

Red dot: true mass;

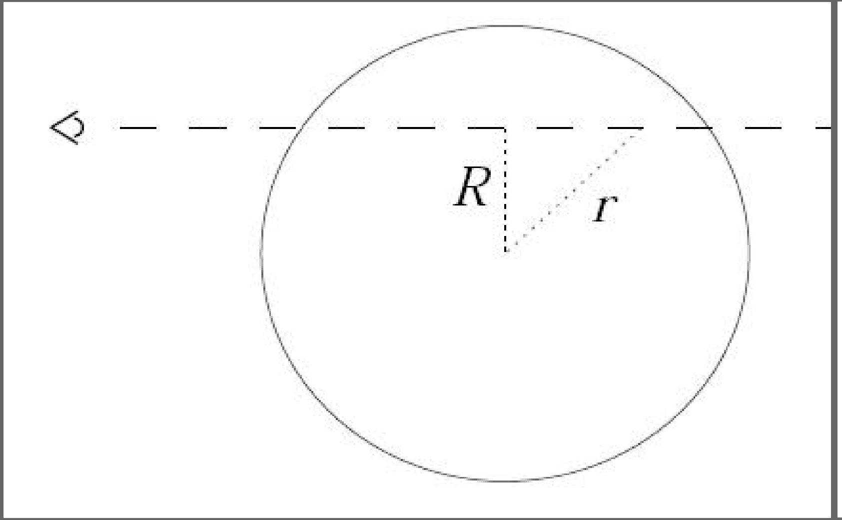
Orange dot: mass needed to keep the galaxy in the inferred orbital configuration

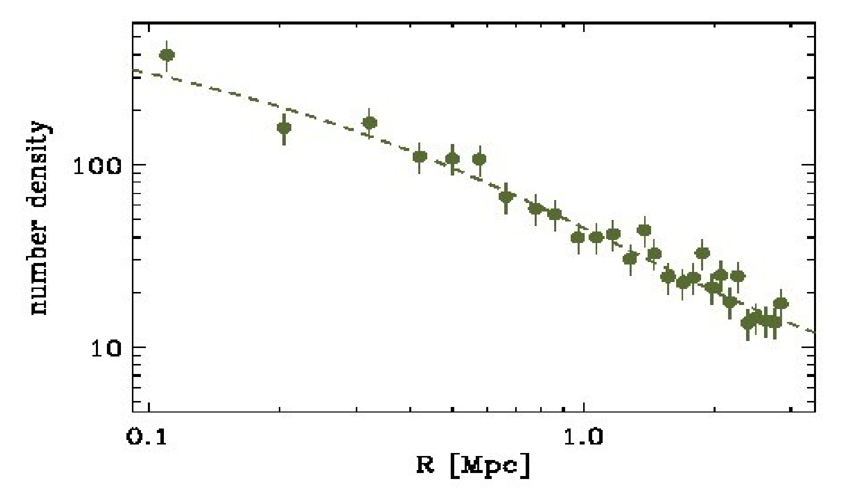
De-projecting the equations

For the **Jeans equation**:

$$GM(r) = -r\sigma_r^2 \left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta \right)$$

n(R): projected number density profile





$$\nu(r) = -\frac{1}{\pi}\int_r^{\infty} \frac{dn}{dR} \frac{dR}{\sqrt{R^2 - r^2}}$$

Abel inversion equation

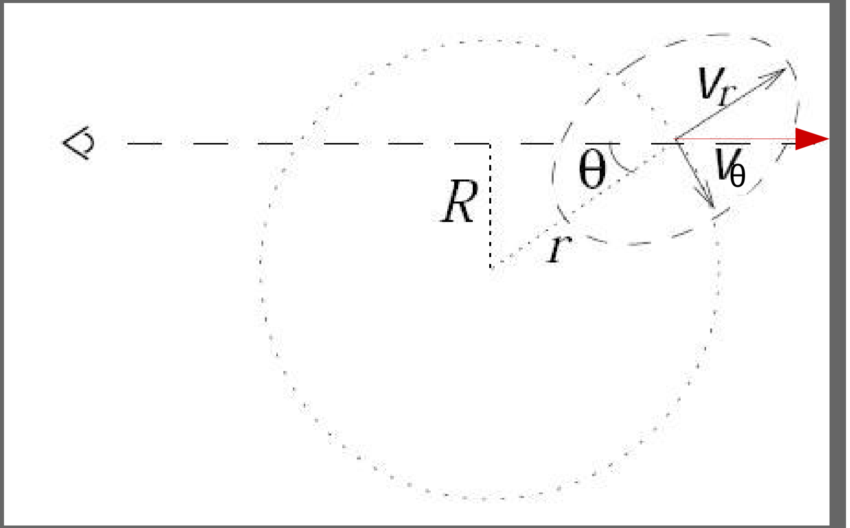
De-projecting the equations

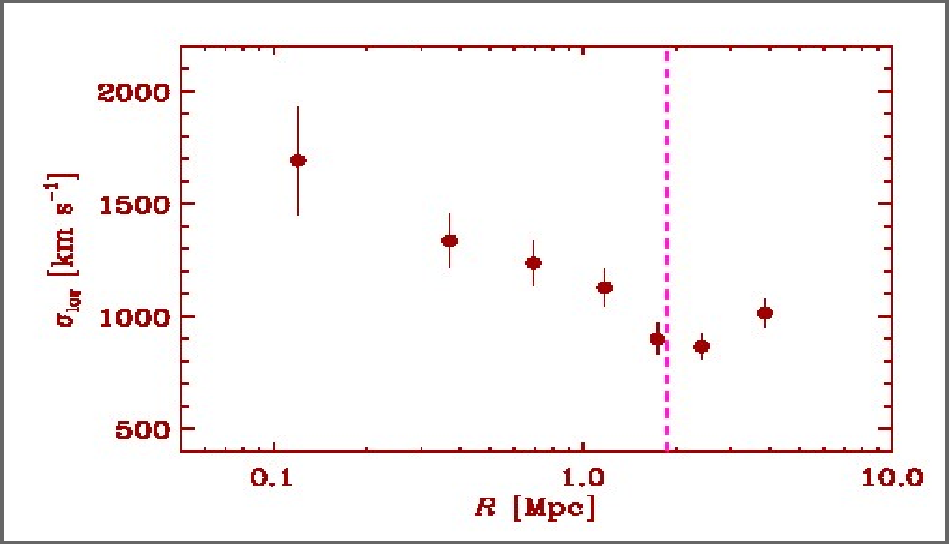
For the Jeans equation:

$$GM(r) = -r\sigma_r^2 \left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta \right)$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

$\sigma_{\rm los}(R)$: line-of-sight velocity dispersion profile





$$\sigma_r^2(r) = -\frac{1}{\pi\nu(r)} \int_r^\infty \frac{d(n\sigma_{\rm los}^2)}{dR} \frac{dR}{\sqrt{R^2 - r^2}}$$

Abel inversion equation, **valid ONLY for β(r)=0**

De-projecting the equations

We observe:
1) the projected number density profile n(R)
2) and the line-of-sight velocity dispersion profile $\sigma_{los}(R)$

but to know the mass profile M(r) we need to know:
1) the 3-d number density profile ν(r),
2) the radial component of the velocity dispersion tensor $\sigma_r(r)$,
3) and the velocity anisotropy profile β(r) – or, equivalently, $\sigma_\theta(r)$

This is also true for the total mass (from the virial theorem)
unless we assume to know the mass distribution

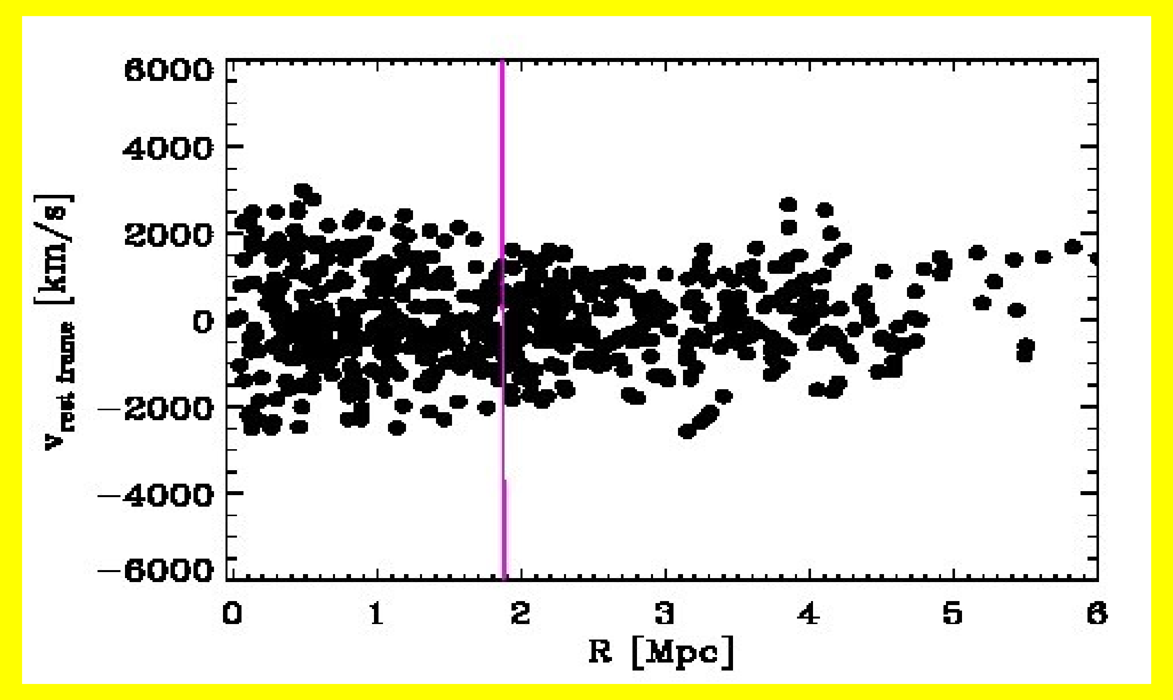
➔ Need to solve this **“mass-orbit degeneracy”**

Several possibilities:

- ➔ Trust cosmological numerical simulations and use their β(r)
- ➔ Solve multiple Jeans/Virial equations separately for ≠ tracers (e.g. ellipticals/spirals) – this works if they have ≠ β(r), since M(r) is unique
- ➔ Go beyond the Jeans equation, considering higher moments of the velocity distribution – several approaches are available; here we consider MAMPOSSt

MAMPOSSt

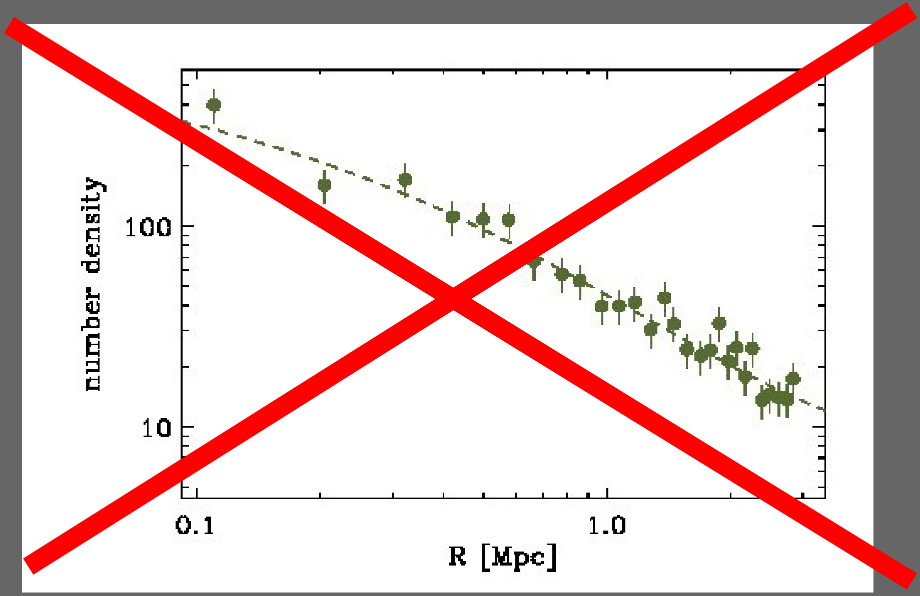
direct
maximum
likelihood
fit to the
phase-space
distribution
of cluster
galaxies
in projection



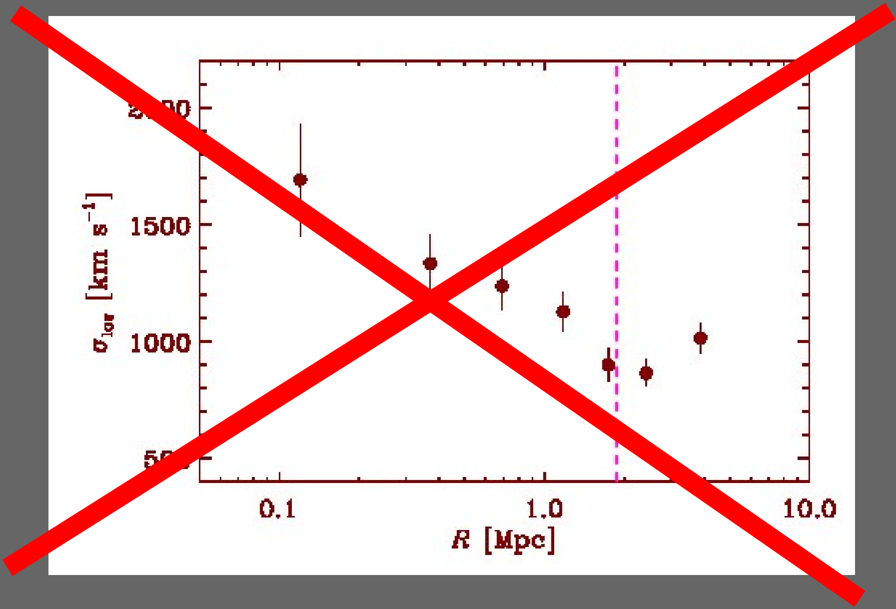
Modelling
Anisotropy and
Mass
Profiles of
Observed
Spherical
Systems

[Mamon, AB, Boué 2013]

projected number density profile n(R)

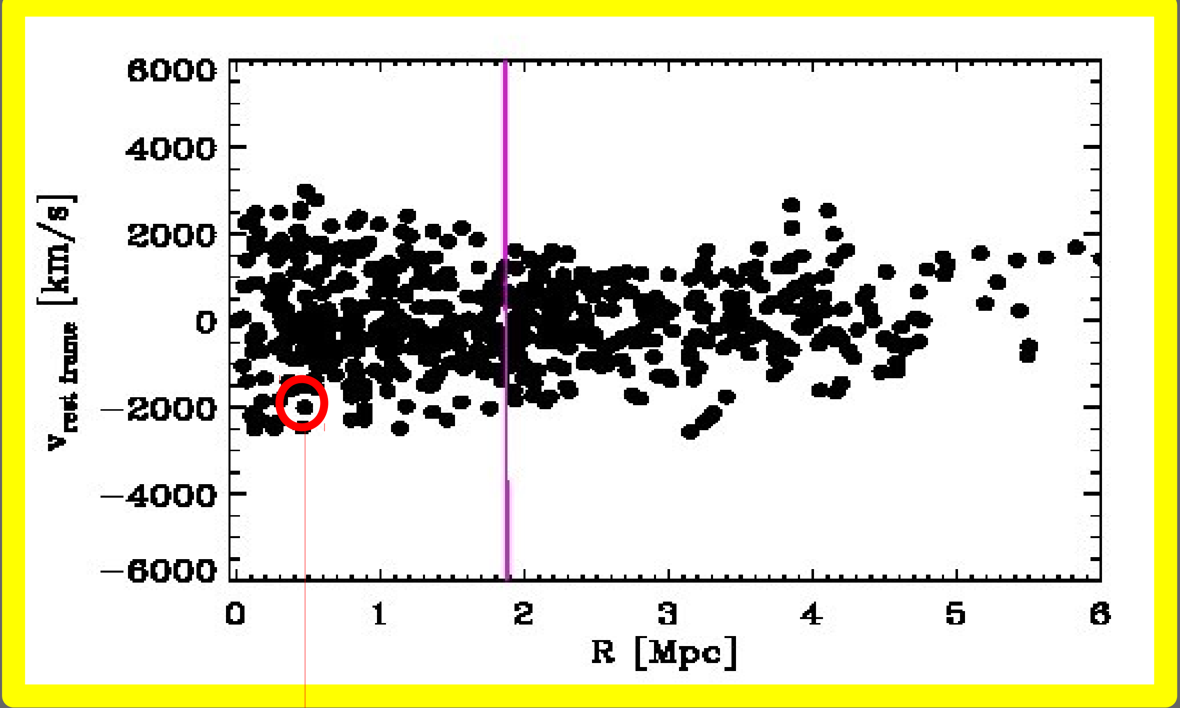


l.o.s. velocity dispersion profile σ_{los}(R)



MAMPOSSt

direct maximum likelihood fit to the phase-space distribution of cluster galaxies in projection



Modelling
Anisotropy and
Mass
Profiles of
Observed
Spherical
Systems

[Mamon, AB, Boué 2013]

Compute the probability p_i of observing a galaxy i at a projected radial distance R_i from the cluster center with a rest-frame line-of-sight velocity v_i, given parameterized models for:

- ➔ the 3D number density profile $\nu(r,\boldsymbol{\kappa})$
- ➔ the mass profile $M(r,\boldsymbol{\lambda})$
- ➔ the velocity anisotropy profile $\beta(r,\boldsymbol{\mu})$

Find the optimal (best-fit) parameters **κ, λ, μ** by maximizing: $\prod_{i=1}^{N} p_i$

The surface density of observed objects in projected phase space is:

MAMPOSSt:

direct maximum likelihood fit to the phase-space distribution of cluster galaxies in projection

$$\begin{aligned} g(R,v_z) &= \mathrm{n}(R)\ \langle h(v_z|R,r)\rangle_{\mathrm{LOS}} \\ &= 2\int_R^\infty \frac{r\,\nu(r)}{\sqrt{r^2-R^2}}\,h(v_z|R,r)\,\mathrm{d}r\,, \qquad (4) \\ &= 2\int_R^\infty \frac{r\,\mathrm{d}r}{\sqrt{r^2-R^2}}\int_{-\infty}^{+\infty}\mathrm{d}v_\perp\int_{-\infty}^{+\infty} f(r,v_z,v_\perp,v_\phi)\,\mathrm{d}v_\phi, \\ & \qquad (5) \end{aligned}$$

Hence, the probability density of observing an object at position (R,v_z) is:

$$\begin{aligned} q(R,v_z) &= \frac{2\pi\,R\,g(R,v_z)}{\Delta N_\mathrm{p}} \\ &= \frac{4\pi\,R}{\Delta N_\mathrm{p}}\int_R^\infty \frac{r\,\nu(r)}{\sqrt{r^2-R^2}}\,h(v_z|R,r)\,\mathrm{d}r \end{aligned}$$

Can be solved by assuming a distribution for 3D galaxy velocities (e.g. Gaussian):

$$h(v_z|R,r) = \frac{1}{\sqrt{2\pi\sigma_z^2(R,r)}}\exp\left[-\frac{v_z^2}{2\,\sigma_z^2(R,r)}\right]$$

$$\sigma_z^2(R,r) = \left[1-\beta(r)\left(\frac{R}{r}\right)^2\right]\sigma_r^2(r).$$

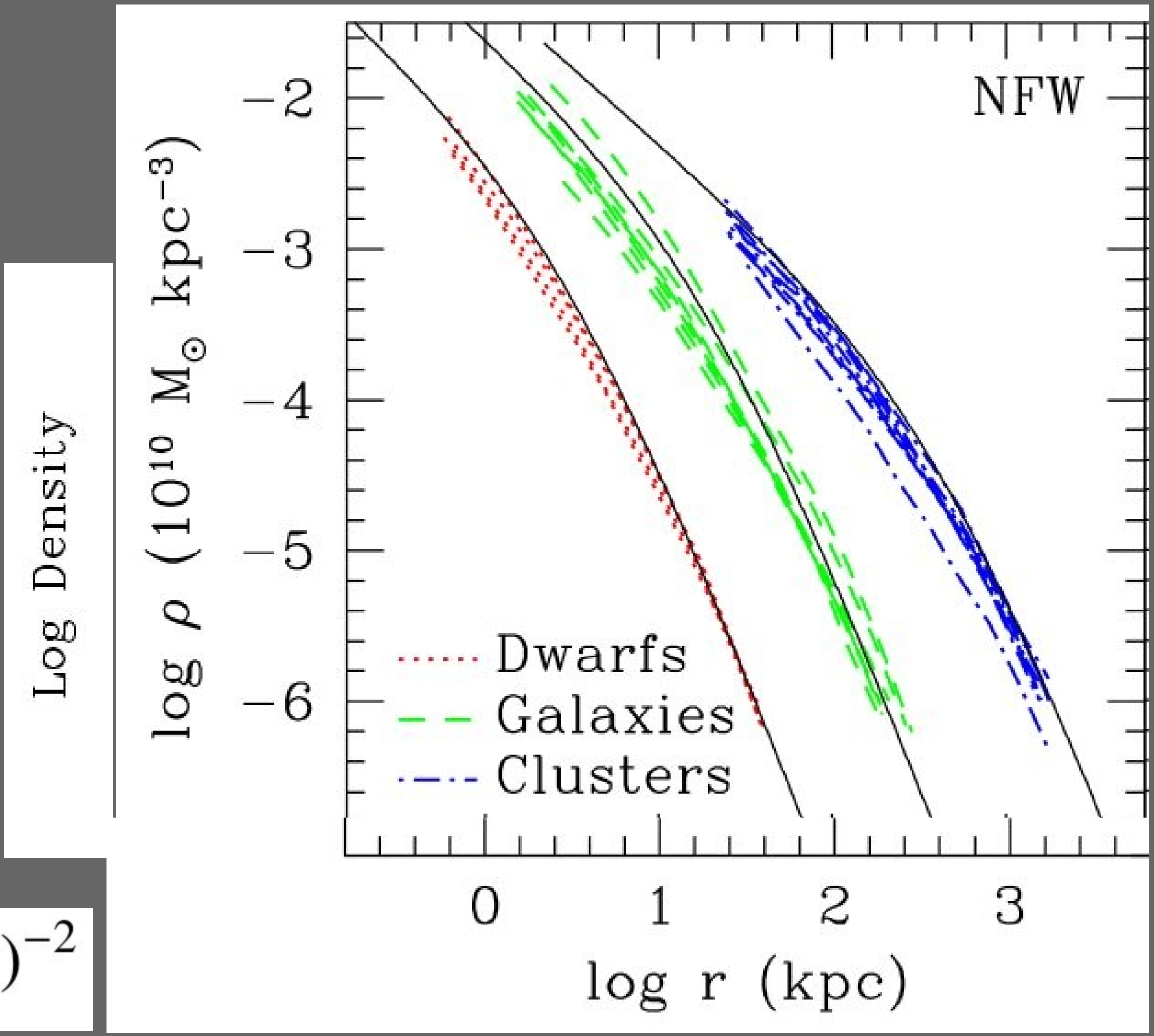
where $\sigma_r^2(r)$ is obtained from the Jeans equation, given M(r) and β(r)

$$\sigma_r^2(r) = \frac{1}{\nu(r)}\int_r^\infty \exp\left[2\int_r^s \boxed{\beta(t)}\frac{\mathrm{d}t}{t}\right]\nu(s)\frac{\boxed{GM(s)}}{s^2}\mathrm{d}s$$

Why do we care about cluster masses?

Navarro, Frenk & White 97:

the mass density profile, ρ(r), of all cosmological halos, is **universal**
- based on (collisionless) Cold DM numerical simulations



$$\rho(r) \propto (r/r_{-2})^{-1} \times (1 + r/r_{-2})^{-2}$$

Inner logarithmic slope γ = -1, asymptotic slope -3 with a change in slope at a characteristic radius r_{-2}

Clusters probe Dark Matter!

Since clusters are DM dominated, their mass distribution should be similar to the NFW shape found in collisionless Cold DM cosmological simulations

If cluster DM distribution deviates from NFW shape,
DM may not be Cold (Warm DM),
or it may be collisional (self-interacting DM)

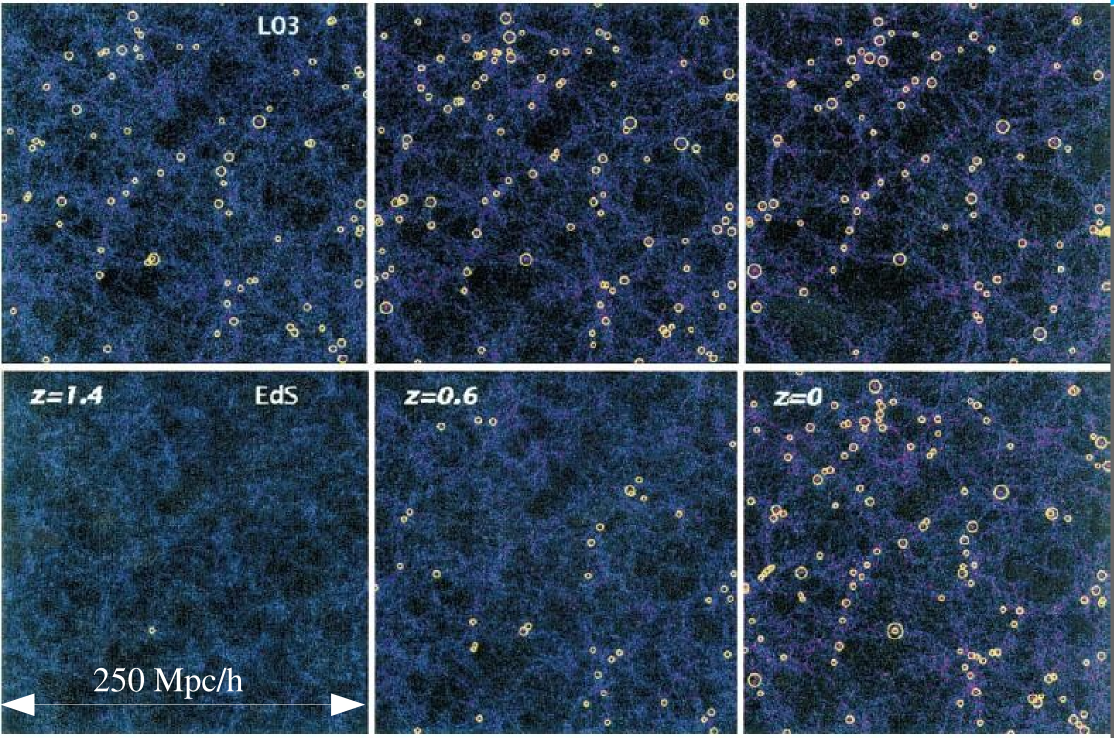
However, other cosmological simulations have shown that the DM distribution can deviate from the NFW shape because of physical processes:

- adiabatic contraction
- mass accretion
- dynamical friction
- AGN feedback

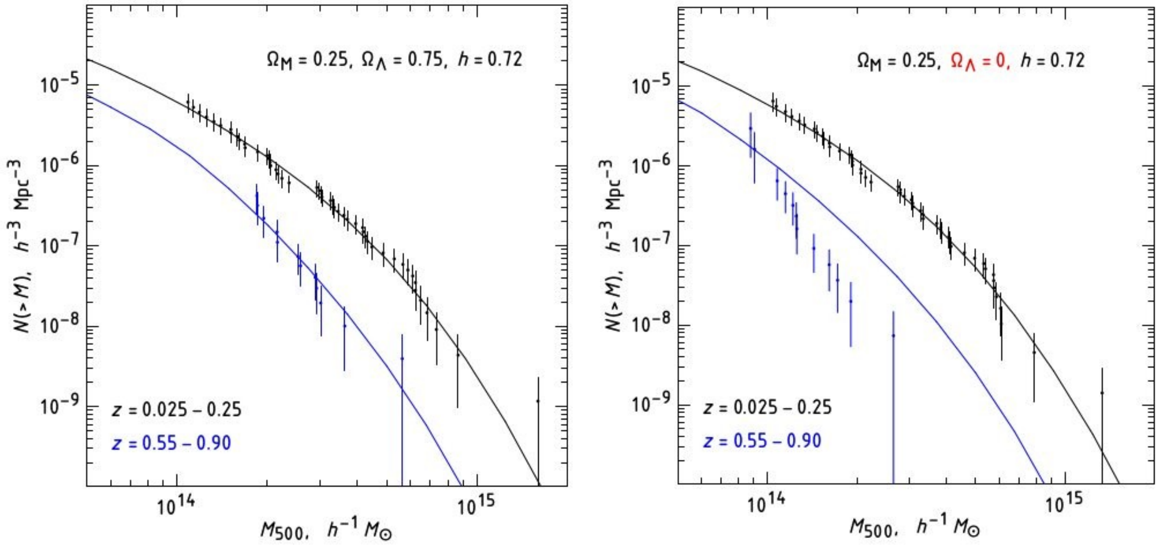
Since these processes have different impact at different cosmological times, while the DM characteristics are not expected to change with cosmological time, **it is important to study the DM distribution in clusters at different redshifts**

Clusters probe cosmology!

Borgani & Guzzo 2001: N-body simulation: DM particles (blue) and X-ray emitting clusters (yellow circles) in two cosmologies



Vikhlinin et al. 09: observed cluster mass function (points) cmpd to two cosmological model predictions (curves)

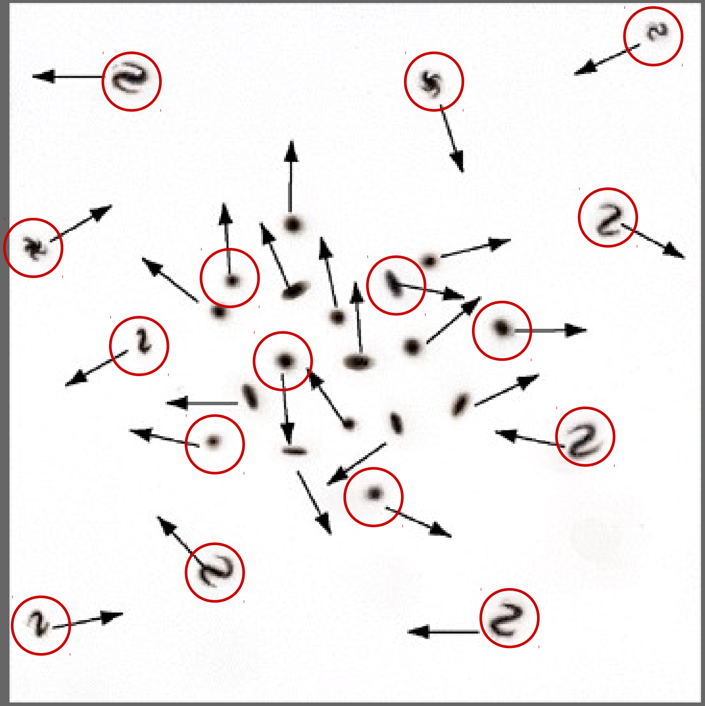


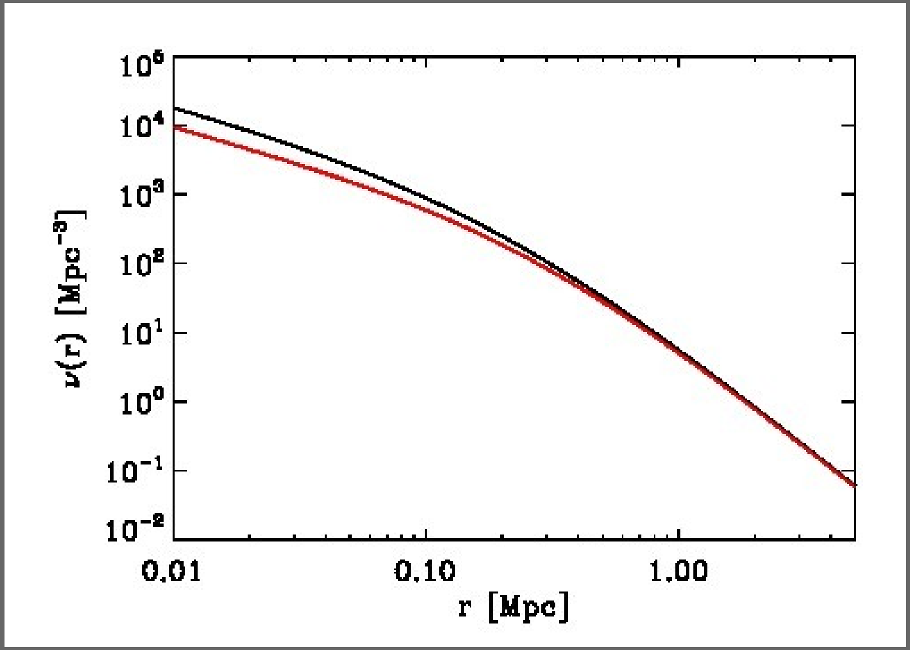
Clusters form late in cosmic history: the evolution of their number density above a given mass is a sensitive probe of cosmological models

Observational problems

I) Spatial incompleteness of the spectroscopic sample

Affects estimates of the spatial distribution of galaxies,
like the harmonic mean radius (virial theorem),
and the number density profile (Jeans equation)





Solutions:

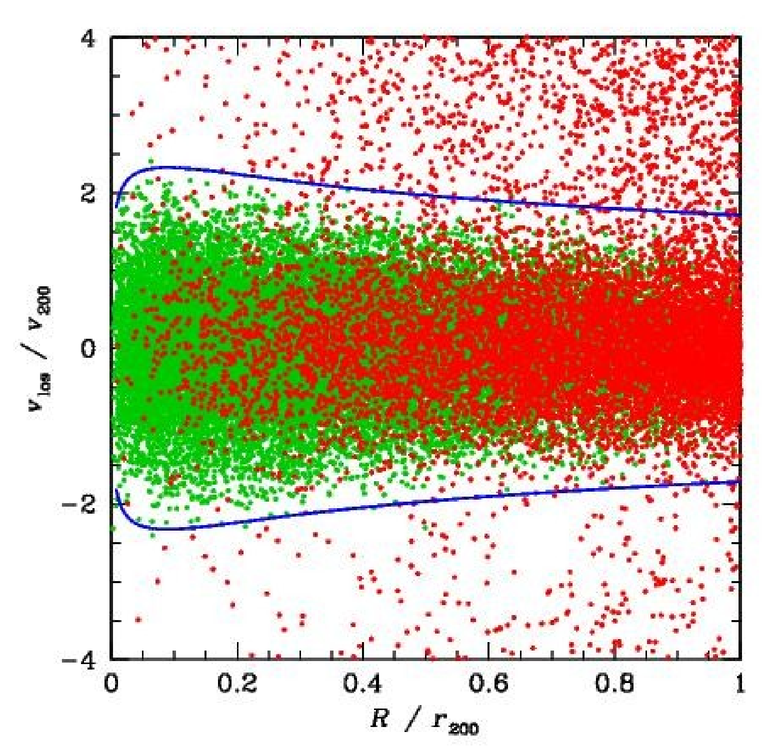
- ✓ Estimate the incompleteness and correct the spectroscopic sample
- ✓ Use a substitute sample that is complete (e.g. photometric sample)

Observational problems

II) Interlopers

≡ Galaxies erroneously identified as cluster members (in projection)

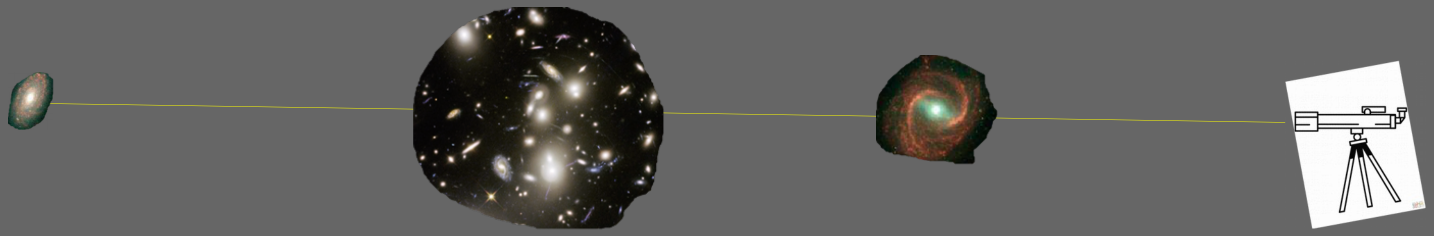
Affect estimates of the spatial and velocity distribution of galaxies



Projected phase-space distribution of a simulated cluster (Mamon, AB, Murante 10)

Green dots: galaxies in r_{200} sphere

Red dots: galaxies ouside r_{200} sphere

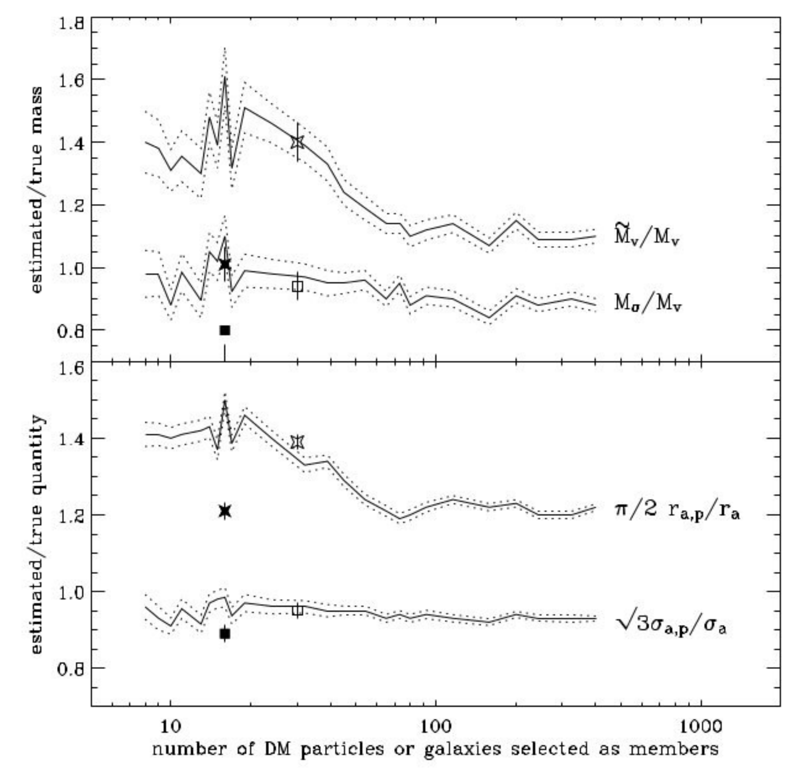


Observational problems

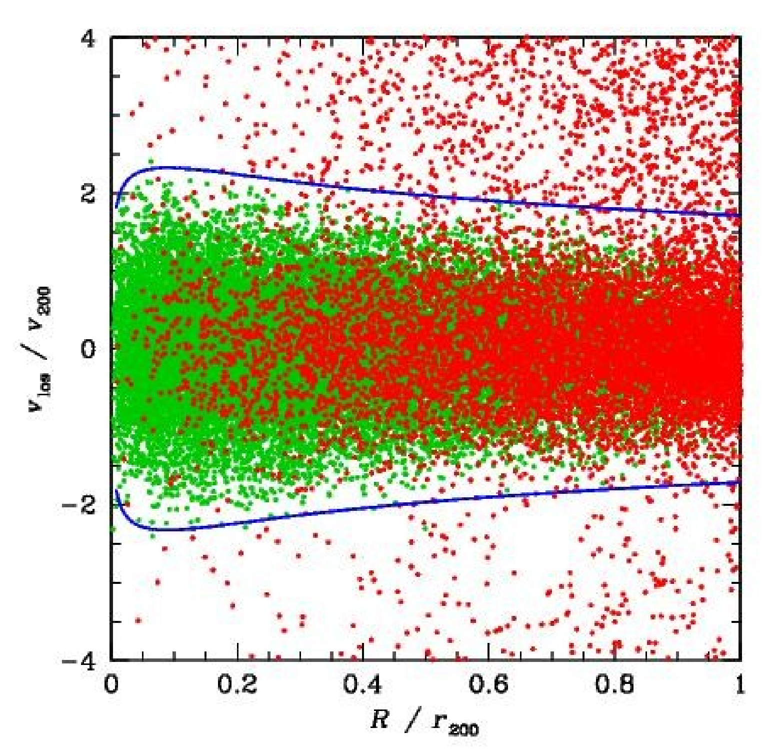
II) Interlopers

≡ Galaxies erroneously identified as cluster members (in projection)

Affect estimates of the spatial and velocity distribution of galaxies



AB et al. 06: ratio of observed to true quantities for simulated clusters



Projected phase-space distribution of a simulated cluster (Mamon, AB, Murante 10)

Green dots: galaxies in r_{200} sphere

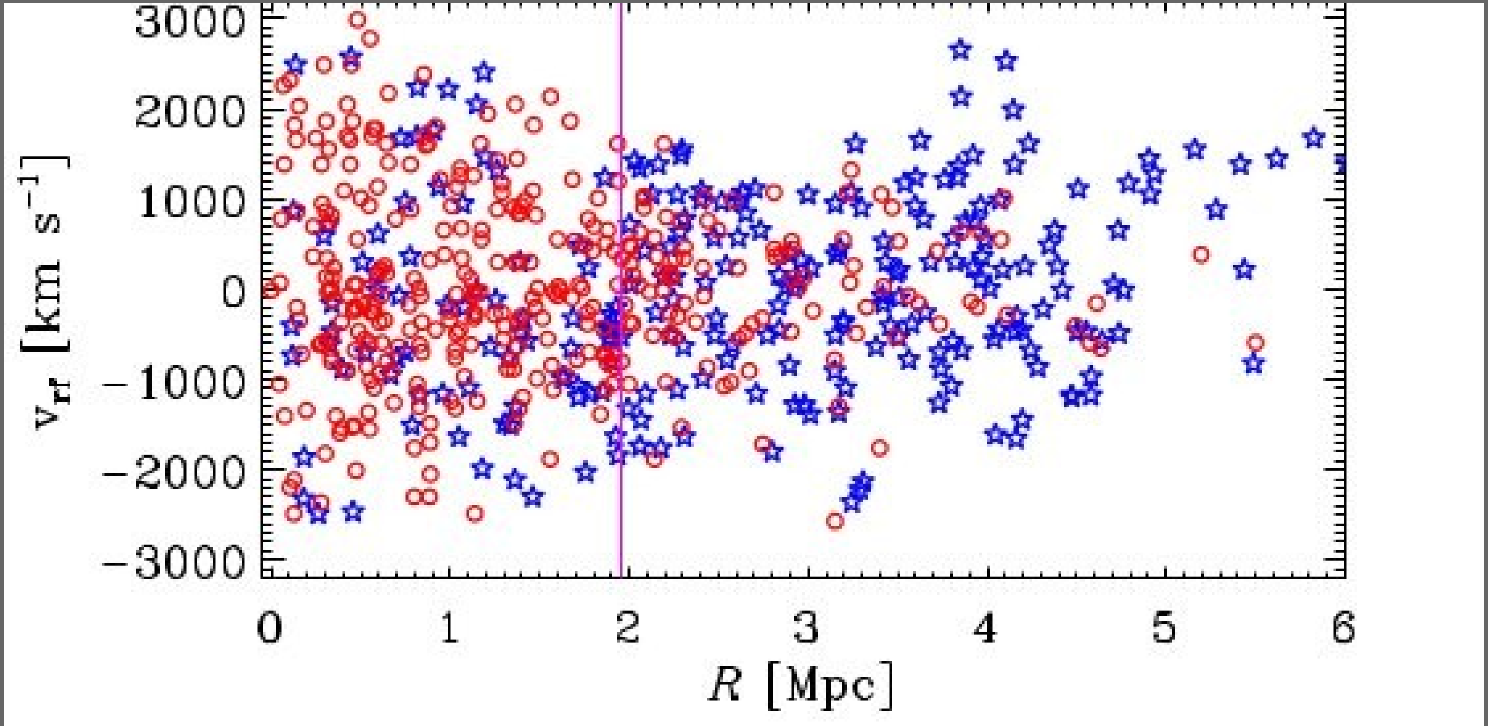
Red dots: galaxies ouside r_{200} sphere

Solutions:

- ✔ Identify cluster members by several comparative techniques
- ✔ Use colors to improve selection (red vs. blue galaxies)
- ✔ Statistical subtraction of interlopers from the sample of cluster members

Observational problems

II) Interlopers: how to remove them



Red circles: red cluster members (passive galaxies)
Blue stars: blue cluster members (star-forming galaxies)

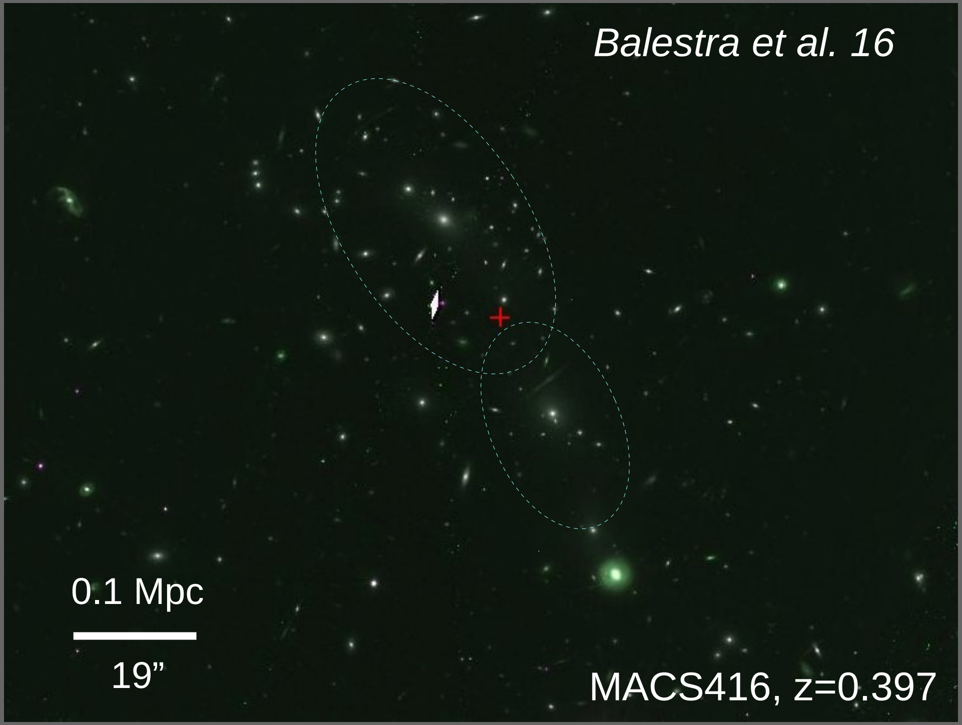
Since the cluster over-density with respect to the field is higher near the center, and red galaxies are more concentrated than blue galaxies, the interlopers are more likely to be blue than red galaxies

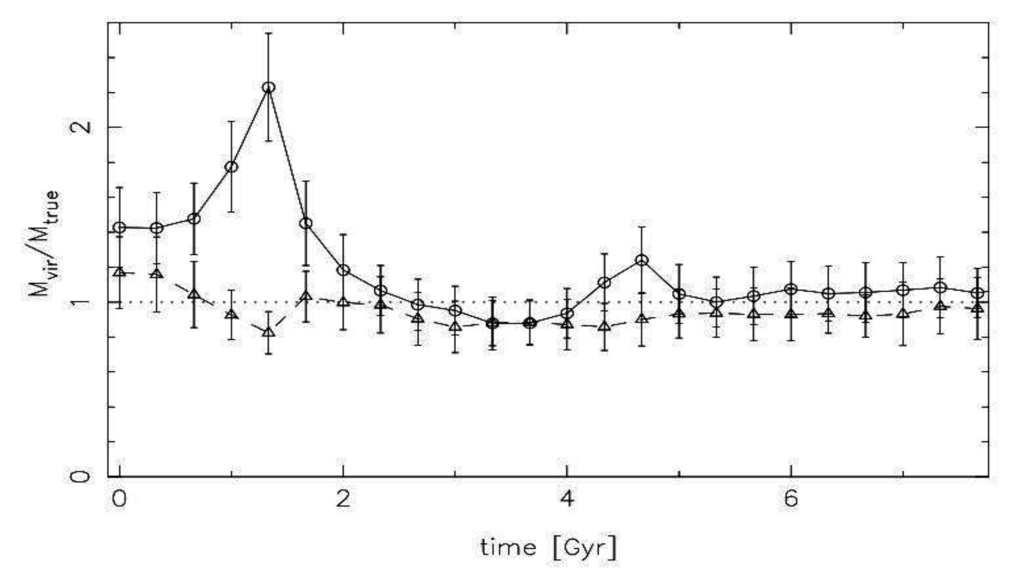
Observational problems

III) Deviation from dynamical equilibrium

Clusters are young cosmic objects, they are still forming by accretion of galaxies and smaller groups of galaxies from the surrounding field

Affect estimates of the spatial and velocity distribution of galaxies





Estimated/True mass of a simulated cluster vs. time during a merger, along two line-of-sight axes *(Takizawa et al. 10)*

Solutions:

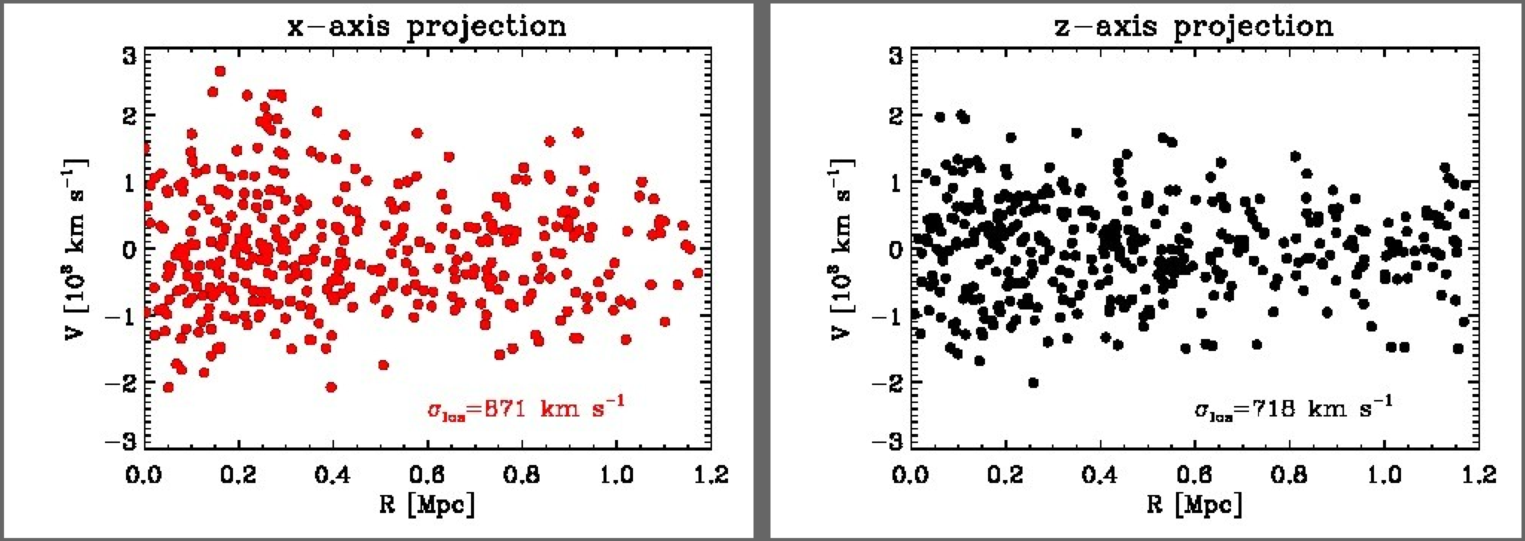
- ✔ Identify substructures (i.e. colliding groups) and remove them from the sample used for the dynamical estimate

Observational problems

IV) Deviation from spherical symmetry

Clusters are triaxial systems, their velocity distributions are wider along their major axes (inertia and velocity tensors are aligned)
[e.g. Kasun & Evrard 05; Wojtak et al. 13]

Affects estimates of the velocity distribution of galaxies,
like the line-of-sight velocity dispersion (virial theorem, Jeans equation)



Same simulated cluster as it would be observed in projected phase-space with the l.o.s. aligned with the minor (x) and major (z) axes.

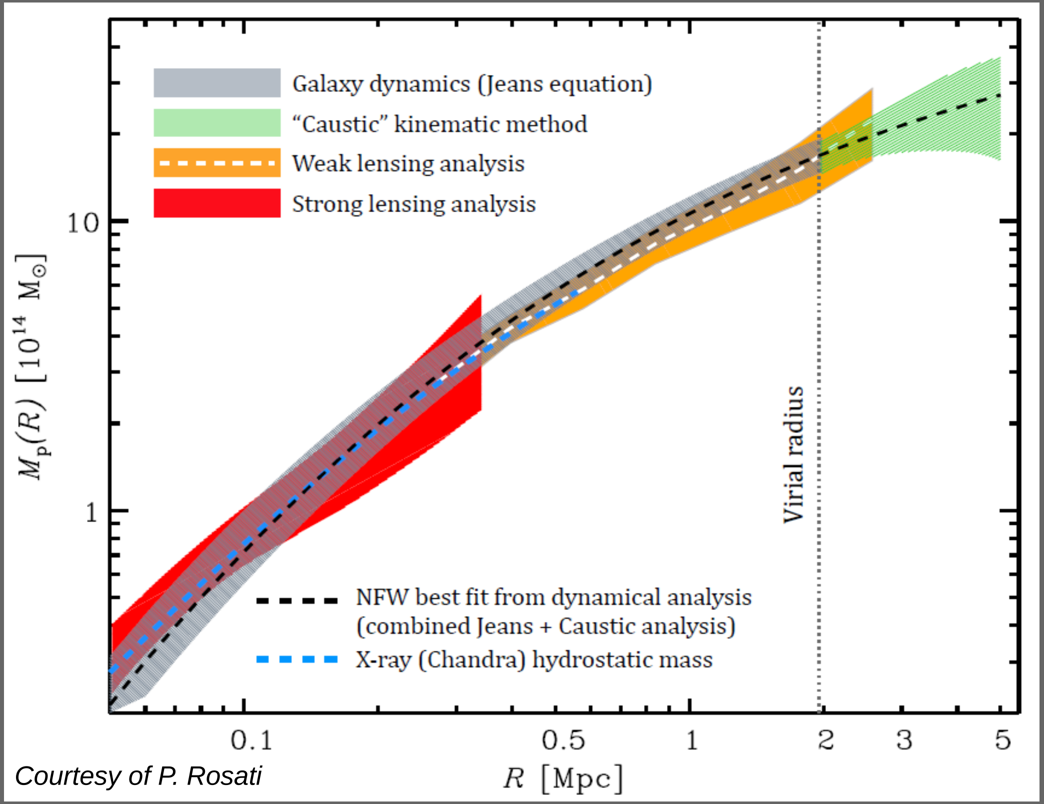
Solutions:

- ✔ Statistical correction of measured σ_{los} using observed distribution of galaxies
- ✔ Stack several clusters irrespective of the viewing angle, creating an effective spherical system

Observational problems

V) Poor statistics

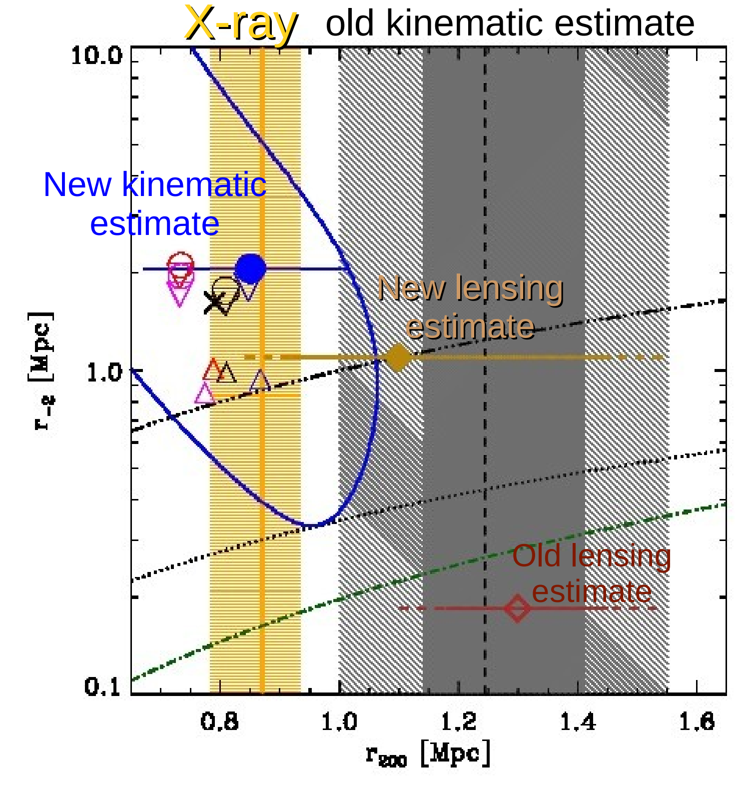
~500 members are needed for a precise sampling of the gravitational potential, comparable to precision attainable by weak lensing *(see, e.g., AB et al. 13)*



Courtesy of P. Rosati

Small-number statistics affects:

- identification of cluster members
- detection of substructures



z=0.17 cluster; from 20 to 200 members *(AB et al. 17)*

Solutions:

- ✓ Use robust statistical estimators (e.g. median instead of mean)
- ✓ Go back to the telescope!

Dynamical mass measurements of clusters of galaxies Summary

Observational quantities:

galaxy positions and redshifts (cluster-centric distances and rest-frame velocities)

Equations:

- ✔ Collision-less Boltzmann (Vlasov)
- ✔ Jeans equation M(r) ↔ $\nu(r)$, $\sigma_r(r)$, $\beta(r)$
  mass profile ↔ number-density, velocity-dispersion, velocity-anisotropy profiles
  $\beta(r) \equiv$ orbits of galaxies in the cluster
- ✔ Virial theorem (total mass, but not mass profile)

Deprojection:

mass-orbit degeneracy; can be solved using higher moments of velocity distribution (i.e. not only the dispersion), e.g. by MAMPOSSt

MAMPOSSt takes **models** for M(r), $\nu(r)$, and $\beta(r)$

Observational issues:

- ✔ spatial incompleteness of the sample (can bias the observed spatial distribution of galaxies)
- ✔ interlopers (must correctly identify cluster members; red are better tracers than blue)
- ✔ deviation from dynamical equilibrium (must find and – if possible - remove colliding groups)
- ✔ deviation from spherical symmetry (stack many clusters to randomize orientation)
- ✔ poor statistics (go to the telescope)