G_{ij} TENSORE DI EINSTEIN

R_{ij} , R , g_{ij}

- SIMMETRICO
- LINEARE NELLE DERIVATE SECONDE DI g_{ij}
- QUADRATICO NELLE " PRIME DI g_{ij}
- DIVERGENZA COVARIANTE NULLA

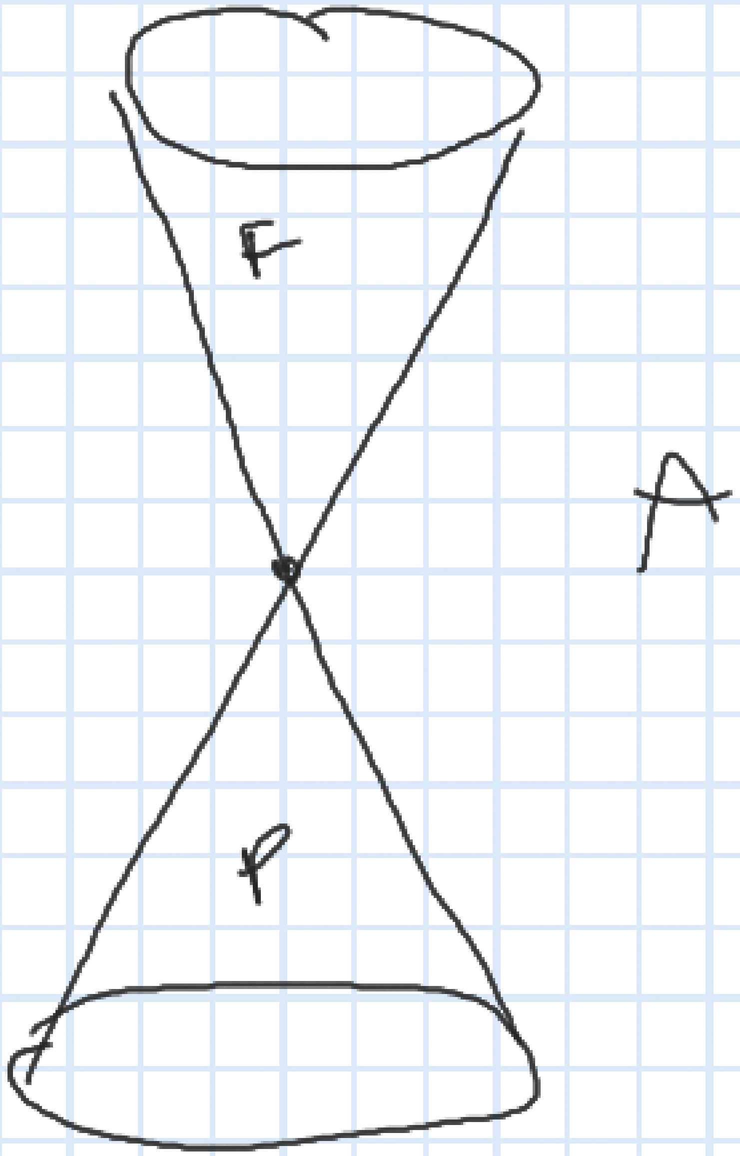
$$ds^2 = c^2 t^2 - (dx^2 + dy^2 + dz^2)$$

$$ds^2 = \eta_{\alpha\beta}\, dx^\alpha dx^\beta \qquad \eta_{\alpha\beta} \to g_{ij}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

PSEUDO EUCLIDEO

$$ds^2 = \begin{cases} \text{TIME LIKE} > 0 \\ \text{SPACE LIKE} < 0 \\ \text{LIGHT LIKE} = 0 \end{cases}$$



$$dx, dy, dz = 0$$

$$d\tau^2 = dt^2/\gamma^2$$

$$\beta = v/c \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$U^\alpha = \frac{dx^\alpha}{ds} \qquad U^\alpha U_\alpha = 1 \qquad \equiv \text{ QUADRIVELOCITÀ}$$

$$P^\alpha = m_0 U^\alpha \qquad \equiv \text{ QUADRIMOMENTO}$$

$P^\alpha = \text{COST} \Rightarrow$ CONSERVAZIONE ENERGIA E DELLA QUANTITÀ DI MOTO

$U^\alpha \Rightarrow$ QUADRIACCELERAZIONE $a^\alpha_c = \dfrac{d^2 x^\alpha}{ds^2} = \dfrac{dU^\alpha}{ds}$

EQ. GEODETICHE

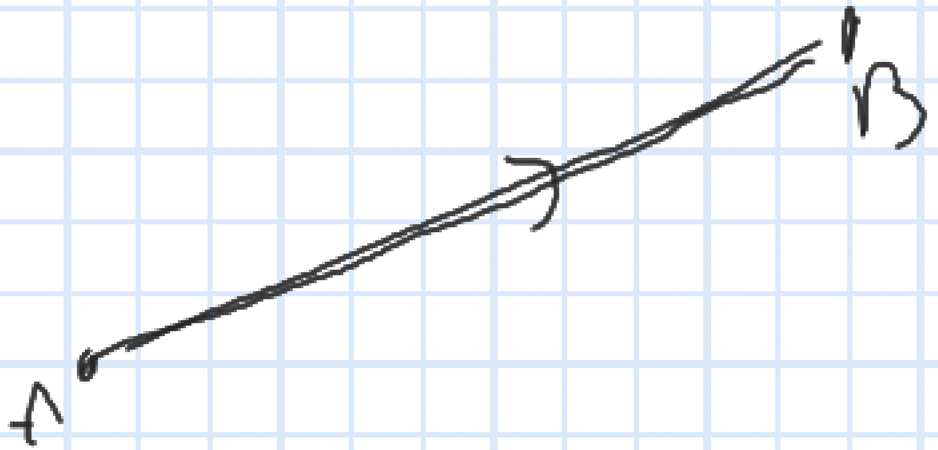
$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds} = 0$$

$$g_{\alpha\beta} \Rightarrow \eta_{\alpha\beta} \Rightarrow \Gamma^\alpha_{\beta\gamma} = 0$$

$$\frac{d^2x^\alpha}{ds^2} = 0 \Rightarrow x^\alpha = a^\alpha \cdot s + b^\alpha$$

$$\begin{cases} ct = a^0 \cdot s + b^0 \\ \bar{x} = \bar{a} \cdot s + \bar{b} \end{cases}$$

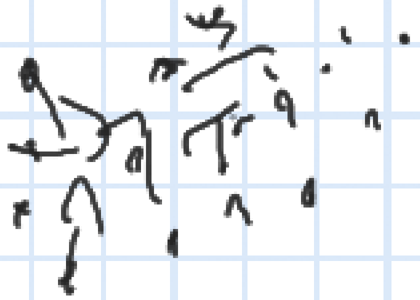
$\Rightarrow$ TRAIETTORIA DI UNA RETTA PERCORSA DA MOTO RETTILINEO UNIFORME

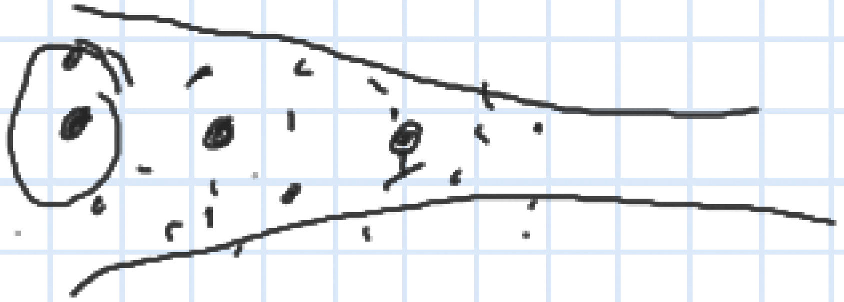


TENSORE ENERGIA - IMPULSO

ρ, v $\Rightarrow$ CONSERVAZIONE DELL'ENERGIA E DELLA QUANTITÀ DI MOTO

("DUST")





$$U^{\alpha} = \gamma\left(1, \bar{v}/c\right) \qquad \rho_0(x)$$

$$\boxed{T^{\alpha\beta} = \rho_0 c^2 U^{\alpha} U^{\beta} = T^{\beta\alpha}} \rightarrow$$ TENSORE DI RANGO 2 SIMMETRICO

$$T^{00} = \rho_0 c^2 \gamma^2 = \rho c^2 \qquad \rho = \gamma^2 \rho_0$$

$$m = \gamma m_0$$

↓

MASSA A RIPOSO

$$\frac{1}{\cancel{\gamma}} \qquad \gamma$$

$$v \Rightarrow \gamma^2$$

$$T^{\alpha\beta} = \rho c^2 \begin{pmatrix} 1 & v_x/c & v_y/c & v_z/c \\ v_x/c & v_x^2/c^2 & v_x v_y/c^2 & v_x v_z/c^2 \\ v_y/c & v_x v_y/c^2 & v_y^2/c^2 & v_y v_z/c^2 \\ v_z/c & v_x v_z/c^2 & v_y v_z/c^2 & v_z^2/c^2 \end{pmatrix}$$

$$\boxed{\partial_\beta T^{\alpha\beta} = 0} \Rightarrow \text{EQ DEL MOTO}$$

$$\boxed{\alpha = 0} \quad \partial_\beta T^{0\beta} = \frac{\partial T^{0,\beta}}{\partial x_\beta} = 0$$

$$\underbrace{\frac{1}{c}\frac{\partial(\rho c^2)}{\partial t} + \frac{\partial(\rho c v_x)}{\partial x} + \frac{\partial(\rho c v_y)}{\partial y} + \frac{\partial(\rho c v_z)}{\partial z}}_{} = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$

EQ. DI CONTINUITÀ PER UN FLUIDO $\Rightarrow$

$\Rightarrow$ CONSERVAZIONE DI MASSA-ENERGIA

$\alpha = 1, 2, 3 \Rightarrow \partial_\beta T^{i\beta} = 0 \Rightarrow \dfrac{\partial T^{i\beta}}{\partial x^\beta} = 0$

$\alpha = 1 \quad 0 = \left(\frac{1}{c} \frac{\partial(\rho c v_x)}{\partial t} + \frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} + \frac{\partial(\rho v_x v_z)}{\partial z} \right) \hat{i}$

$\alpha = 2 \quad 0 = \left[\frac{1}{c} \frac{\partial(\rho c v_y)}{\partial t} + \frac{\partial(\rho v_y v_x)}{\partial x} + \frac{\partial(\rho v_y v_y)}{\partial y} + \frac{\partial(\rho v_y v_z)}{\partial z} \right] \hat{j}$

$\alpha = 3 \quad 0 = \left[\frac{1}{c} \frac{\partial(\rho c v_z)}{\partial t} + \frac{\partial(\rho v_z v_x)}{\partial x} + \frac{\partial(\rho v_z v_y)}{\partial y} + \frac{\partial(\rho v_z v_z)}{\partial z} \right] \hat{k}$

$\alpha = 1 \quad \hat{i}$ VERSORE RELATIVO ALL'ASSE x

$\alpha = 2 \quad \hat{j} \to y \quad \alpha = 3 \quad \hat{k} \to z$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \frac{\partial(\rho v_x \vec{v})}{\partial x} + \frac{\partial(\rho v_y \vec{v})}{\partial y} + \frac{\partial(\rho v_z \vec{v})}{\partial z}$$

$$\rho\frac{\partial\vec{v}}{\partial t} + \vec{v}\frac{\partial\rho}{\partial t} + \rho v_x\frac{\partial\vec{v}}{\partial x} + \vec{v}\frac{\partial(\rho v_x)}{\partial x} + \rho v_y\frac{\partial\vec{v}}{\partial y} + \vec{v}\frac{\partial(\rho v_y)}{\partial y} + \rho v_z\frac{\partial\vec{v}}{\partial z} + \vec{v}\frac{\partial(\rho v_z)}{\partial z}$$

$$* \quad \vec{v}\cdot\vec{\nabla}(\rho\vec{v})$$

$$* \quad \rho(\vec{v}\cdot\vec{\nabla})\vec{v}$$

$$\rho\frac{\partial\vec{v}}{\partial t} + \vec{v}\underbrace{\left[\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v})\right]}_{=0} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} = 0$$

$$\boxed{\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = 0 \quad \mathrm{I} \qquad \Updownarrow \qquad \rho\frac{d\vec{v}}{dt} = 0 \quad \mathrm{II}}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\partial \vec{x}}{\partial t}\frac{\partial}{\partial x}$$

CONSERVAZIONE DELLA QUANTITÀ DI MOTO

I $\Rightarrow$ DESCRIVE COME EVOLVE IL MOTO IN UN PUNTO FISSO DELLO SPAZIO (EULERIANO)

II $\Rightarrow$ DESCRIVE COME VARIANO LE PROPRIETÀ DELLE MEZZO SEGUENDO IL MOTO DELLE PARTICELLE (LAGRANGIAN)

$T^{\alpha\beta}$ ⇒ TUTTE LE PROPRIETÀ ENERGETICHE E DINAMICHE PER UN FLUSSO DI "POLVERE"

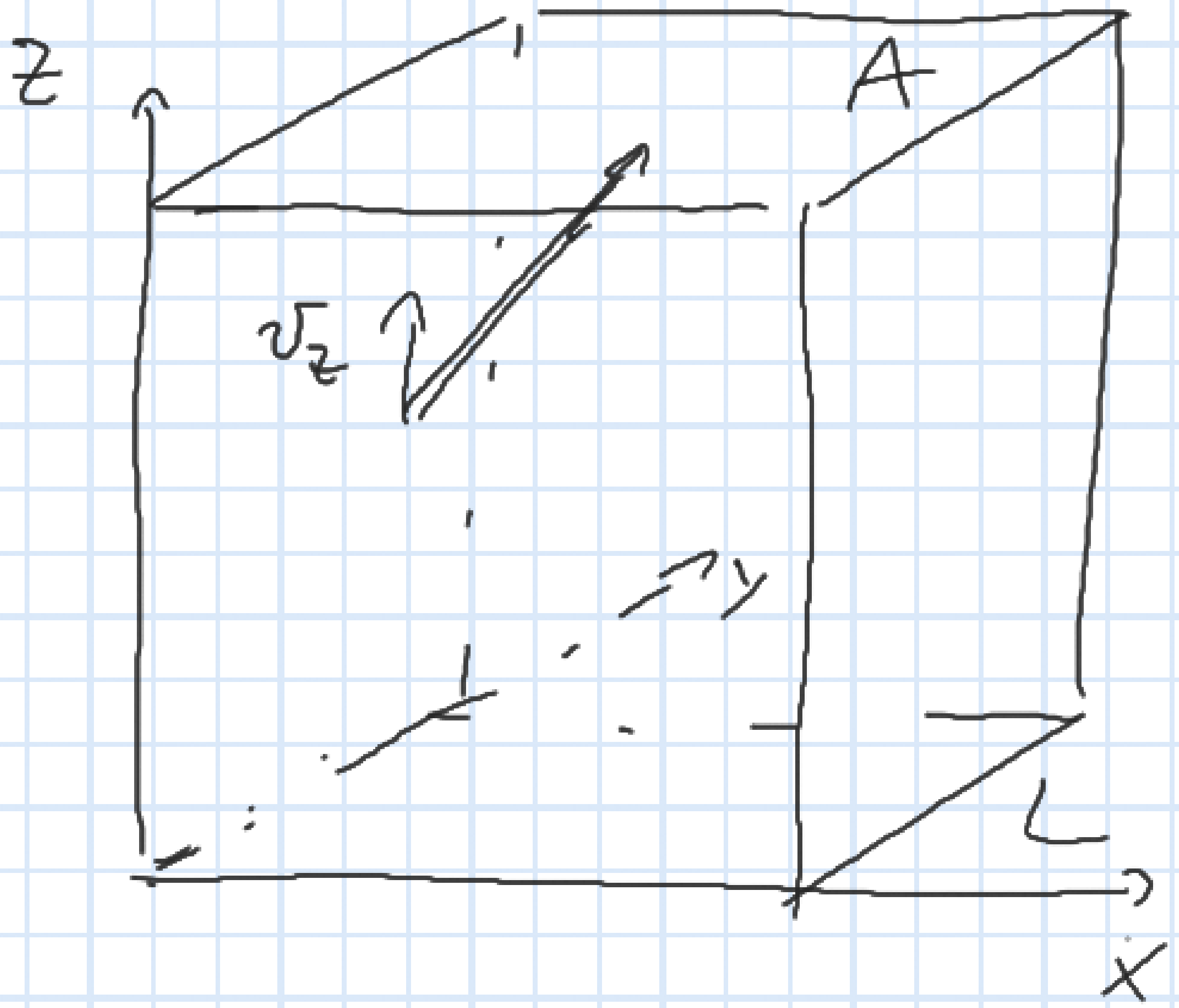
≡ $\boxed{\text{TENSORE ENERGIA - IMPULSO}}$

SILQ ⇒ SISTEMA INERZIALE LOCALMENTE IN QUIETE

$$U^\alpha = (1, 0, 0, 0)$$

$$T^{\alpha\beta}_{SILQ} = \begin{pmatrix} \rho_0 c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ X POLVERE

SILQ $\Rightarrow$ FLUIDO PERFETTO



$\bar{v}$ $\bar{P}$

VEL

QUANTITÀ DI MOTO

$$f_z = \frac{\Delta P}{\Delta t} = \frac{1}{L} P_z v_z$$

PRESSIONE

$$N \Rightarrow F_z = \frac{N}{L} P_z v_z = \frac{N}{L^3} P_z v_z \left(L^2 = p \cdot A \right)$$

$$p = \frac{N}{L^3} \langle P_z v_z \rangle = n \langle P_z v_z \rangle$$

$$\vec{P}\cdot\vec{v} = P_x v_x + P_y v_y + P_z v_z = 3 P_z v_z$$

$$p = \frac{n}{3}\langle \vec{P}\cdot\vec{v}\rangle = \langle n P_x v_x\rangle = \langle n m v_x^2\rangle = \langle \rho\, v_x^2\rangle$$

FLUIDO PERFETTO

$$T^{\alpha\beta}_{SILQ} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

DERIVA DA

$$U^\alpha_{SILQ} \equiv (1,0,0,0)$$

$$\boxed{T^{\alpha\beta}_{SILQ} = (p + \rho c^2)\, U^\alpha U^\beta - p\,\eta^{\alpha\beta}}$$

$$T^{\alpha\beta} = \rho c^2 \begin{pmatrix} 1 & \langle v_x \rangle / c & \langle v_y \rangle / c & \langle v_z \rangle / c \\ & \langle v_x^2 \rangle / c^2 & \langle v_x v_y \rangle / c^2 & \langle v_x v_z \rangle / c^2 \\ & & \langle v_y^2 \rangle / c^2 & \langle v_y v_z \rangle / c^2 \\ & & & \langle v_z^2 \rangle / c^2 \end{pmatrix}$$

$$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$$

$$\langle v_x v_y \rangle = \langle v_x \rangle \cdot \langle v_y \rangle = 0$$

$$T^{\alpha\beta} = \left(p + \rho c^2\right) u^\alpha u^\beta - p\,\eta^{\alpha\beta}$$

$$\boxed{T_{\alpha\beta} = \left(p + \rho c^2\right) u_\alpha u_\beta - p\, g_{\alpha\beta}}$$