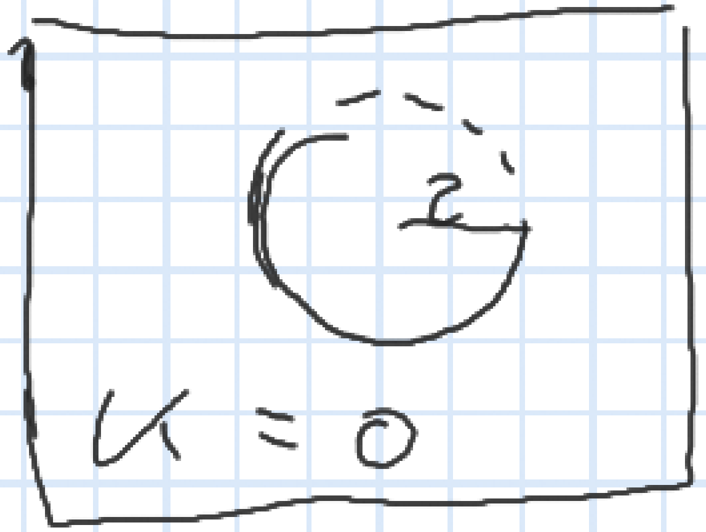


$C = 2\pi r$

$C < 2\pi r$

$C > 2\pi r$

$\boxed{K}$ → PROPRIETÀ INTRINSECA ≡ NON DIPENDE DAL SISTEMA DI CORDINATE

→ PROPRIETÀ LOCALE

$\boxed{K \leftrightarrow g_{ij}}$ $\quad X_0 \quad$ $g_{ij} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ LOCALMENTE EUCLIDEO

$$g_{ij,k} \Rightarrow \frac{\partial g_{ij}}{\partial u^k} = 0$$

$$g'_{kl} = \frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij} \qquad \boxed{g'_{kl}(x_0)}$$

$$g'_{kl}(x) = g'_{kl}(x_0) + g'_{kl,m}(x_0)(x^m - x_0^m) + \frac{1}{2} g'_{kl,mn}(x_0)(x^m - x_0^m) \cdot (x^n - x_0^n) + \dots$$

- $$g'_{kl}(x_0) = \left[\frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij}\right]_{x_0}$$

- $$g'_{kl,m}(x_0) = \left(\frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij,m}\right]_{x_0} + \left[\frac{\partial^2 u^i}{\partial u'^k \partial u'^m} \frac{\partial u^j}{\partial u'^l} g_{ij}\right]_{x_0} + \left[\frac{\partial^2 u^j}{\partial u'^l \partial u'^m} \frac{\partial u^i}{\partial u'^k} g_{ij}\right]_{x_0}$$

$$\overset{!}{=} \left(\frac{\partial u^i}{\partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij,m}\right]_{x_0} + \left(2 \frac{\partial^2 u^i}{\partial u'^m \partial u'^k} \frac{\partial u^j}{\partial u'^l} g_{ij}\right]_{x_0}$$

$$\bullet\; g'_{kl,mn}(x) = \left[\frac{\partial u^i}{\partial u'^k}\frac{\partial u^j}{\partial u'^l}\, g_{ij,mn}\right]_{x_0} + 2\left[\frac{\partial^3 u^i}{\partial u'^m\,\partial u'^n\,\partial u'^k}\frac{\partial u^j}{\partial u'^l}\, g_{ij}\right]_{x_0}$$

$+$ derivate I, II, $\underline{\text{III}}$

		2D	3D	4D	ND
# PARAMETRI COEFFICIENTI	$\left(\dfrac{\partial u^i}{\partial u'^k}\right)_{x_0}$	$2\times 2=4$	9	16	N^2
	$\left(\dfrac{\partial^2 u^i}{\partial u'^m \partial u'^k}\right)_{x_0}$	$2\times 3=6$	18	40	$\dfrac{N^2(N+1)}{2}$
	$\left(\dfrac{\partial^3 u^i}{\partial u'^m \partial u'^n \partial u'^k}\right)_{x_0}$	$2\times 4=8$	30	80	$\dfrac{N^2(N+1)(N+2)}{6}$
# EQ INDIPENDENTI	$g'_{kl}(x_0)$	3	6	10	$\dfrac{N(N+1)}{2}$
	$g'_{kl,m}(x_0)$	6	18	40	$\dfrac{N^2(N+1)}{2}$
	$g'_{kl,mn}(x_0)$	9	36	100	$\left[\dfrac{N(N+1)}{2}\right]^2$

- DERIVATA COVARIANTE

$$\phi \Rightarrow \boxed{\frac{\partial \phi}{\partial u^i}} \Rightarrow \text{VETTORE COVARIANTE}$$

$A_i(u^k)$ $\qquad dA_i$

$$A_i = \frac{\partial u'^k}{\partial u^i} A'_k$$

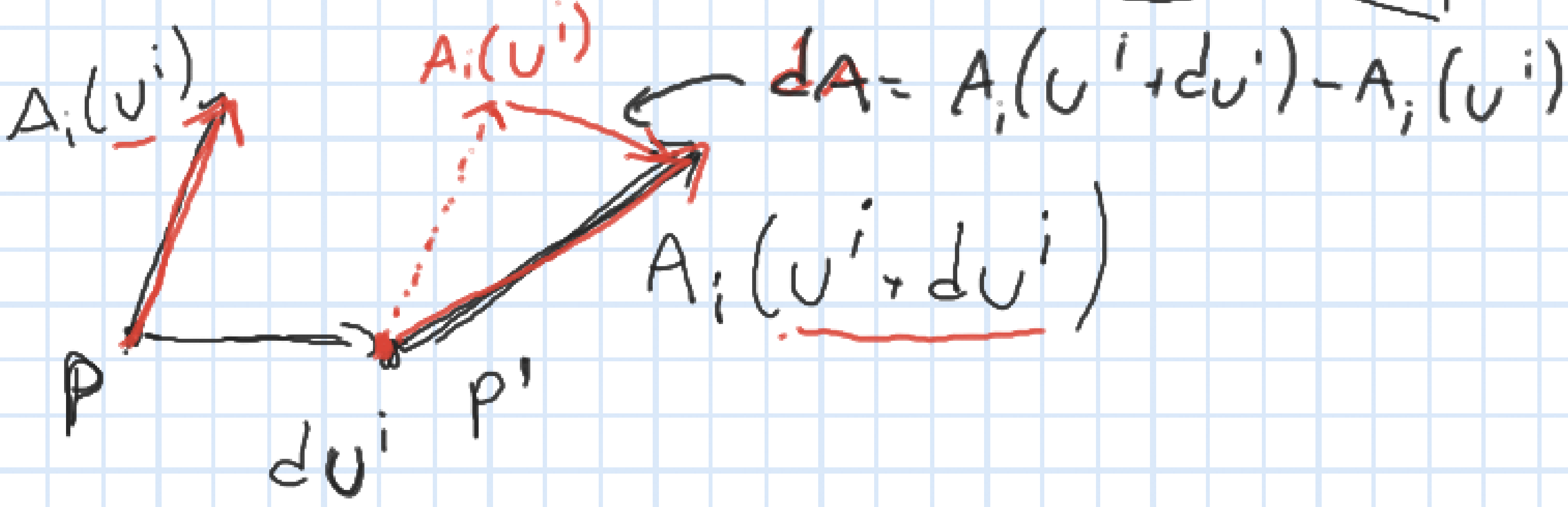
$$\boxed{dA_i} = \frac{\partial u'^k}{\partial u^i} dA'_k + A'_k \, d\left(\frac{\partial u'^k}{\partial u^i}\right) = \boxed{\frac{\partial u'^k}{\partial u^i} \cdot dA'_k} + \overbrace{\frac{\partial^2 u'^k}{\partial u^i \partial u^l} A'_k \, du^l}^{=0}$$

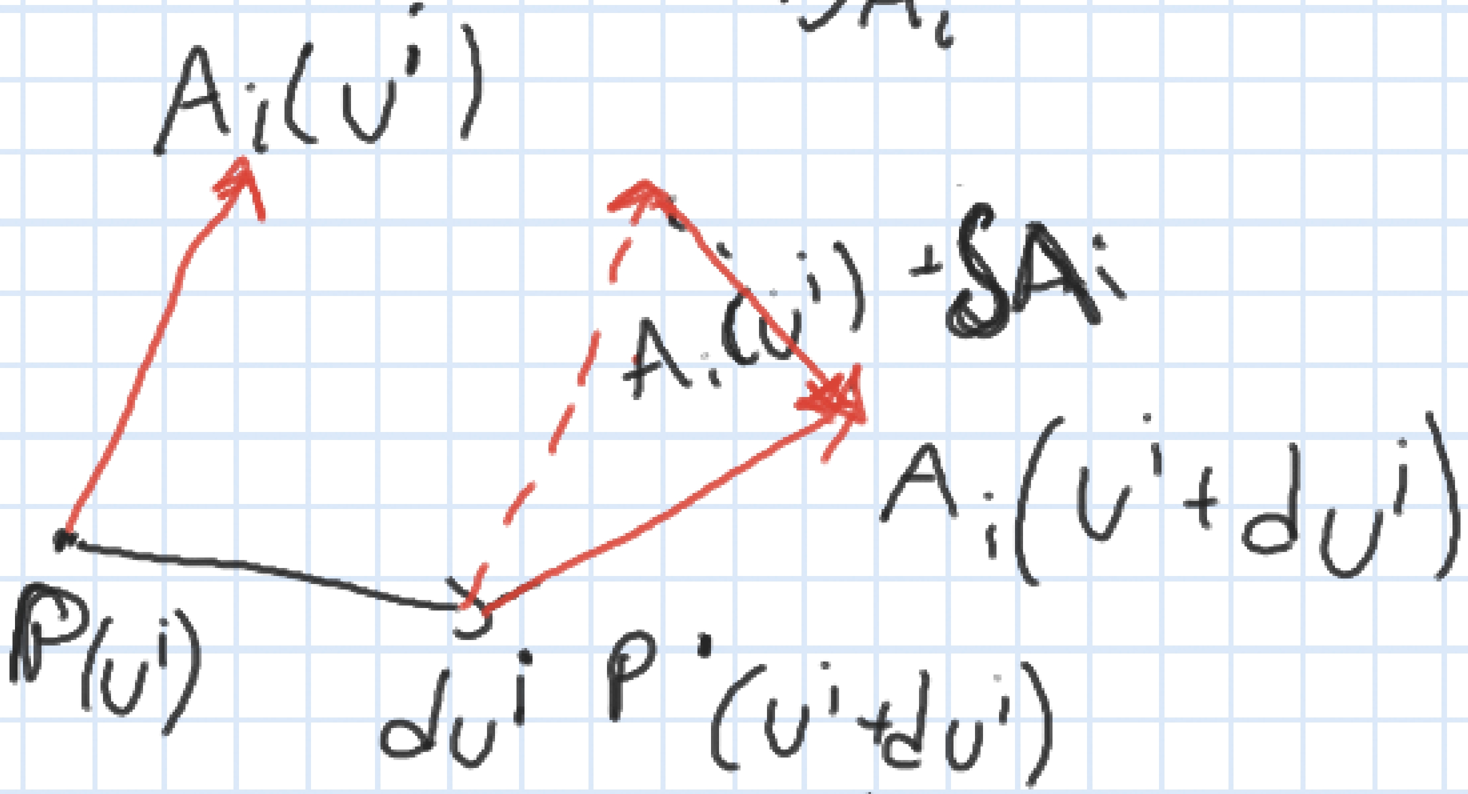
SOLO SE $\frac{\partial^2 u'^k}{\partial u^i \partial u^l} = 0 \Rightarrow u'^i$ È LINEARE u^i

dA_i = NON È UN VETTORE

$$\boxed{dA_i = A_i(u^i + du^i) - A_i(u^i)}$$

DA_i





EUCLIDEO

$A_i(u^i)$

GENERALE

$$\lim_{du^i \to 0} \frac{A_i(u^i + du^i) - A_i(u^i)}{du^i}$$

$$\boxed{\delta A = 0}$$

$P' \rightarrow A_i + dA_i = A_i(u^i + du^i)$ $\qquad \delta A_i = ?$

$\boxed{DA_i} = (A_i + dA_i) - (A_i + \delta_{A_i}) = dA_i - \boxed{\delta A_i} =$ È UN VETTORE

DERIVAZIONE (DIFFERENZIALE ASSOLUTO)

$\delta_{A_i} \propto A_i, du^i \Rightarrow \delta A_i = \boxed{\Delta^m_{il}} A_m du^l$

$\boxed{\Delta^m_{il} = 0}$ NEL SISTEMA LOCALMENTE EUCLIDEO

, MA DIPENDE DAL SISTEMA DI CORDINATE

$\Rightarrow \Delta^m_{il}$ <u>NON</u> È UN TENSORE

$$\boxed{\Delta^m_{il} \stackrel{!}{=} \Gamma^m_{il}}$$

$$DA_i = dA_i - \delta A_i = \frac{\partial A_i}{\partial u^l} du^l - \Gamma^m_{il} A_m du^l$$

DERIVATA COVARIANTE

$$\frac{DA_i}{\partial u^l} \equiv A_{i;l} = \frac{\partial A_i}{\partial u^l} - \Gamma^m_{il} A_m$$

$$T_{ik} \equiv A_i B_k$$

$$T_{ik;l} = B_k \cdot A_{i;l} + A_i \cdot B_{k;l} = B_k \left(\frac{\partial A_i}{\partial u^l} - \Gamma^m_{il} A_m \right) +$$

$$+ A_i \left(\frac{\partial B_k}{\partial u^l} - \Gamma^m_{kl} B_m \right) =$$

$$= B_k \frac{\partial A_i}{\partial u^l} + A_i \frac{\partial B_k}{\partial u^l} - \Gamma^m_{il} A_m B_k - \Gamma^m_{kl} A_i B_m$$

$$\stackrel{!}{=} \frac{\partial T_{ik}}{\partial u^l} - \Gamma^m_{il} T_{mk} - \Gamma^m_{kl} T_{im}$$

$$\boxed{g_{ij}} \Rightarrow \quad A_{i;l} = (g_{ik} A^k)_{;l} = g_{ik;l} A^k + g_{ik} A^k_{;l}$$

$$A_{i;l} \stackrel{!}{=} g_{ik} \cdot A^k_{;l} \Rightarrow \quad g_{ik;l} = 0$$

$$g_{ik;l} = 0 \Rightarrow \frac{\partial g_{ik}}{\partial u^l} - \Gamma^m_{il} g_{mk} - \Gamma^m_{kl} g_{im} = 0 \quad (1)$$

i, k, l

$$(2) \quad \frac{\partial g_{kl}}{\partial u^i} - \Gamma^m_{ki} g_{ml} - \Gamma^m_{li} g_{km} = 0$$

i, k, l

$$(3) \quad \frac{\partial g_{li}}{\partial u^k} - \Gamma^m_{lk} g_{mi} - \Gamma^m_{ik} g_{lm} = 0$$

$$(1) + (3) - (2) \Rightarrow \frac{\partial g_{ik}}{\partial u^l} + \frac{\partial g_{li}}{\partial u^k} - \frac{\partial g_{kl}}{\partial u^i} - \ast$$

$$* \; -\Gamma^m_{il}\cancel{g_{mk}} - \Gamma^m_{kl} g_{im} \qquad - \Gamma^m_{lk} g_{mi} - \cancel{\Gamma^m_{lk} g_{lm}} + \cancel{\Gamma^m_{ki} g_{ml}}$$

$$+ \cancel{\Gamma^m_{li} g_{km}} = 0$$

$$\Gamma^m_{il} \qquad g_{ik} \qquad \boxed{\Delta^m_{il} \equiv \Gamma^m_{il}}$$

$$\Rightarrow \frac{\partial g_{ik}}{\partial u^l} + \frac{\partial g_{li}}{\partial u^k} - \frac{\partial g_{kl}}{\partial u^i} - 2\Gamma^m_{kl} g_{im} = 0 \quad , \; \frac{1}{2} g^{ij}$$

$$\boxed{\frac{1}{2} g^{ij}\left(\frac{\partial g_{ik}}{\partial u^l} + \frac{\partial g_{li}}{\partial u^k} - \frac{\partial g_{kl}}{\partial u^i}\right) = \Gamma^m_{kl} g_{im} g^{ij} = \Gamma^m_{kl} \delta^j_m = \Gamma^j_{kl}}$$

$A_i B^i$ È SCALARE $\Rightarrow$ NON CAMBIA PER TRASPORTO PARALLELO

$$\delta(A_i B^i) = 0$$

$$B^i \delta A_i + A_i \delta B^i = 0 \quad \Rightarrow \quad A_i \delta B^i = - B^i \boxed{\delta A_i}$$

$$\delta A_i = \Gamma^m_{il} A_m du^l$$

$$A_i \delta B^i = - B^m \Gamma^i_{ml} A_i du^l$$

$$\delta B^i = - \Gamma^i_{ml} B^m du^l$$

$$\Rightarrow \frac{DB^i}{\partial u^l} \equiv B^i_{;l} = \frac{\partial B^i}{\partial u^l} + \Gamma^i_{ml} B^m$$

$+\Gamma$ PER INDICI CONTROVARIANTI

$-\Gamma$ PER INDICI COVARIANTI