$$du'^j = \sum_i \frac{\partial u'^j}{\partial u^i} \cdot du^i \quad \Rightarrow \quad \frac{\partial u'^j}{\partial u^i} du^i = du'^j$$

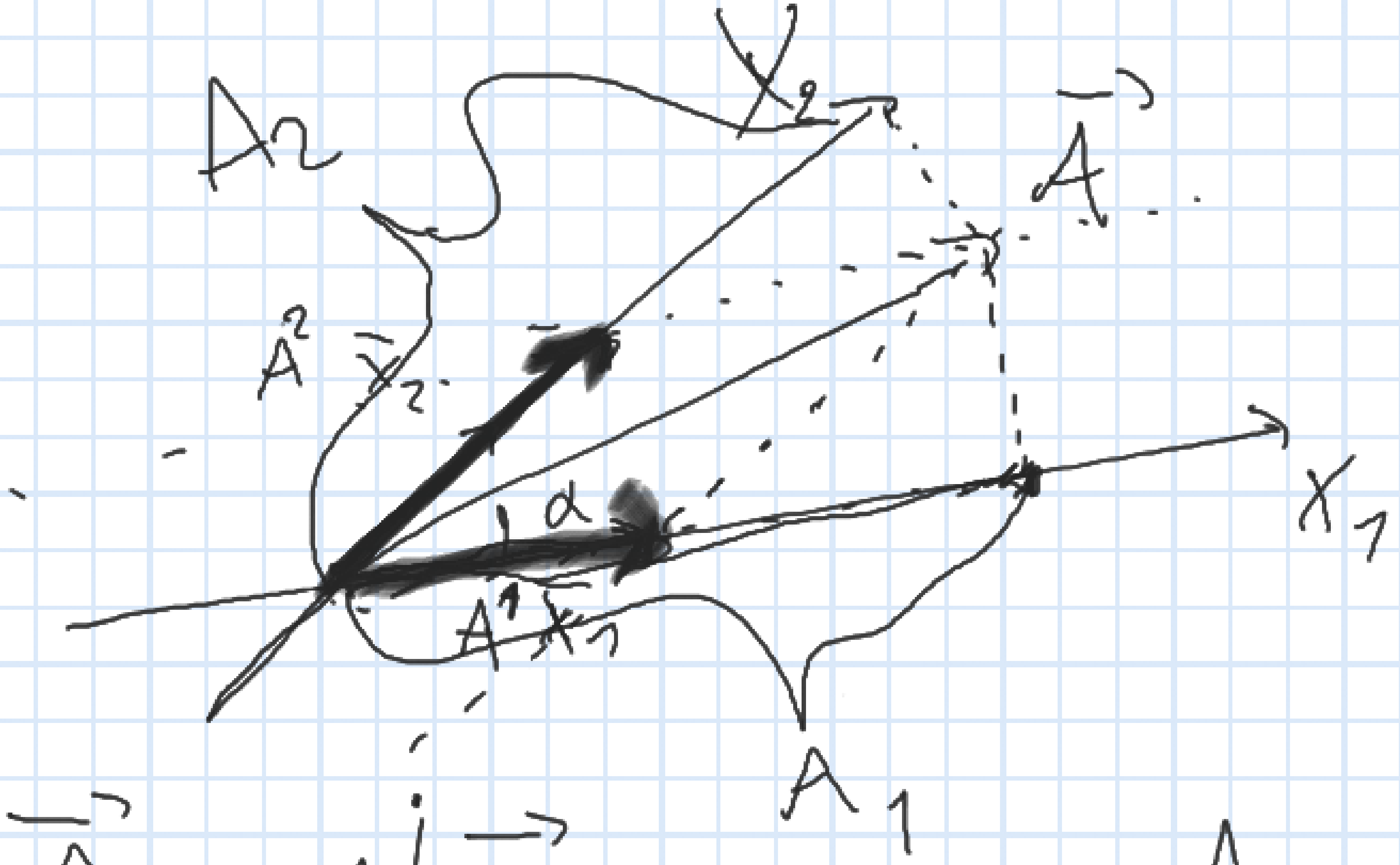
$$V^i(u) \longrightarrow V'^j(u')$$

$$\boxed{V'^j = \frac{\partial u'^j}{\partial u^i} V^i} = \text{CONTRO VARIANTI}$$

$$\phi \Rightarrow \frac{\partial \phi}{\partial u^i} \quad \Rightarrow \quad \frac{\partial \phi'}{\partial u'^j} = \sum_i \frac{\partial \phi}{\partial u^i} \cdot \frac{\partial u^i}{\partial u'^j} = \partial \phi_j' \frac{\partial u^i}{\partial u'^j}$$

$$w_j' = w_i \, \partial u^i / \partial u'^j \Rightarrow \boxed{\text{COVARIANTI}}$$

$$V^j = g^{ij} V_i \qquad V_j = g_{ij} V^j$$



$x_1 X_2$

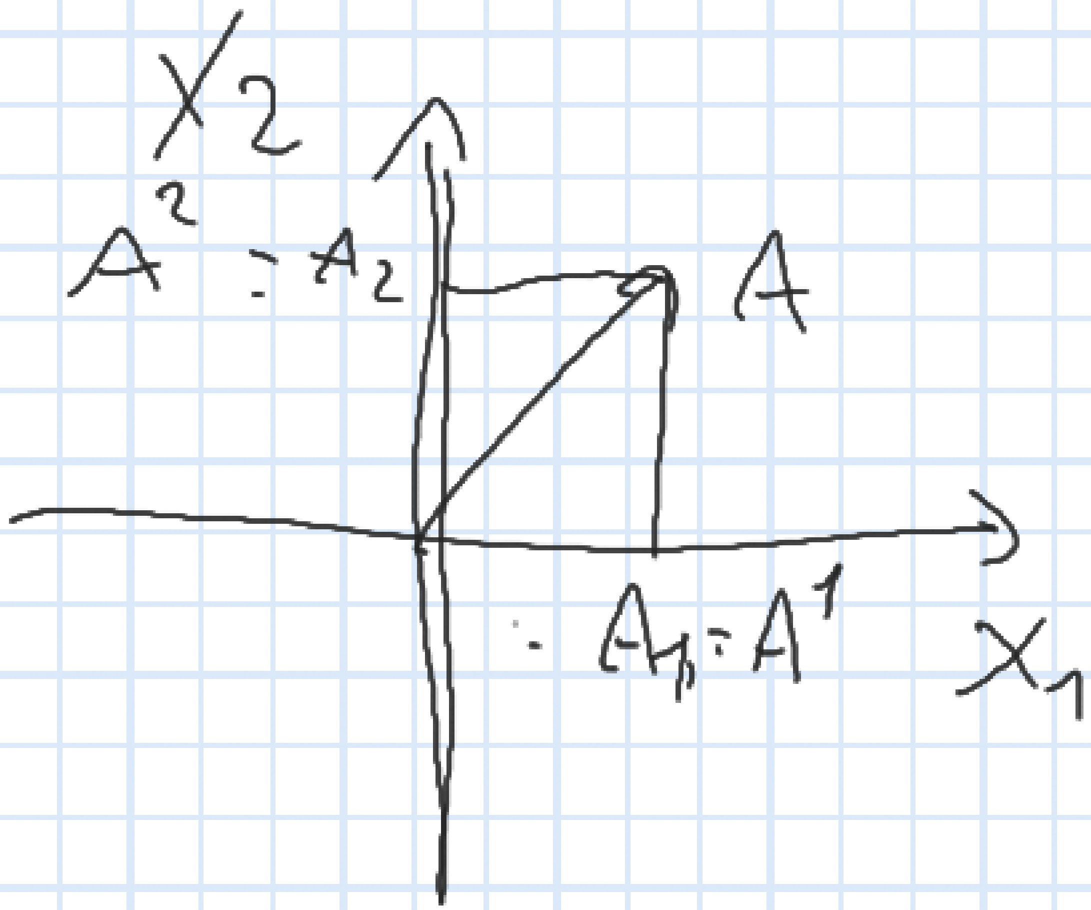
$\bar{x}_1$

$$\vec{A} = A^i \vec{x}_i \qquad A_i = \vec{A} \cdot \bar{x}_i$$

$$A_i = g_{ij} A^j$$

A^i = SONO LE PROIEZIONI PARALLELE A $\bar{x}_i$

A_j = " " " PERPENDICOLARI



$\Gamma^i_{jk} \neq$ TENSORE

$$W^{P}{}_{Q} \to W'^{m}{}_{n} = \frac{\partial U'^{m}}{\partial U^{P}} \frac{\partial U^{Q}}{\partial U'^{n}} W^{P}{}_{Q}$$

$$= \sum_{P} \sum_{Q} \frac{\partial U'^{m}}{\partial U^{P}} \frac{\partial U^{Q}}{\partial U^{n}} W^{P}{}_{Q}$$

$$W'_{mn} = \frac{\partial U^{Q}}{\partial U'^{m}} \frac{\partial U^{P}}{\partial U'^{n}} W_{PQ}$$

$$V^{i\cdots}{}_{\cdots j} + W^{i\cdots}{}_{\cdots j} = T^{i\cdots}{}_{\cdots j}$$

$$V^i (W_j) = T^i{}_j \qquad V^i W_j = T^{ij}$$

$$T^{ij}$$

$$V^m W^n = T^{mn}$$

$$! \; W^n V^m \neq V^n W^m$$

$$T^{mn} \neq T^{nm}$$

$$S \cdot V^{m} = T^{m}$$

$$V'^{m} W'_{m} = \sum_{m} V'^{m} \cdot W'_{m} = \text{SCALARE}$$

$$= \sum \frac{\partial u'^{m}}{\partial u^{a}} \cdot \frac{\partial u^{a}}{\partial u'^{m}} \cdot V^{a} W_{a}$$

$$= V^{a} W_{a}$$

OGNI EQ. SARÀ INVARIANTE RISPETTO A UNA TRASFORMAZIONE DI COORDINATE SE $A^{\alpha}_{\beta\gamma} = B^{\alpha}_{\beta\gamma}$

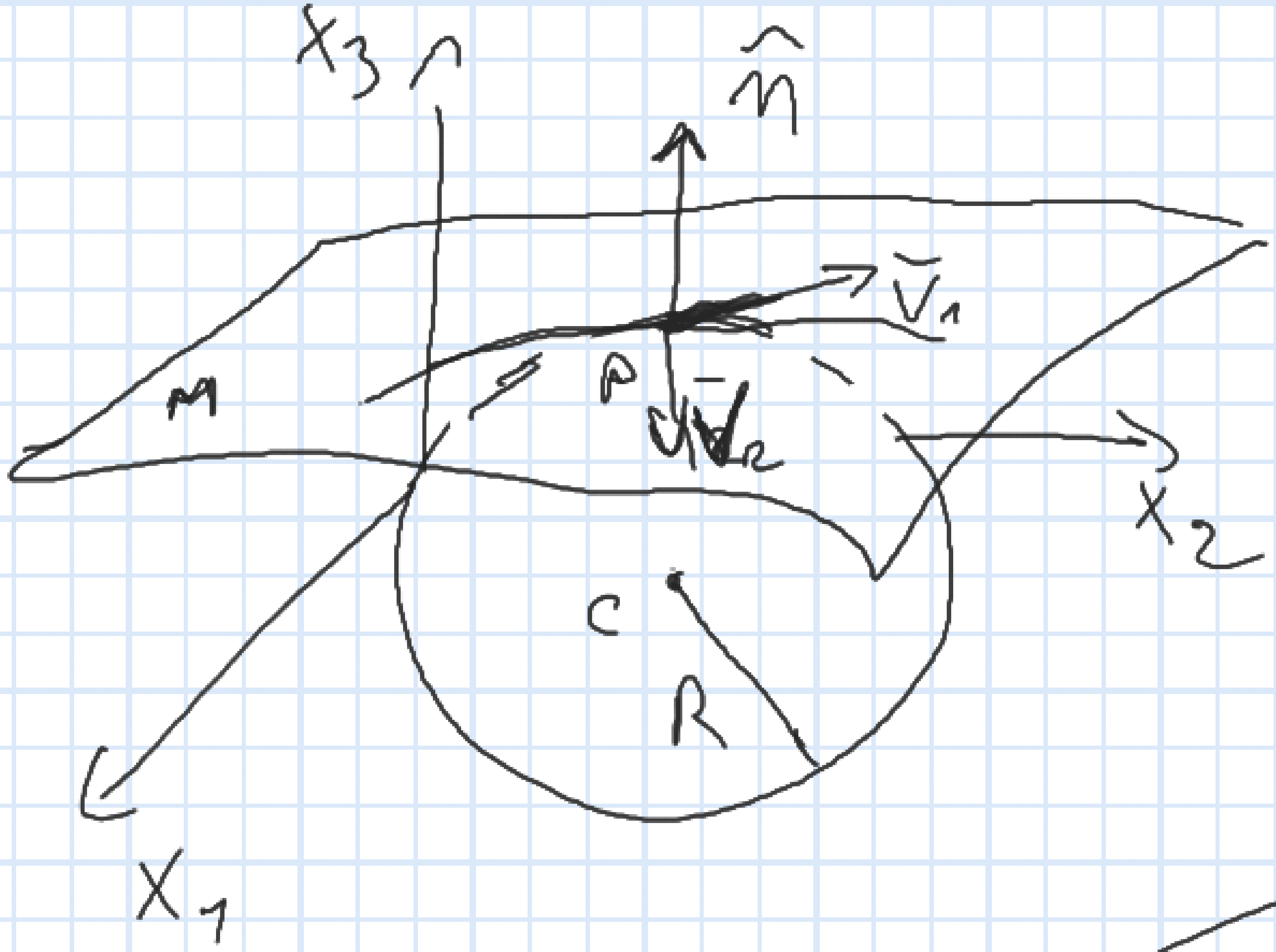
$$A^{\alpha}_{\beta\gamma} = B^{\alpha}_{\beta\gamma} \rightarrow A'^{\alpha}_{\beta\gamma} = B'^{\alpha}_{\beta\gamma} \Rightarrow \boxed{A'^{\alpha}_{\beta\gamma} - B'^{\alpha}_{\beta\gamma}} = 0$$

$$T^{\alpha}_{\beta\gamma}$$

$$\boxed{T_{\mu\nu} = 5 \qquad V^{i} = B_{i}}$$

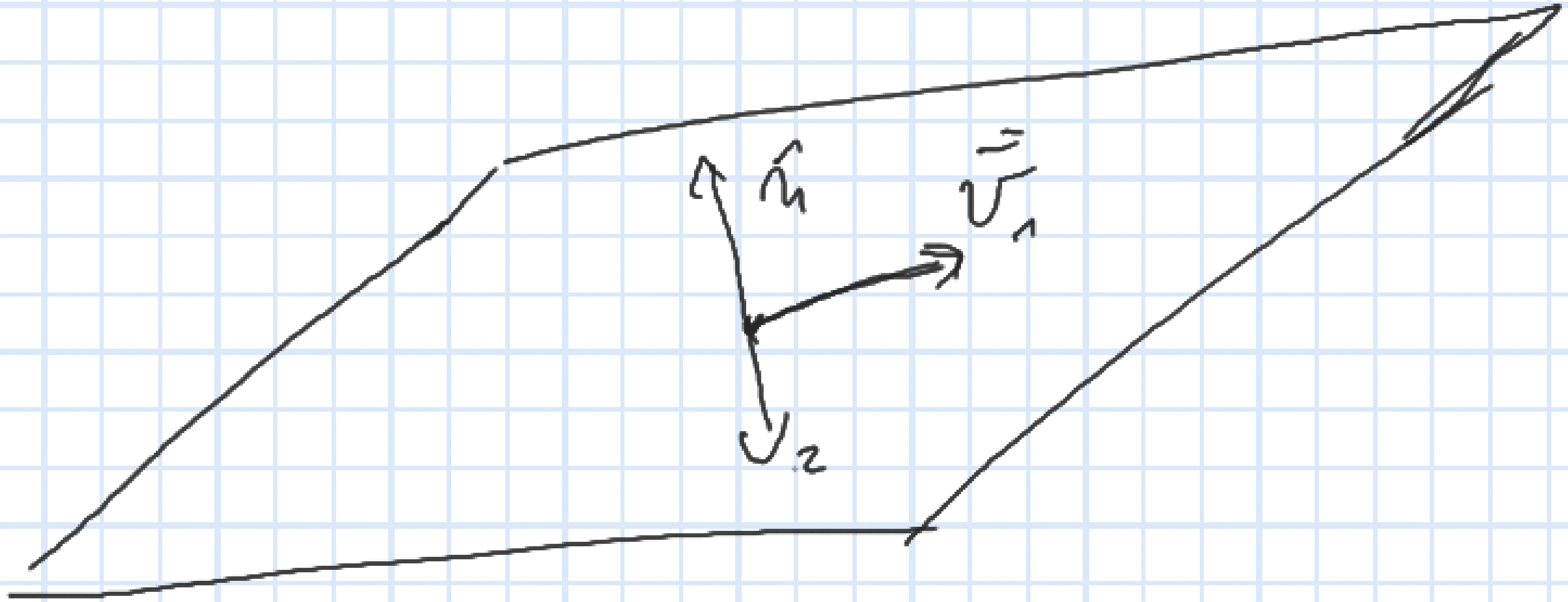
CURVATURA DI SUPERFICIE

E^3



$$P \Rightarrow \left(K_{1-2} = \pm \frac{1}{R_{1-2}} \right.$$

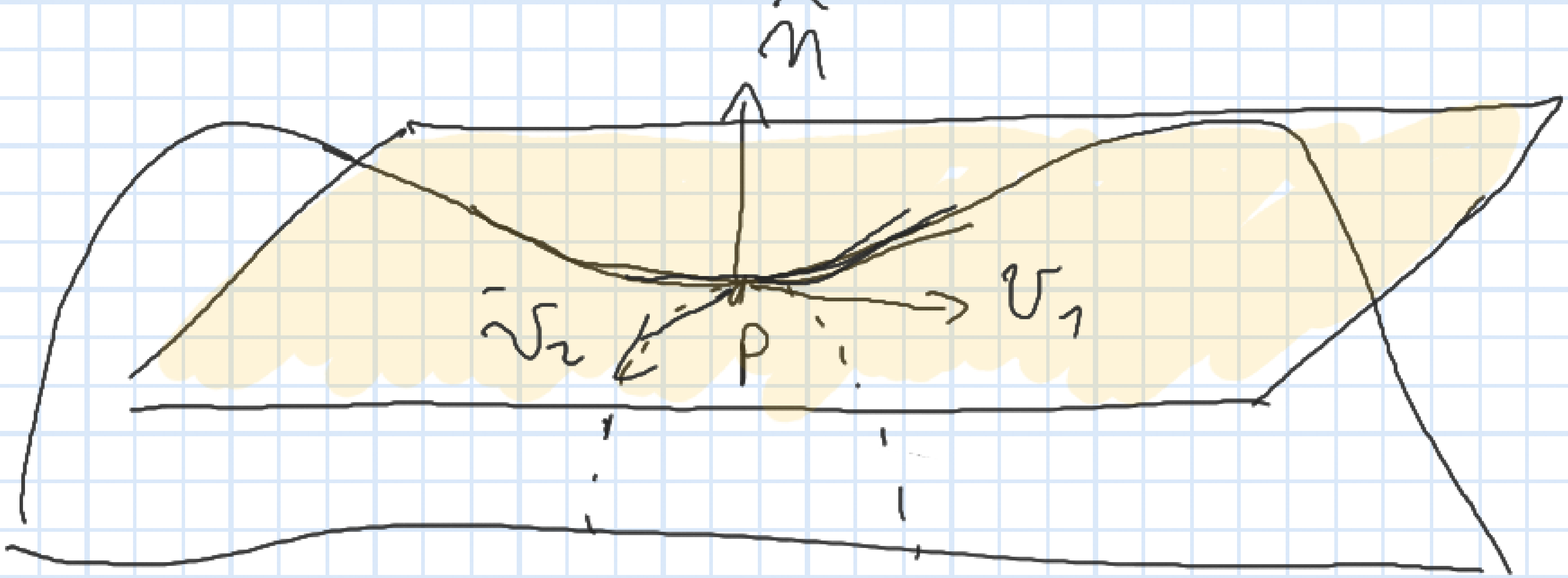
$K > 0$

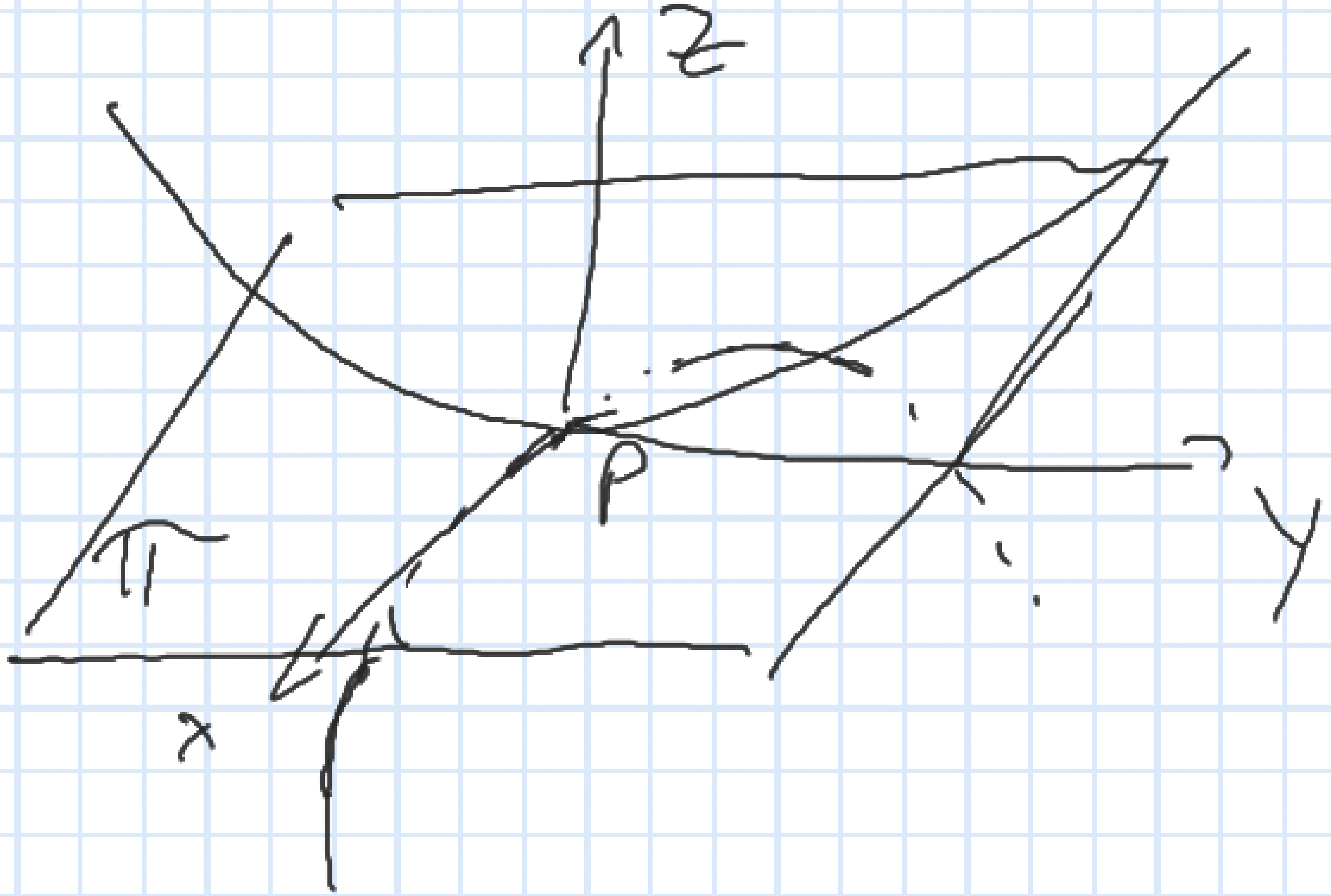


$R = \infty$

$K = 0$

DEF CURVATURA DI GAUSS $= k_1 \cdot k_2 = \boxed{K}$





$$z = \frac{\partial f}{\partial x}\Big|_p x + \frac{\partial f}{\partial y}\Big|_p y + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Big|_p x^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}\Big|_p y^2 + \frac{\partial^2 f}{\partial x \partial y}\Big|_p x \cdot y + O^3$$

$$= \frac{1}{2}\left[\frac{\partial f}{\partial x^2} x^2 + \frac{\partial^2 f}{\partial y^2} y^2 + 2\frac{\partial f}{\partial x \partial y} xy\right] + O(3)$$

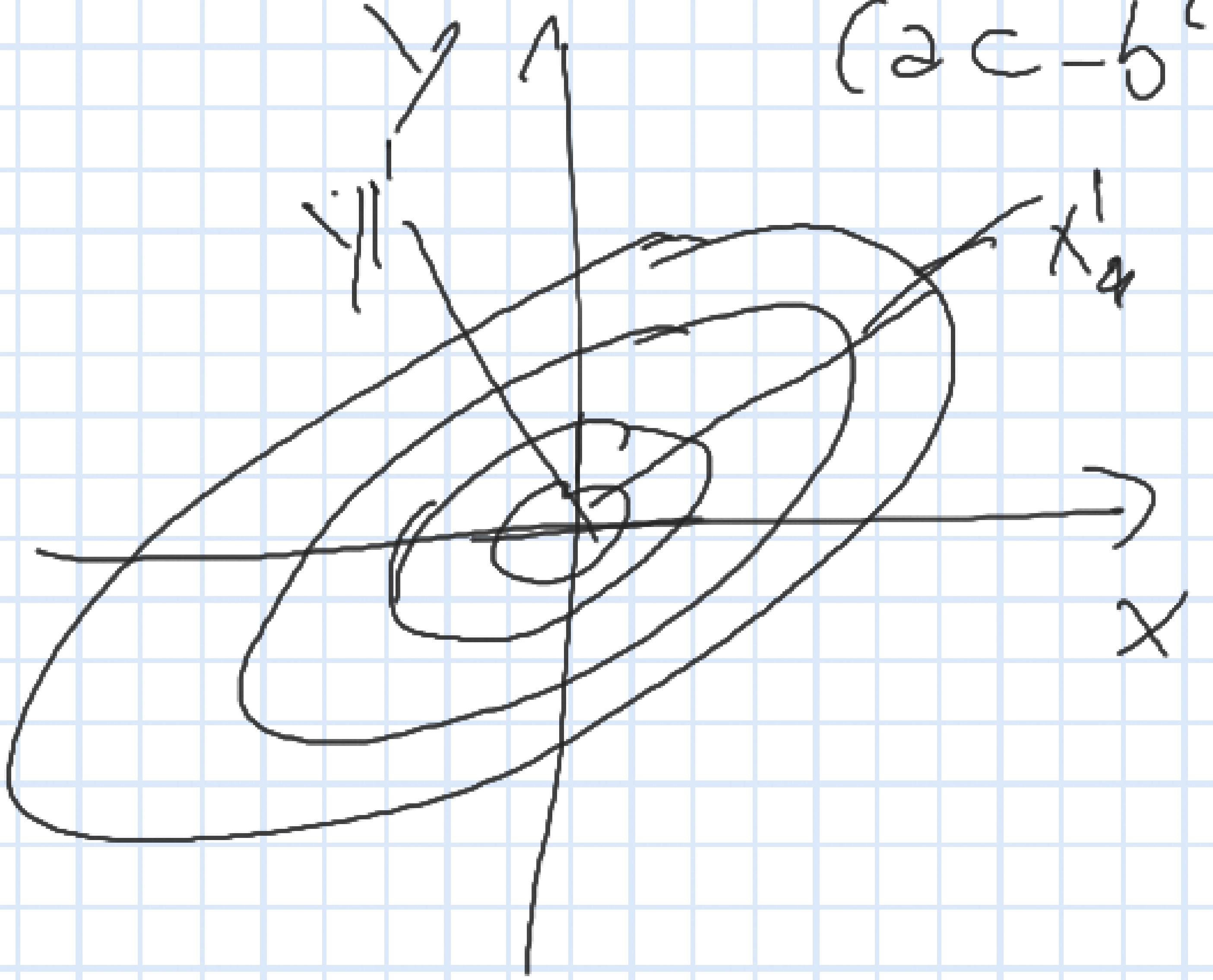
$$\Rightarrow z = \frac{1}{2}\left[a x^2 + 2bxy + cy^2\right]$$

$(ac - b^2) > 0 \Rightarrow$ ellisse

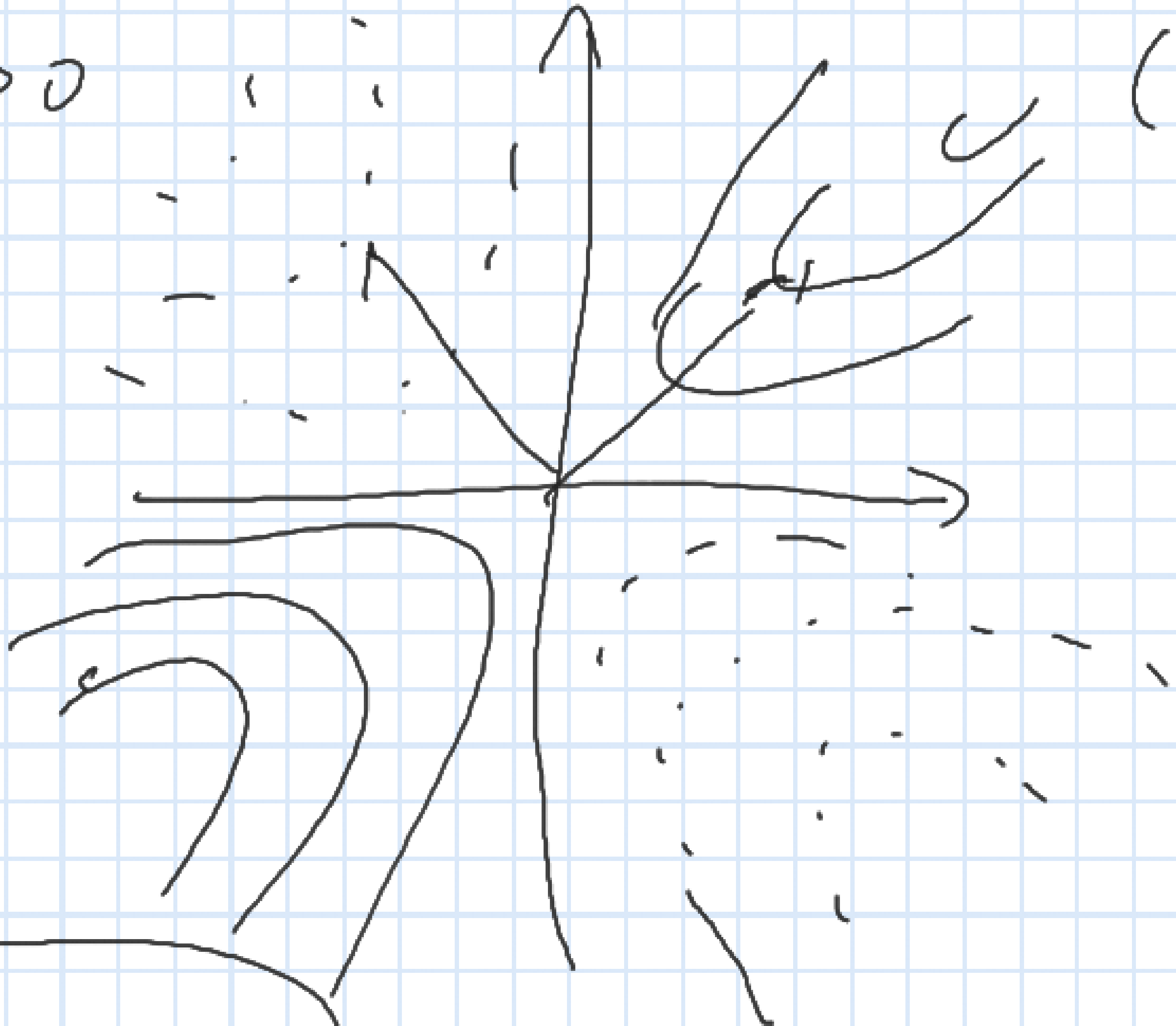
$(ac - b^2) = 0 \Rightarrow$ parabole

$(ac - b^2) < 0 \Rightarrow$ iperbole

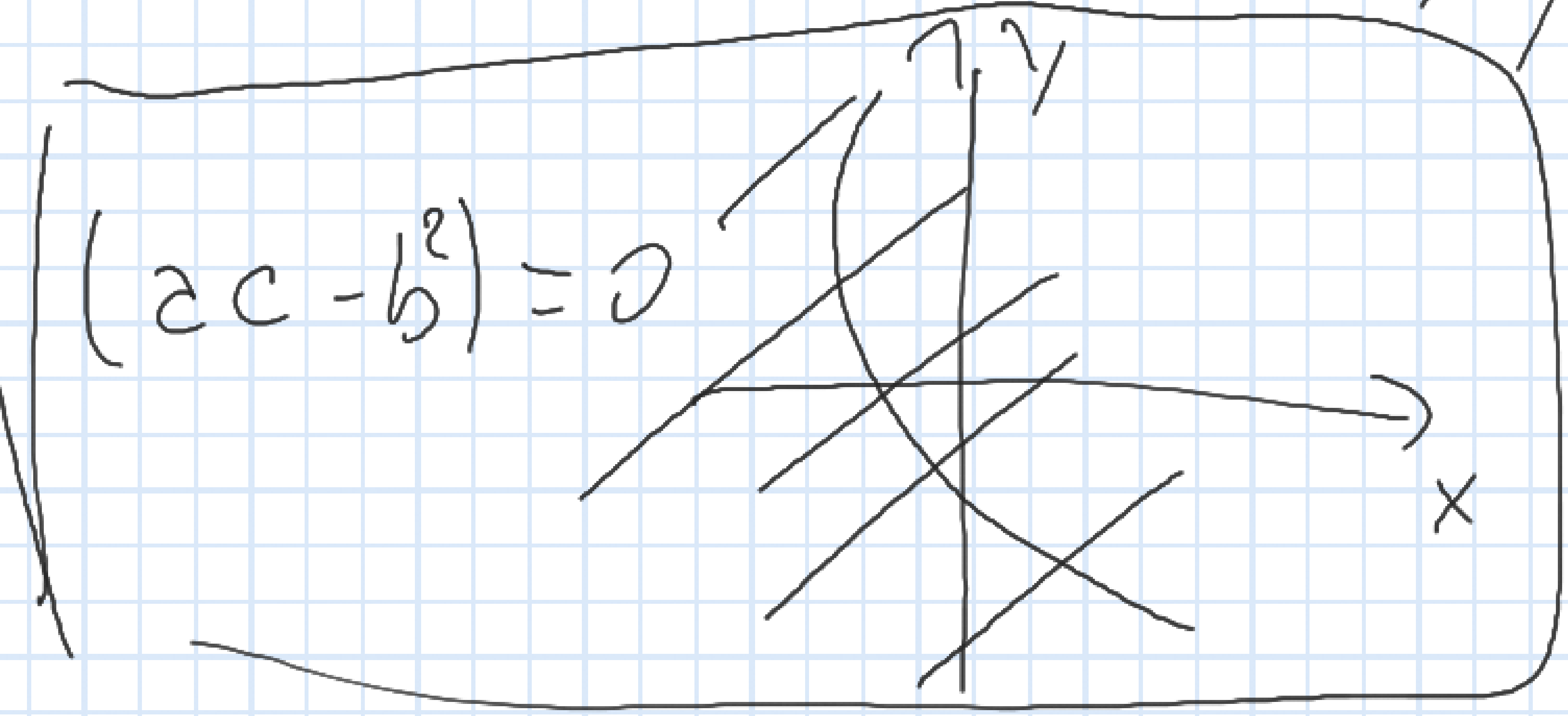
$(ac-b^2)>0$

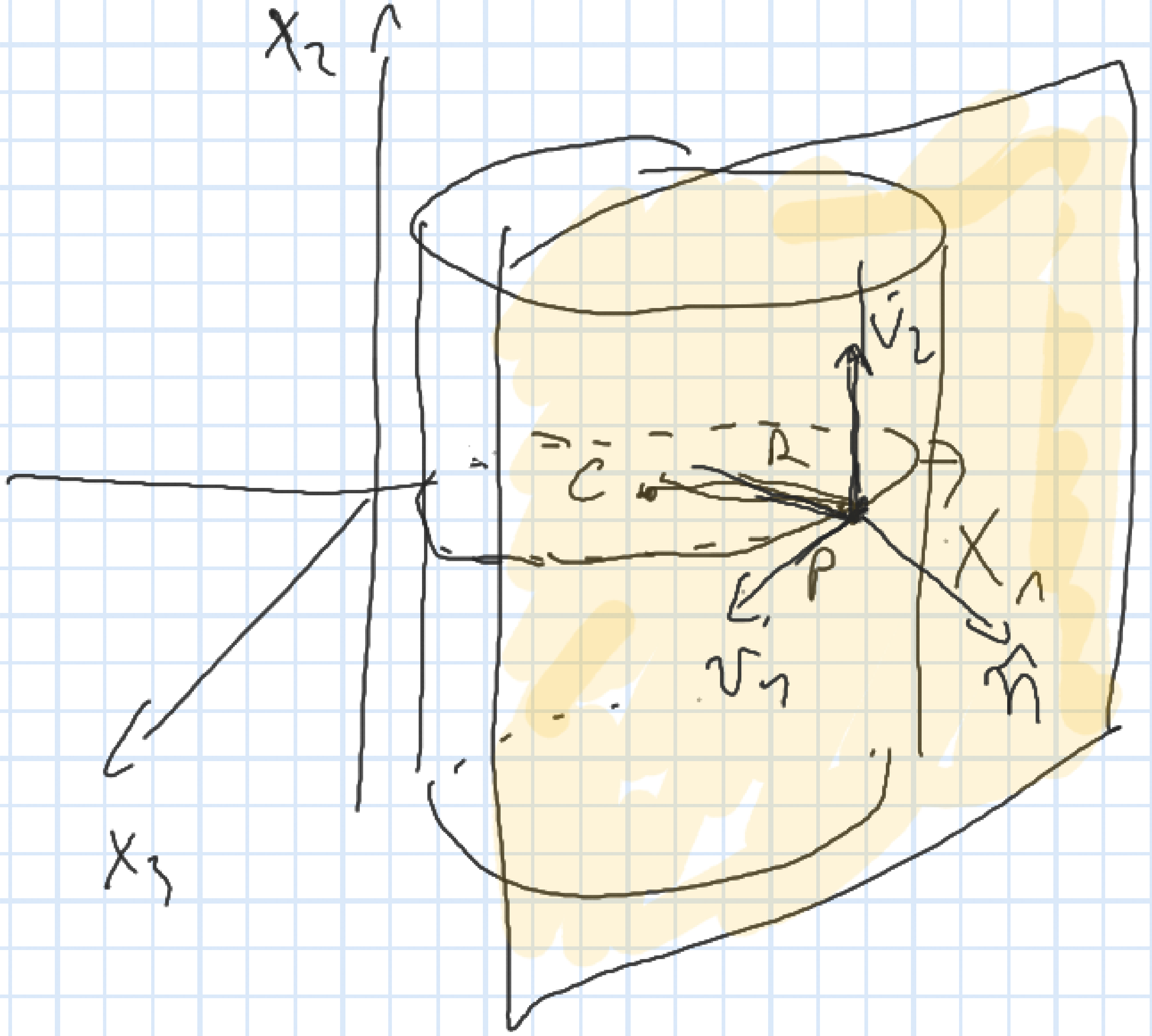


$(ac-b^2)<0$



$(ac-b^2)=0$



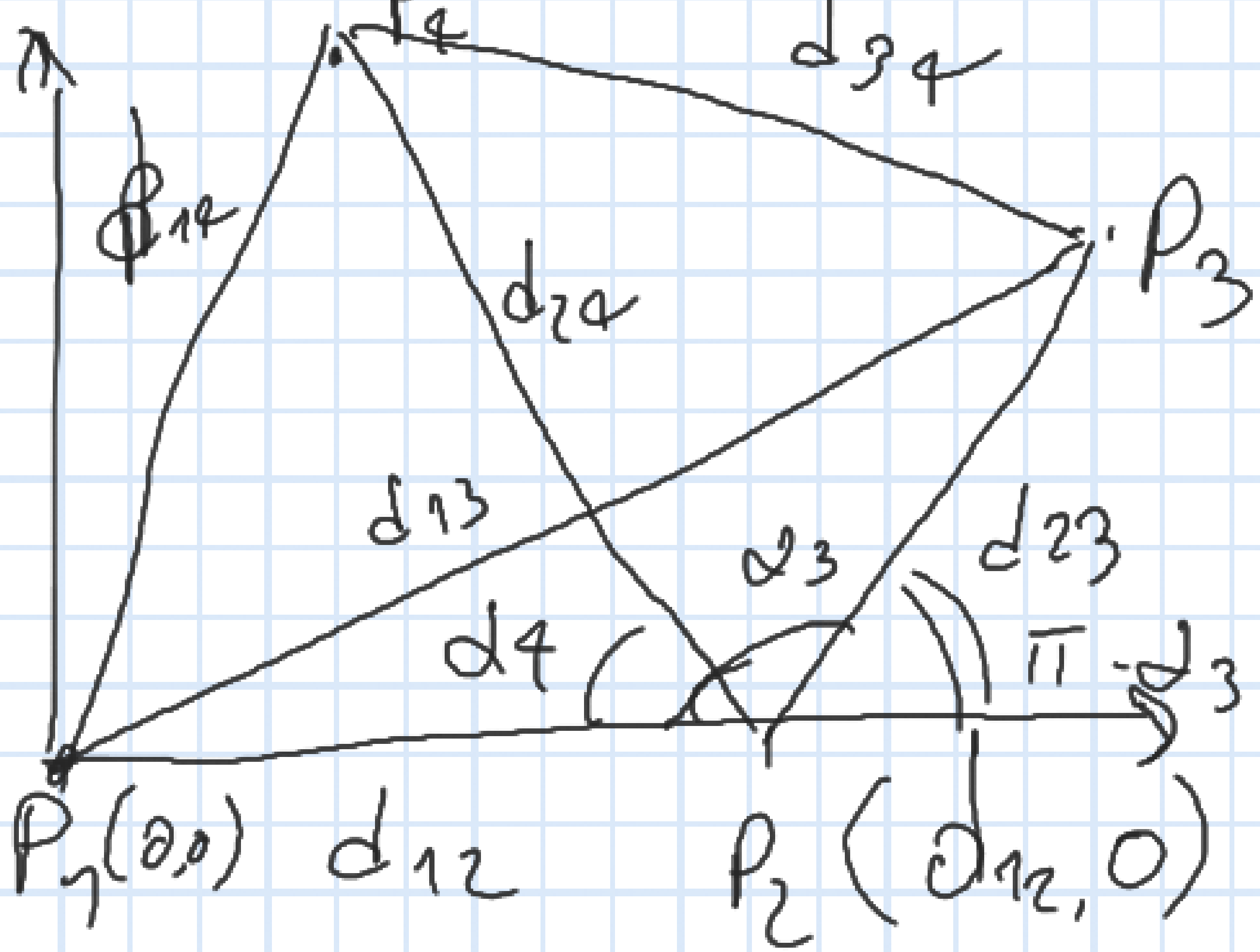


$$K_1 = \frac{1}{R}$$

$$K_2 = 0 \quad : \quad R_2 = \infty$$

$$K = K_1 \cdot K_2 = 0$$

LA CURVATURA DI GAUSS È UNA PROPRIETÀ INTRINSECA DELLA SUPERFICIE E SI PUÒ DETERMINARE CONOSCENDO g_{ij} $\Rightarrow$ $\boxed{\text{TEOREMA EGREGIUM}}$



P_1, P_2, P_3, P_4

$d_{12} = 780$ Km

$d_{13} = 1498$ Km

$d_{14} = 1112$ Km

$d_{23} = 735$ Km

$d_{24} = 960$ Km

$d_{34} = 813$ Km

CARNOT

$$d_{13}^2 = d_{12}^2 + d_{23}^2 - 2 d_{12} d_{23} \cos\alpha_3$$

$$\cos\alpha_3 = \frac{d_{12}^2 + d_{23}^2 - d_{13}^2}{2 d_{12} d_{23}}$$

$$\Rightarrow \cos\alpha_4 = \frac{d_{12}^2 + d_{24}^2 - d_{14}^2}{2 d_{12} d_{24}}$$

$$P_3 = \left(d_{12} + d_{23}\cos(\pi - \alpha_3),\ d_{23}\sin(\pi - \alpha_3)\right)$$

$$\stackrel{!}{=} \left(d_{12} - d_{23}\cos\alpha_3,\ d_{23}\sin\alpha_3\right)$$

$$P_4 = \left(d_{12} - d_{24}\cos\alpha_4,\ d_{24}\sin\alpha_4\right)$$

$$d_{34}^2 = \left[\cancel{d_{12}} - d_{23}\cos\alpha_3 - \cancel{d_{12}} + d_{24}\cos\alpha_4\right]^2 +$$

$$+ \left[d_{23}\sin\alpha_3 - d_{24}\sin\alpha_4\right]^2 = d_{23}^2 + d_{24}^2 - 2 d_{23} d_{24}\cos(\alpha_3 - \alpha_4)$$

$$\Rightarrow d_{34} = 1147.6 \text{ Km} \Rightarrow 813 \text{ Km}$$