$$R^{i}_{klm} = \frac{\partial \Gamma^{i}_{km}}{\partial u^{l}} - \frac{\partial \Gamma^{i}_{kl}}{\partial u^{m}} + \Gamma^{n}_{km}\Gamma^{i}_{nl} - \Gamma^{n}_{kl}\Gamma^{i}_{nm}$$

$$R_{jklm} = g_{ji}\, R^{i}_{klm}$$

(1) SIM $\quad R_{jklm} = R_{lmjk}$

(2) ANTISIM $\quad R_{jklm} = -R_{kjlm} = -R_{jkml} = R_{kjml}$

(3) CICLICITÀ $\quad R_{jklm} + R_{jmkl} + R_{jlmk} = 0$

$$R^i_{\,klm;j} + R^i_{\,kmj;l} + R^i_{\,kjl;m} = 0 \equiv \text{IDENTITÀ BIANCHI}$$

$$R^i_{\,klm} = R^i_{\,kim} = R_{km} \equiv \text{TENSORE DI RICCI}$$

$$R^i_{\,klm} = R^i_{\,kli} = -R^i_{\,kil} = -R_{kl}$$

TENSORE DI RICCI È L'UNICO TENS. DI RANGO 2 CON SIM $\Rightarrow$ $R_{mk} = R^i_{\,mik} = g^{ir} R_{rmik} = g^{ir} R_{ikrm} =$

$$= R^r_{\,krm} = R_{km}$$

$$R = g^{km} R_{km}$$ SCALARE DI RICCI O CURVATURA

È L'UNICO SCALARE CHE SI PUÒ OTTENERE DAL TENSORE DI CURVATURA $\boxed{R^{i}_{\ klm}}$

N DIMENSIONI $\boxed{\mathcal{N} = \dfrac{N^2(N^2-1)}{12}}$

1D $\Rightarrow N = 1 \Rightarrow \mathcal{N} = 0$ $\quad R_{1111} \equiv 0$

2D $\Rightarrow N = 2 \Rightarrow \mathcal{N} = 1$ $\quad R_{1212}$

$$K = \frac{R_{1212}}{g}$$

TENSORE DI RICCI

$$R_{11} = g^{il} R_{i1l1} = g^{11} \underset{=0}{R_{1111}} + g^{12} \underbrace{R_{1121}}_{R_{1121} = -R_{1121} = 0} + g^{21} \underset{=0}{R_{2111}} + g^{22} R_{2121}$$

$$= g^{22} R_{2121} = g^{22} R_{1212}$$

$$R_{12} = R_{21} = g^{il} R_{i1l2} = g^{11} \overset{0}{R_{1112}} + g^{12} \overset{0}{R_{1122}} + g^{21} R_{2112} + g^{22} \overset{0}{R_{2122}}$$

$$= g^{21} R_{2112} = -g^{21} R_{1212}$$

$$R_{22} = g^{il} R_{i2l2} = g^{11} R_{1212} + \underbrace{g^{12} R_{1222}}_{\stackrel{!}{=}0} + \underbrace{g^{21} R_{2212}}_{\stackrel{!}{=}0} + \underbrace{g^{22} R_{2222}}_{\stackrel{!}{=}0}$$

$$\stackrel{!}{=} g^{11} R_{1212}$$

SCALARE DI RICCI

$$R = g^{ik} R_{ik} = g^{11} g^{22} R_{1212} - 2 g^{12} g^{21} R_{1212} + g^{22} g^{11} R_{1212}$$

$$= 2 R_{1212} \left(g^{11} g^{22} - g^{12} g^{21} \right) = 2 R_{1212} \det(g^{ij})$$

$$g^{ij} = \begin{pmatrix} \frac{g_{22}}{g} & -\frac{g_{21}}{g} \\ -\frac{g_{12}}{g} & \frac{g_{11}}{g} \end{pmatrix} \Rightarrow \det(g^{ij}) = \frac{1}{g^2}\left(g_{22}\, g_{11} - g_{12} g_{21} \right)$$

$$= \frac{g}{g^2} = \frac{1}{g} \quad \Rightarrow \quad R = 2\, \frac{R_{1212}}{g}$$

$$\boxed{\frac{R}{2} = \frac{R_{1212}}{g} = K_{GAUSS}}$$

$3D \Rightarrow N = 3$ $\qquad \mathcal{N} = 6$

$$R_{ij} : \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ & R_{22} & R_{23} \\ & & R_{33} \end{pmatrix}$$

$4D \Rightarrow N = 4$ $\qquad \mathcal{N} = 20$

$$R_{km} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ & R_{22} & R_{23} & R_{24} \\ & & R_{33} & R_{34} \\ & & & R_{44} \end{pmatrix}$$

$$R_{iklm\,j\,s} - R_{kimsjl} - R_{ikls\,j\,m} = 0 \qquad g^{il} g^{km}$$

$$R_{js} - R^{l}_{\;sjl} - R^{m}_{\;sjm} = 0 = R_{js} - 2R^{l}_{\;sjl} = 0$$

$$R^{l}_{\;sjl} = \frac{1}{2} R_{js} = \frac{1}{2} \frac{\partial R}{\partial u^{s}} \Rightarrow$$

DIVERGENZA (COVARIANTE) DEL TENSORE DI RICCI

$$\boxed{R^l_{\ j} - \frac{1}{2}\delta^l_j R}$$

$$R^l_{\ j;l} - \underbrace{\frac{1}{2}\delta^l_{j;l}}_{\stackrel{!}{=}0} R - \frac{1}{2}\delta^l_j \frac{\partial R}{\partial u^l}$$

$$\delta^l_{j;l} = \underbrace{\frac{\partial \delta^l_j}{\partial u^l}}_{\downarrow\ 0} + \Gamma^l_{lk}\delta^k_j - \Gamma^m_{jl}\delta^l_m = \Gamma^l_{lj} - \Gamma^m_{jm}$$

$$\Big| \ \underbrace{\Gamma^l_{lj} - \Gamma^l_{jl}}_{=0}$$

$$\Gamma^l_{lj} = \Gamma^l_{jl}$$

$$R^{l}{}_{jjl} - \frac{1}{2}\delta^{l}_{j}\frac{\partial R}{\partial u^{l}} = R^{l}{}_{jjl} - \frac{1}{2}\frac{\partial R}{\partial u^{j}} = 0$$

$$\downarrow$$

$$\frac{1}{2}\frac{\partial R}{\partial u^{j}}$$

$$g_{il}R^{l}{}_{j} - \frac{1}{2}g_{il}\delta^{l}_{j}R = \boxed{R_{ij} - \frac{1}{2}g_{ij}R = G_{ij}}$$

$\uparrow$

TENSORE DI EINSTEIN

$G_{ij} =$ 1 È SIMMETRICO

2 HA DIVERGENZA NULLA

3 È DERIVATO DAL TENSORE DI RIEMANN $\rightarrow$ CONTIENE TERMINI LINEARI NELLE DERIVATE SECONDE E QUADRATICI NELLE DERIVATE PRIME DEL TENSORE METRICO

SPAZIO DI MINKOWSKI

$$ds^2 = c^2 dt^2 - \left(dx^2 + dy^2 + dz^2\right) \equiv$$

È INVARIANTE PER SISTEMA DI RIFERIMENTO

$x^0 = ct$

$x^1 = x$

$x^2 = y$

$x^3 = z$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = \eta_{\alpha\beta}\, dx^\alpha dx^\beta -$$

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$(1, -1, -1, -1)$

$(-1, 1, 1, 1)$

$\alpha, \beta, \gamma, \dots \qquad 0 \dots 3$

$i, j, k \dots \qquad 1 \dots 3$

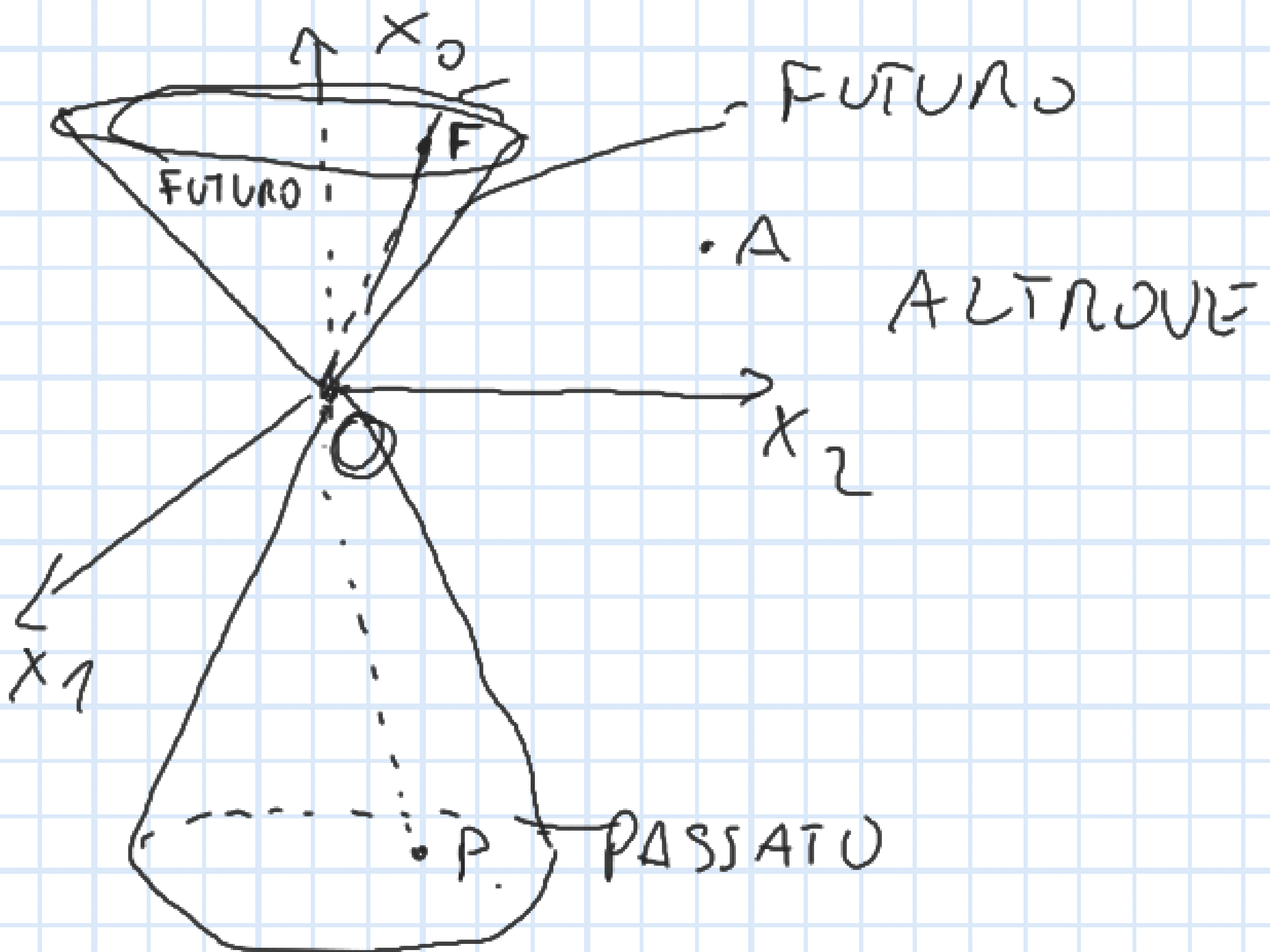
$\eta_{\alpha,\beta} \equiv$ PSEUSO EUCLIDEO SPAZIO PIATTO

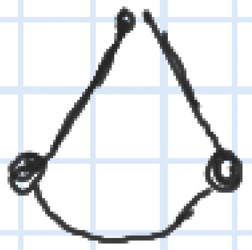
$\eta_{\alpha,\beta} = \text{COST} \Rightarrow \Gamma^{i}_{jk} = 0 \Rightarrow R^{h}_{ijk} = 0 \Rightarrow$

$ds^2 > 0$ = TIME LIKE $v < c$

$ds^2 = 0$ = LIGHT LIKE $v = c$
(NULL)

$ds^2 < 0$ = SPACE LIKE $v > c$





τ

$$dx = dy = dz = 0$$

$$ds^2 = c^2 d\tau^2$$

$d\ell \qquad d\tau$

$$ds^2 = (c\,dt)^2 - (|d\vec{\ell}|)^2 = (c\,d\tau)^2$$

$$d\tau^2 = dt^2\left(1 - \frac{1}{c^2}\,\frac{d\bar{\ell}}{dt}\cdot\frac{d\bar{\ell}}{dt}\right) = dt^2\left(1 - \frac{v^2}{c^2}\right) = \frac{dt^2}{\gamma^2}$$

$$\beta = \frac{v}{c} \quad \Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad dt = \gamma\, d\tau$$

$$(\gamma \geq 1)$$

$$U^\alpha \equiv \left(\frac{dx^\alpha}{ds}\right) \equiv \text{QUADRIVELOCITÀ} \qquad U^\alpha = \gamma\left(\cancel{\gamma}\ \beta_0\ \beta_1\ \beta\right)$$

$$U^0 = \frac{dx^0}{ds} = \frac{d(ct)}{c\,d\tau} = \frac{dt}{d\tau} = \gamma$$

$$U^i = \frac{dx^i}{c\,d\tau} = \frac{1}{c}\frac{dx^i}{dt}\frac{dt}{d\tau} = \gamma\frac{1}{c}v^i = \gamma\beta^i$$

$$U^\alpha = \gamma(1, \bar{\beta}) \qquad U^\alpha = (1, 0, 0, 0)$$

$$U^\alpha \cdot U_\alpha = \eta_{\alpha\beta}U^\alpha U^\beta = U^0U^0 - (U^1U^1 + U^2U^2 + U^3U^3) =$$

$$= \gamma^2 - \left(\gamma^2\frac{v^2}{c^2}\right) = \gamma^2\left(1 - \frac{v^2}{c^2}\right) = \gamma^2(1-\beta^2) = \cancel{\gamma^2}/\gamma^2 = 1$$

$U^\alpha \equiv$ QUADRIVELOCITÀ $\equiv$ VERSORE TANGENTE ALLA TRAIETTORIA

QUADRIMOMENTO $\quad p^\alpha = m_0 U^\alpha \quad m_0 \equiv$ MASSA A RIPOSO

$\vec{P} = m\vec{v} = \gamma m_0 \vec{v}$

$E = mc^2 = m_0 c^2 \gamma$

$$p^0 = \gamma m_0 = \frac{E}{c^2} \qquad p^i = \gamma m_0 \frac{v^i}{c} = m \frac{v^i}{c}$$

$$p^\alpha p_\alpha = \gamma^2 m_0^2 - \gamma^2 m_0^2 \frac{v^2}{c^2} = \gamma^2 m_0^2 \left(1 - \frac{v^2}{c^2}\right) = \gamma^2 m_0^2 \frac{1}{\gamma^2} = m_0^2$$

$$p^\alpha p_\alpha = m_0^2 = \frac{E^2}{c^4} - \frac{1}{c^2}\vec{p}\cdot\vec{p}$$

$$m_0^2 c^2 = \frac{E^2}{c^2} - |\bar{p}|^2$$

$p^\alpha = \text{COST} \implies E \Rightarrow \text{COST} \Rightarrow \vec{p} = \text{COST}$

$p^\alpha = \text{COST} \Rightarrow$ CONSERVAZIONE DELL'ENERGIA E DELLA QUATITÀ DI MOTO