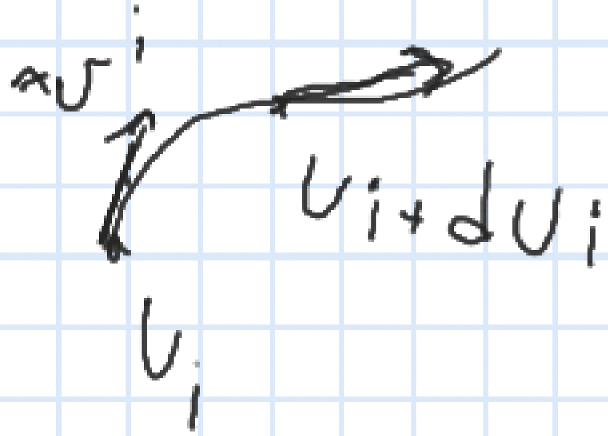
DERIVATA COVARIANTE

$$\frac{DA_i}{\partial u^l} = A_{i;l} = \frac{\partial A_i}{\partial u^l} - \Gamma^{m}_{il} A_m$$

$$\frac{DB^i}{\partial u^l} = B^i_{;l} = \frac{\partial B^i}{\partial u^l} + \Gamma^{i}_{ml} B^m$$

TRASPORTO PARALLELO E TENSORE DI CURVATURA



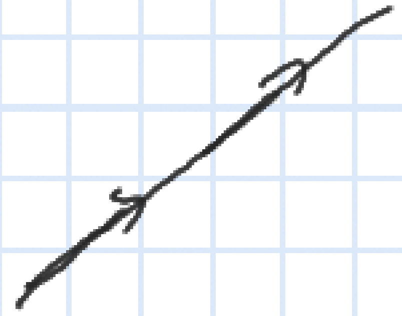
$$u^i = u^i(s)$$

$$du^i \Rightarrow \boxed{\frac{du^i}{ds} \equiv v^i}$$

$$|v^i| = 1 \qquad v_i v^i = g_{ij} v^i v^j = g_{ij} \frac{du^i}{ds} \frac{du^j}{ds} = 1$$

$$ds^2 = g_{ij} \, du^i du^j$$

$$\boxed{\text{EUCLIDEO} \qquad \frac{dv^i}{dS} = 0}$$



$$\frac{Dv^i}{ds} = 0 \Rightarrow \frac{Dv^i}{ds} = \frac{Dv^i}{du^l}\frac{du^l}{ds} = \frac{du^l}{ds}\left(\frac{\partial v^i}{\partial u^l} + \Gamma^i_{ml} v^m\right) = 0$$

$$\left(\frac{\partial v^i}{\partial u^l}\frac{du^l}{ds}\right) + \Gamma^i_{ml} v^m \frac{du^l}{ds} = 0$$

$$\frac{dv^i}{ds} + \Gamma^i_{ml} v^m v^l = 0$$

$$v^i = \frac{du^i}{ds}$$

$$\Rightarrow \boxed{\frac{d^2u^i}{ds^2} + \Gamma^i_{ml}\frac{du^m}{ds}\frac{du^l}{ds} = 0}$$

LUNGO LA GEODETICA

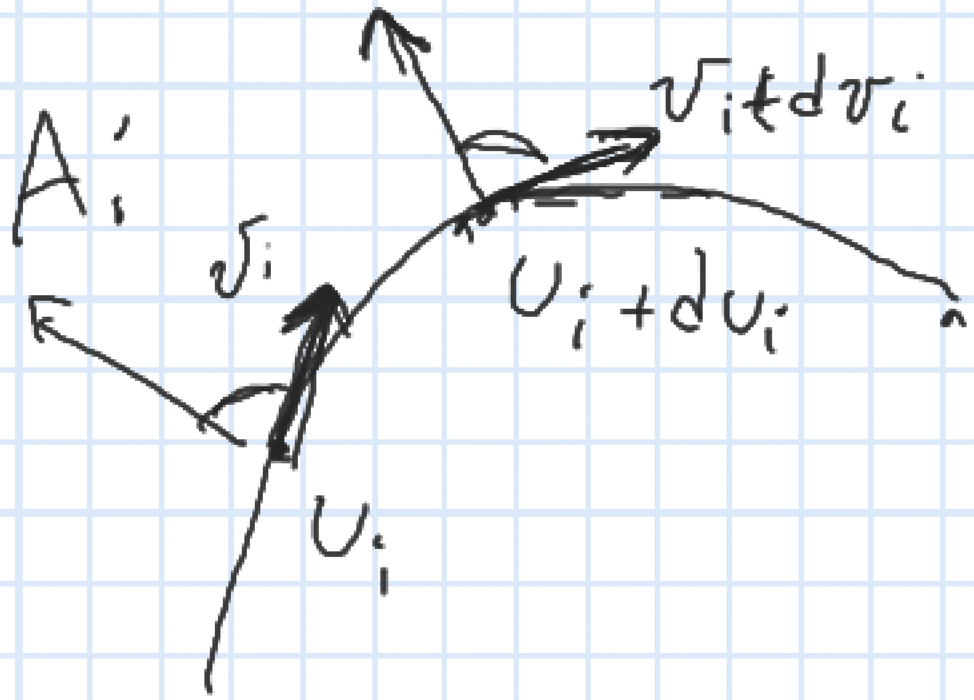
$$DA_i = dA_i - \delta A_i$$

$Dv^i = 0$

$\Rightarrow dv^i = \delta v^i$

$v^i(u^i)$

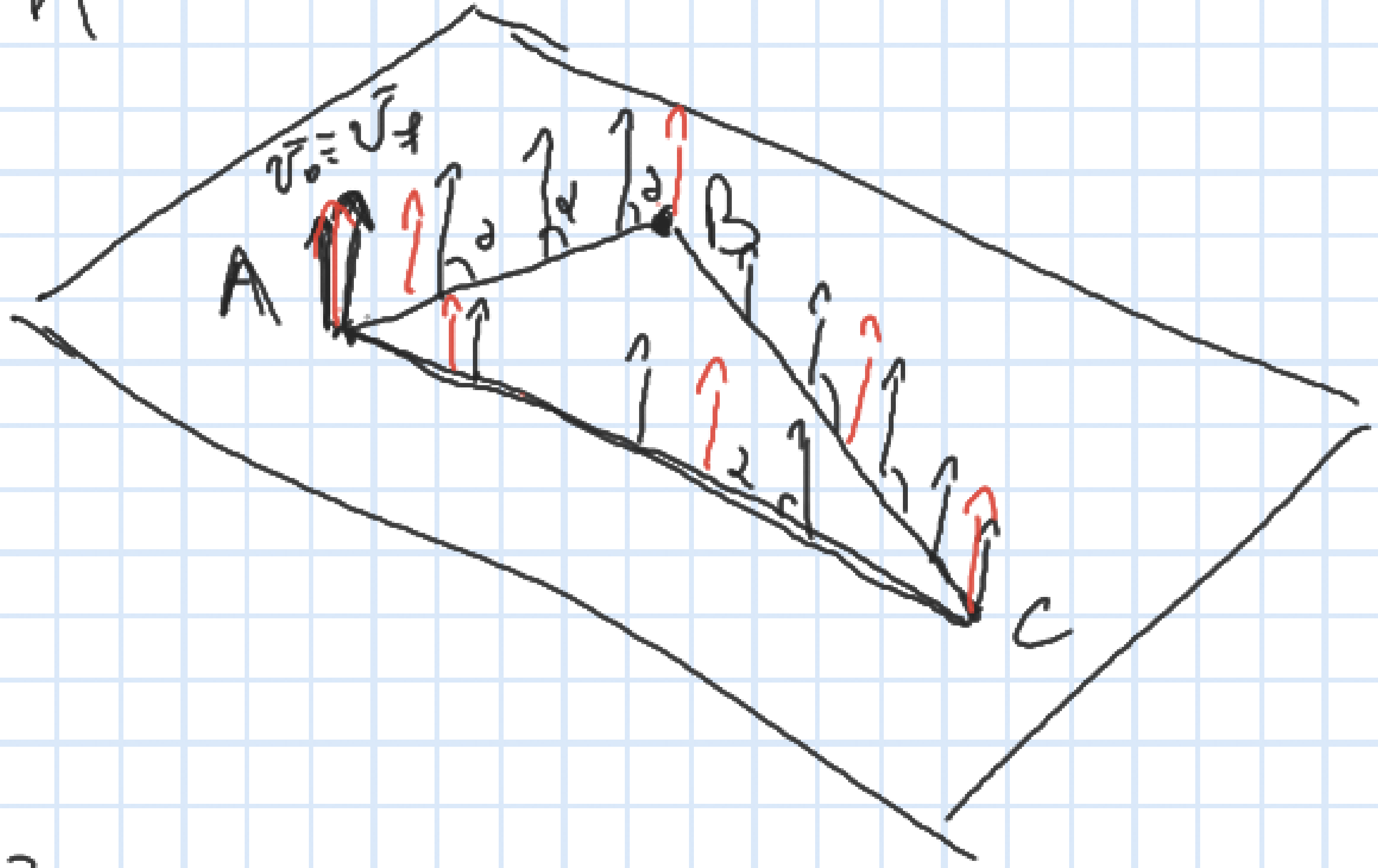
$v_i + dv_i$



$\boxed{A_i \cdot v^i} =$

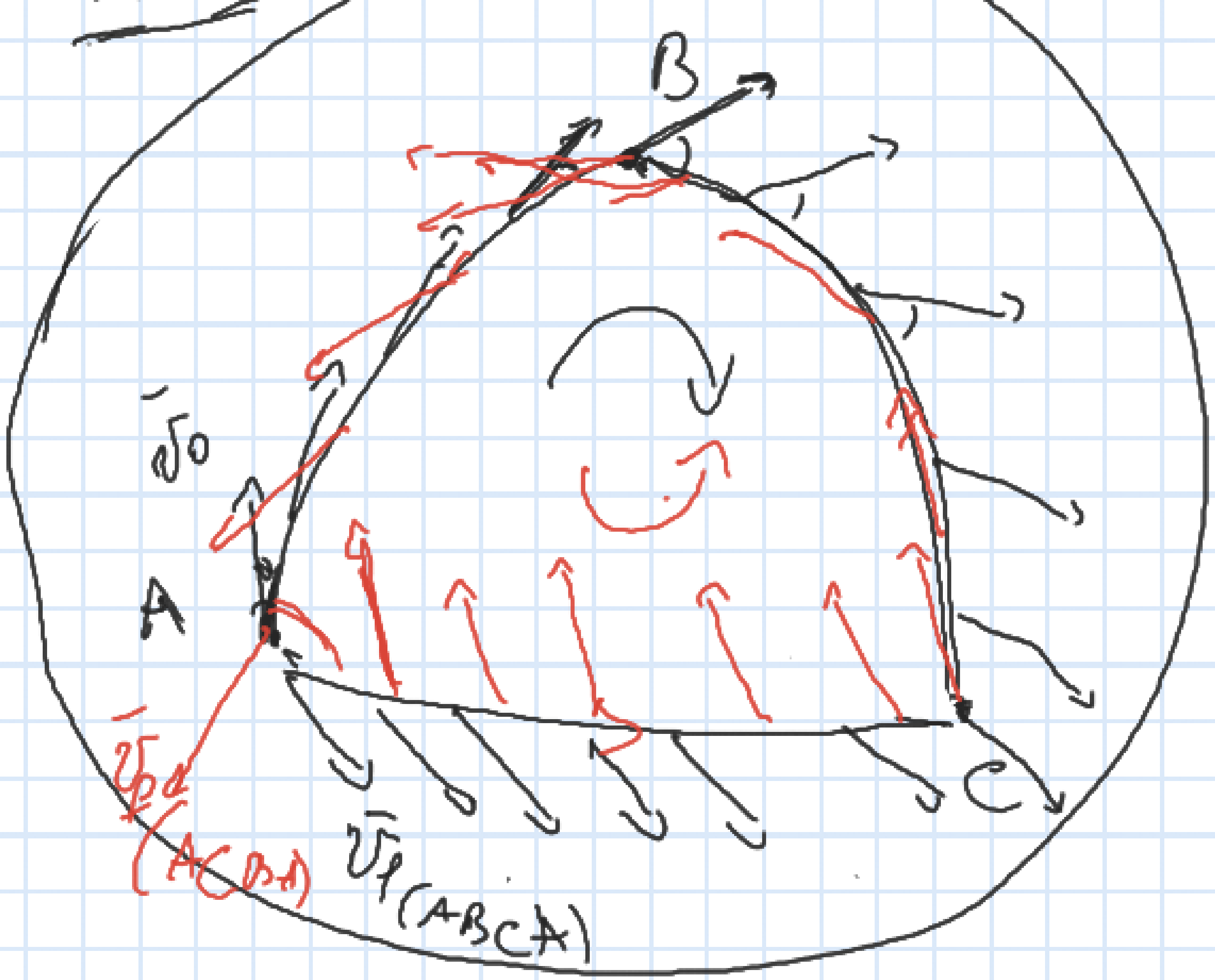
- UN VETTORE A_i VIENE TRASPORTATO PARALLELAMENTE LUNGO UNA GEODETICA SE FORMA SEMPRE LO STESSO ANGOLO CON LA TANGENTE ALLA CURVA

$K=0$



$$\bar{v}_0 = \bar{v}_{f(ABCA)} = \bar{v}_{f(ACBA)}$$

$K>0$

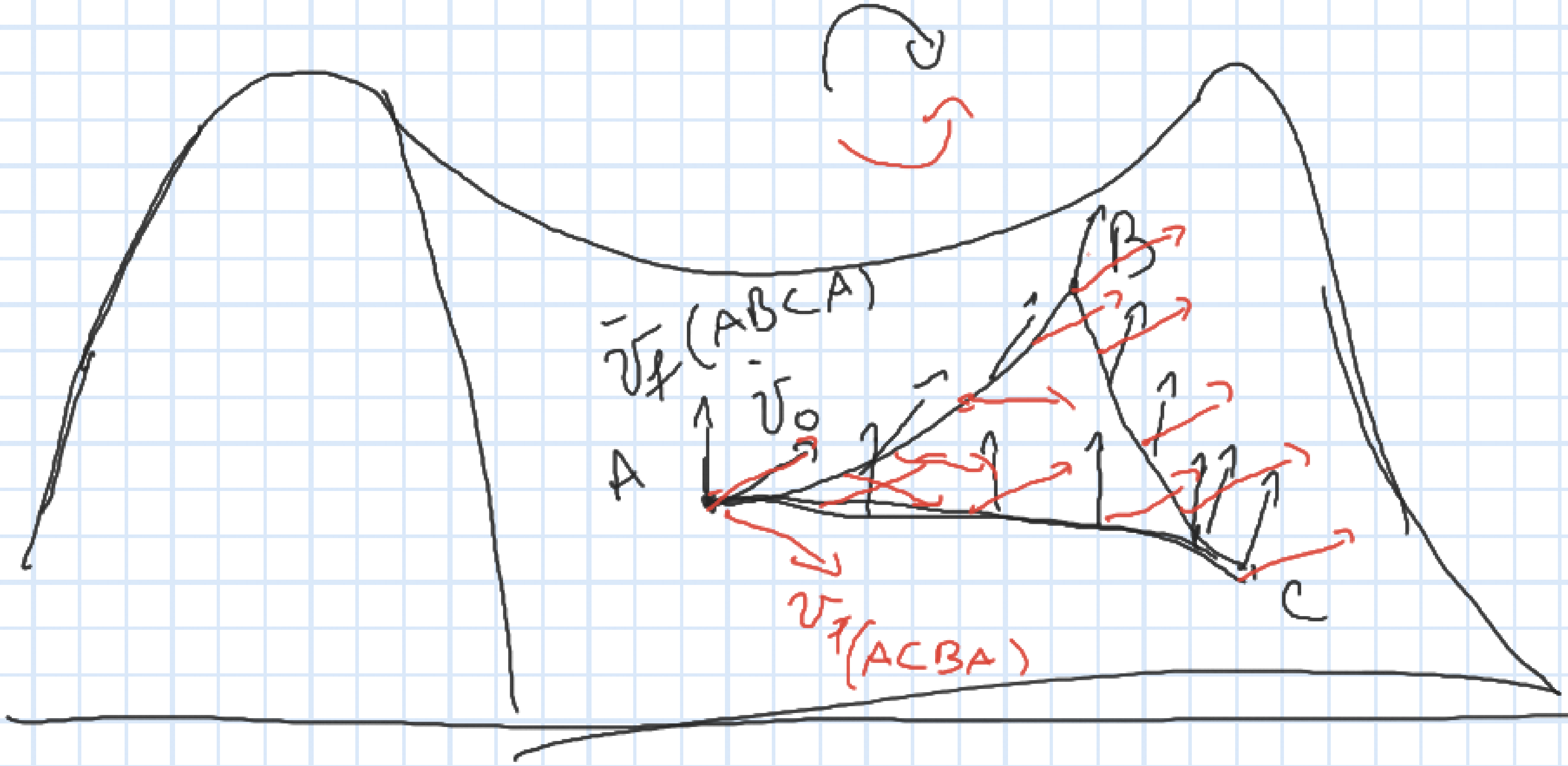


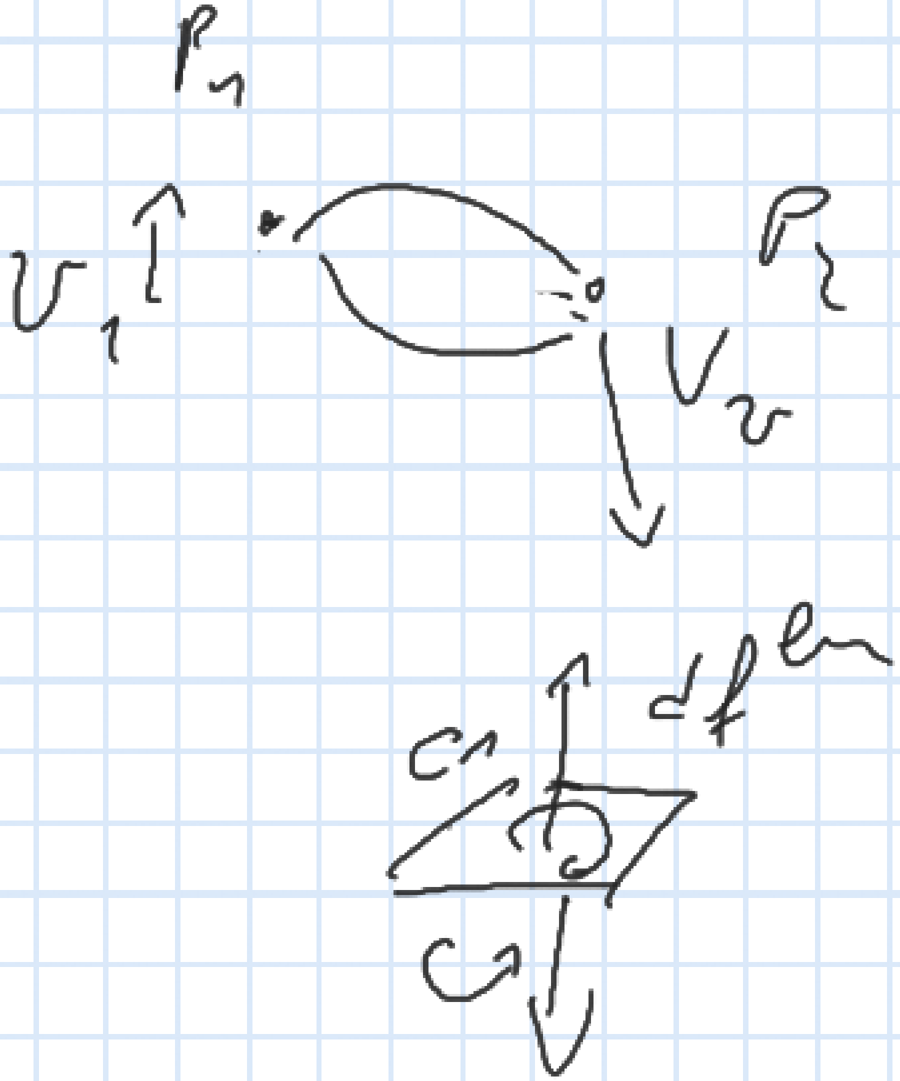
PER $K>0$

IL VETTORE TRASPORTATO //

RUOTA NEL VERSO DI PERCORRENZA

K < 0

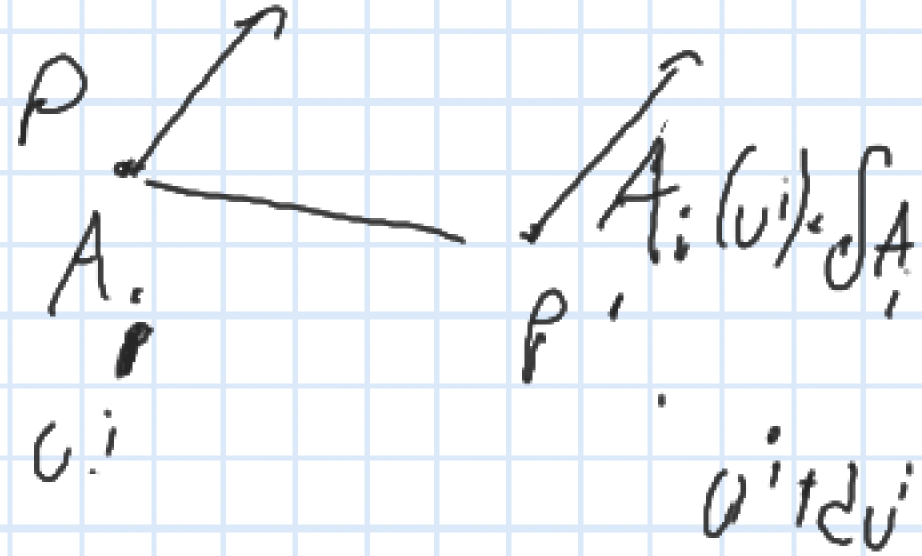




PER K < 0 IL VETTORE TRASPORTATO //
RUOTA NEL VERSO OPPOSTO DI PERCORRENZA

ΔA_k

$A_k \to A_k + \Delta A_k$ $\quad C_1$



$$\boxed{\Delta A_k = \oint_{C_1} \delta A_k} = \oint_{C_1} \Gamma^i_{km} A_i \, du^m$$

DA_i

TEOREMA DI STOKES (ROTORE)

$$\oint_{C_1} \vec{F} \cdot d\vec{F} = \int_{\Sigma} \vec{\nabla} \times \vec{F} \cdot d\Sigma$$

$$\oint_{C_1} A_i\, du^i = \frac{1}{2} \int_{\text{SUPERFICIE}} \left(\frac{\partial A_m}{\partial u^l} - \frac{\partial A_l}{\partial u^m} \right) df^{lm} \qquad df^{lm} = -df^{ml}$$

$$A_m\, du^m \to \Gamma^i_{km} A_i\, du^m$$

$$\Delta A_k = \frac{1}{2} \int_{\text{SUP}} \left[\frac{\partial (\Gamma^i_{km} A_i)}{\partial u^l} - \frac{\partial (\Gamma^i_{kl} A_i)}{\partial u^m} \right] df^{lm}$$

$$\Delta A_k = \frac{1}{2} \Delta f^{lm} \left[\frac{\partial \Gamma^i_{km}}{\partial u^l} A_i + \Gamma^i_{km} \frac{\partial A_i}{\partial u^l} - \frac{\partial \Gamma^i_{kl}}{\partial u^m} A_i - \Gamma^i_{kl} \frac{\partial A_i}{\partial u^m} \right]$$

$$DA_i = dA_i - \delta A_i = \frac{\partial A_i}{\partial u^l} du^l - \Gamma^m_{il} A_m du^l \Rightarrow \underline{\underline{DA_i = 0}}$$

$$\Rightarrow \frac{\partial A_i}{\partial u^l} = \frac{\delta A_i}{\partial u^l} = \Gamma^m_{il} A_m$$

$$\Rightarrow \Delta A_k = \frac{1}{2} \Delta f^{lm} \left[\frac{\partial \Gamma^i_{km}}{\partial u^l} A_i - \frac{\partial \Gamma^i_{kl}}{\partial u^m} A_i + \Gamma^i_{km}\Gamma^n_{il} A_n - \Gamma^i_{kl}\Gamma^n_{im} A_n \right]$$

$$= \frac{1}{2}\Delta f^{lm} A_i \underbrace{\left[\frac{\partial \Gamma^i_{km}}{\partial u^l} - \frac{\partial \Gamma^i_{kl}}{\partial u^m} + \Gamma^n_{km}\Gamma^i_{nl} - \Gamma^n_{kl}\Gamma^i_{nm}\right]}_{R^i_{klm}}$$

$$\Gamma^i_{km}\Gamma^i_{nl} A_i \qquad \Gamma^i_{kl}\Gamma^i_{nm} A_i$$

$R^i_{klm} \equiv$ TENSORE DI RIEMANN-CHRISTOFFEL

$\parallel$

TENSORE DI CURVATURA

$$\boxed{R^{i}_{klm} = \frac{\partial \Gamma^{i}_{km}}{\partial u^{l}} - \frac{\partial \Gamma^{i}_{kl}}{\partial u^{m}} + \Gamma^{n}_{km}\Gamma^{i}_{nl} - \Gamma^{n}_{kl}\Gamma^{i}_{nm}}$$

$R^{i}_{klm} = 0 \Rightarrow \Delta A_k = 0$ SPAZIO PIATTO

$g_{ij} = \text{COST}$

$\boxed{R^{i}_{klm} \neq 0} \Rightarrow$ SPAZIO CURVO

PROPRIETÀ DEL TENSORE DI CURVATURA

- R^i_{klm} È L'UNICO TENSORE CHE PUÒ ESSERE COSTRUITO A PARTIRE DAL TENSORE METRICO E DALLE SUE DERIVATE PRIME E SECONDE CHE È LINEARE NELLE DERIVATE SECONDE E QUADRATICO NELLE DERIVATE PRIME

- $R^i_{kem} \cdot g_{ij} = R_{jkem}$

$u^i \Rightarrow$ LOCALMENTE EUCLIDEO $\quad \dfrac{\partial g_{ij}}{\partial u^k} = 0$

$\Gamma = 0$

(1) $R^i_{kem;j} = \dfrac{\partial}{\partial u^j}\left(R^i_{kem}\right) = \dfrac{\partial^2 \Gamma^i_{km}}{\partial u^j \partial u^l} - \dfrac{\partial^2 \Gamma^i_{ke}}{\partial u^j \partial u^m}$

l, m, j

(2) $R^i_{kmj;l} = \dfrac{\partial^2 \Gamma^i_{kj}}{\partial u^l \partial u^m} - \dfrac{\partial^2 \Gamma^i_{km}}{\partial u^l \partial u^j}$

(3) $$R^i_{kjl;m} = \frac{\partial^2 \Gamma^i_{kl}}{\partial u^m \partial u^j} - \frac{\partial^2 \Gamma^i_{kj}}{\partial u^m \partial u^l}$$

(1) + (2) + (3) = 0

$$\boxed{R^i_{klm;j} + R^i_{kmj;l} + R^i_{kjl;m} = 0}$$

↑

IDENTITÀ DI BIANCHI

$$\boxed{R_{iklm;j} + R_{ikmj;l} + R_{ikjl;m} = 0}$$

$$R_{jk\ell m} = R^{i}_{\ k\ell m} \cdot g_{ij}$$

- SIMMETRIA $\qquad R_{jk\ell m} = R_{\ell m jk}$

- ANTISIMMETRIA $\qquad R_{jk\ell m} = -R_{kj\ell m} = -R_{jkm\ell} = R_{kjm\ell}$

- CICLICITÀ $\qquad R_{jk\ell m} + R_{jmk\ell} + R_{j\ell mk} = 0$

$$R^{i}_{\ kim} \Rightarrow R_{km} \equiv \text{TENSORE DI RANGO 2}$$

TENSORE DI RICCI

$$R^{i}_{\;kli} = -R^{i}_{\;kil} = -R_{kl}$$

RICCI $\equiv R^{i}_{\;kim} \Rightarrow$ PROPRIETÀ

SCALARE DI RICCI

$$\boxed{R \equiv g^{km} R_{km}}$$