TENSORI

g_{ij}, du^i, du^j

$u^i \ (i = 1, 2, \dots) \rightarrow u'^j \ (j = 1, 2 \dots N)$

$$\boxed{du'^j = \sum_i \frac{\partial u'^j}{\partial u^i} du^i} \quad (i, j = 1, 2, \dots N)$$

$$\boxed{V^j} \qquad \boxed{V'^j = \sum_i \frac{\partial u'^j}{\partial u^i} V^i}$$

$\Downarrow$

TENSORI CONTROVARIANTE

ANCHE du^i e $\dot{u}^i$ SONO CONTROVARIANTI

VETTORI SONO TENSORI DI ORDINE 1

$$\phi(u^i) \Rightarrow \phi(u'^j)$$

$$\frac{\partial \phi}{\partial u'^j} = \sum_i \frac{\partial \phi}{\partial u^i} \cdot \frac{\partial u^i}{\partial u'^j} = \sum_i \frac{\partial u^i}{\partial u'^j} \frac{\partial \phi}{\partial u^i}$$

$$\frac{\partial \phi}{\partial u^i} = \text{VETTORE COVARIANTE}$$

$$\Rightarrow \partial_i \phi = \frac{\partial \phi}{\partial u^i}$$

$$V'^j = \frac{\partial u'^j}{\partial u^i} V^i \quad e \quad \frac{\partial \phi}{\partial u'^j} = \frac{\partial u^i}{\partial u'^j} \frac{\partial \phi}{\partial u^i}$$

$$ds^2 = g_{ij} \, du^i du^j$$

$$ds^2 = g'_{kl} \, du'^k du'^l = g_{ij} \, du^i du^j = g_{ij} \frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} \cdot du'^k du'^l$$

$$g'_{kl} = \frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij}$$

g_{ij} = TENSORE METRICO = TENSORE COVARIANTE DI RANGO 2

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g^{ij} = TENSORE CONTROVARIANTE DI RANGO 2

C^j $\quad \boxed{D_i = g_{ij} C^j} \left[\;\cdot\; / D^i\right] \Rightarrow \boxed{D_i D^i} = g_{ij} C^j D^i$

SCALARE $\quad \boxed{\bar{C}, \bar{D}}$

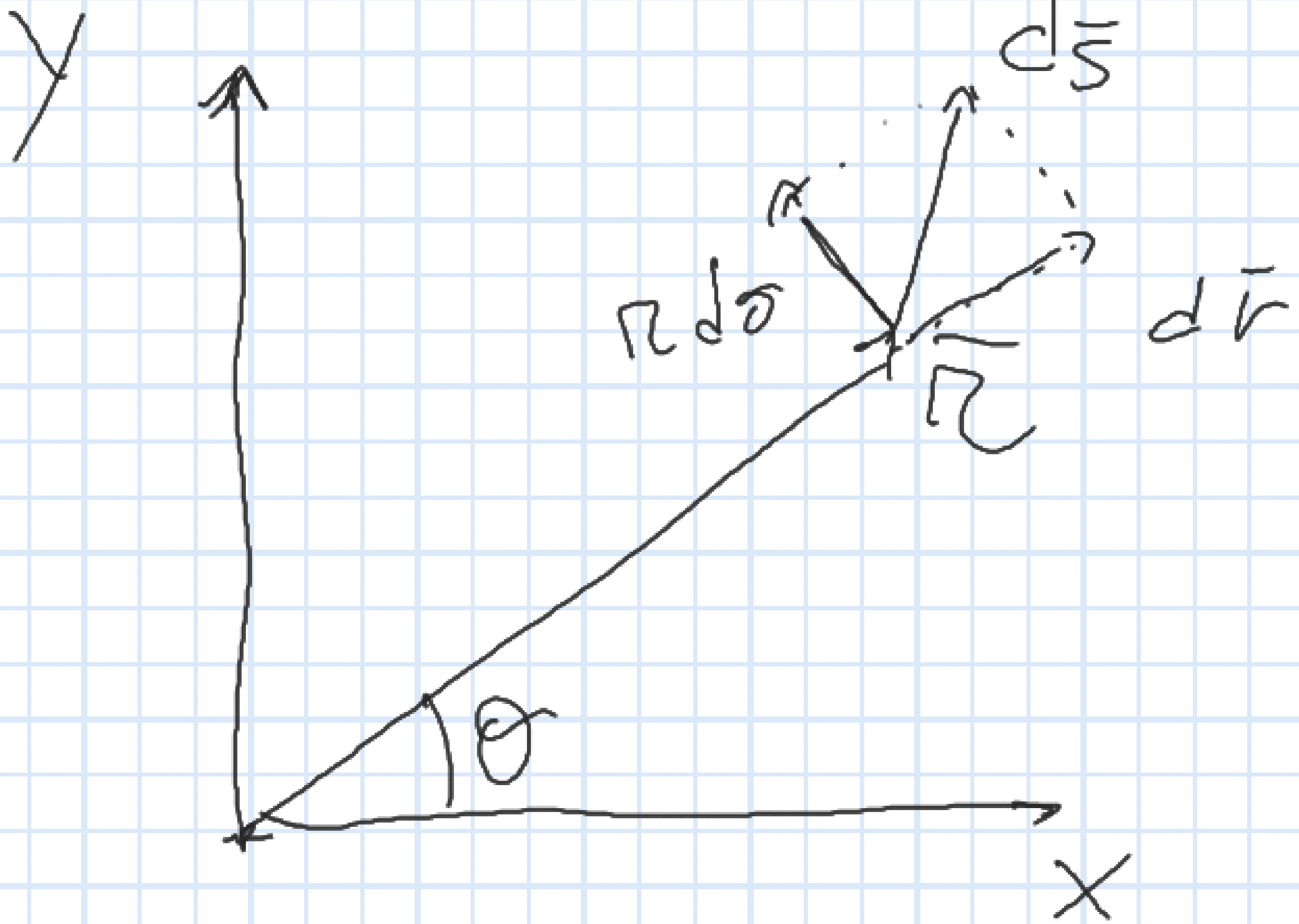
$\bar{C} \equiv \bar{D}$

$D_i = g_{ij} D^j \qquad D^i = g^{ij} D_j$

CONTROVARIANTE

COVARIANTE

IL TENSORE METRICO g_{ij} SERVE A TRASFORMARE LE COMPONENTI CONTROVARIANTI IN COVARIANTI (E VICEVERSA



$$(x, y) = (r\cos\theta, r\sin\theta)$$

$$u^1 = r \qquad u^2 = \theta$$

$$\begin{pmatrix} g_{12} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$$

dS

$$g_{11} = \bar{x}_1 \cdot \bar{x}_1 \qquad g_{22} = \bar{x}_2 \cdot \bar{x}_2$$

$$g_{12} = g_{21} = \bar{x}_1 \cdot \bar{x}_2$$

$$\bar{x}_1 = (\cos\theta, \sin\theta) \qquad \bar{x}_2 = (r\sin\theta, -r\cos\theta)$$

$$g_{11} = \cos^2\theta + \sin^2\theta = 1$$

$$g_{22} = r^2\sin^2\theta + r^2\cos^2\theta = r^2$$

$$g_{12} = r\sin\cancel{\theta}\cos\theta - r\sin\cancel{\theta}\cos\theta = 0$$

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \qquad g^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1/r^2 \end{pmatrix}$$

$$g = r^2 \Rightarrow \sqrt{g} = r \Rightarrow \boxed{dS^2 = \sqrt{g}\, du^1 du^2 = r\, dr\, d\theta}$$

$$A_i = (5, \; 9) \qquad B^i = (3, 7) \qquad g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

$$g^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{r^2} \end{pmatrix}$$

$$A_i B^i = A_1 B^1 + A_2 B^2 = 5 \cdot 3 + 7 \cdot 9 = 78$$

$$A^i = g_{ij} A_j \Rightarrow A^1 = g^{11} A_1 + g^{12} A_2 = 1 \cdot 5 + 0 \cdot 9 = 5$$

$$A^2 = g^{21} \cdot A_1 + g^{22} A_2 = 0 \cdot 5 + \frac{9}{r^2} = 9/r^2$$

$$B_i = g_{ij} B^i \to B_1 = g_{11} B^1 + g_{12} B^2 = 3 + 0 = 3$$

$$B_2 = g_{21} B^1 + g_{22} B^2 = 7 r^2$$

$$\boxed{A^i B_i} = A^1 B_1 + A^2 B_2 = 5 \cdot 3 + \frac{9}{\cancel{k^2}} \cdot 7 \cancel{k^2} = 78 = A_i B^i$$

$$A^i B_i = g_{ij} A^i B^j = g^{ij} A_j B_i = A_i B^i = \text{INVARIANTE}$$

$$\bar{v} = v^i \bar{x}_i \qquad v_k = \bar{v} \cdot \bar{x}_k = v^i \bar{x}_i \cdot \bar{x}_k =$$

$$= v^i g_{ij} = g_{ik} v^i = g_{ki} v^i$$