$T^{\alpha\beta}$

$$\frac{\partial T^{\alpha\beta}}{\partial x_\beta} = 0$$

FLUIDO PERFETTO

$$T_{\alpha\beta} = \left(P + \rho c^2\right) u_\alpha u_\beta - P\, g_{\alpha\beta}$$

$$\partial_\beta T^{\alpha\beta} \Rightarrow U_\alpha \frac{\partial T^{\alpha\beta}}{\partial x^\beta}$$

$$U_\alpha U^\alpha = 1$$

$$\frac{\partial}{\partial x^\beta}(U^\alpha U_\alpha) = U^\alpha \frac{\partial U_\alpha}{\partial x^\beta} + U_\alpha \frac{\partial U^\alpha}{\partial x^\beta} = \eta^{\alpha\gamma} U_\gamma \frac{\partial U^\alpha}{\partial x^\beta} + U_\alpha \frac{\partial U^\alpha}{\partial x^\beta}$$

$$= U_\gamma \frac{\partial U^\gamma}{\partial x^\beta} + U_\alpha \frac{\partial U^\alpha}{\partial x^\gamma} = 2 U_\alpha \frac{\partial U^\alpha}{\partial x^\beta} = 0 \Rightarrow \frac{\partial U^\alpha}{\partial x^\beta} = 0$$

$$U_\alpha \frac{\partial T^{\alpha\beta}}{\partial x^\beta} = U_\alpha \frac{\partial}{\partial x^\beta}\left[(p+\rho c^2)\,U^\alpha U^\beta\right] - \frac{\partial p}{\partial x^\beta}\eta^{\alpha\beta}U_\alpha = 0$$

$$\Rightarrow \; U_\alpha \left\{ U^\alpha \frac{\partial}{\partial x^\beta}\left[(p+\rho c^2)\,U^\beta\right] + (p+\rho c^2)\,U^\beta \underbrace{\frac{\partial U^\alpha}{\partial x^\beta}\Big\}}_{=0} \right. - \frac{\partial p}{\partial x^\beta} U^\beta = 0, \qquad \underbrace{U_\alpha U^\alpha}_{=1}$$

$$\Rightarrow \; \frac{\partial}{\partial x^\beta}\left[(p+\rho c^2)\,U^\beta\right] - U^\beta \frac{\partial p}{\partial x^\beta} = 0$$

$$(p+\rho c^2)\frac{\partial U^\beta}{\partial x^\beta} + U^\beta \frac{\partial}{\partial x^\beta}(p+\rho c^2) - U^\beta \frac{\partial p}{\partial x^\beta} = 0$$

CONSERVAZIONE DEL NUMERO DI PARTICELLE

$$\frac{\partial(n u^\beta)}{\partial x^\beta} = 0 \Rightarrow n \frac{\partial u^\beta}{\partial x^\beta} + u^\beta \frac{\partial n}{\partial x^\beta} = 0 \Rightarrow \frac{\partial u^\beta}{\partial x^\beta} = -\frac{u^\beta}{n} \frac{\partial n}{\partial x^\beta}$$

$$u^\beta \left\{ -\frac{(p + \rho c^2)}{n} \frac{\partial n}{\partial x^\beta} + \frac{\partial (p + \rho c^2)}{\partial x^\beta} - \frac{\partial p}{\partial x^\beta} \right. = 0$$

$$\frac{\partial}{\partial x^\beta}\left(\frac{p+\rho c^2}{n}\right) = \frac{1}{n^2}\left[\frac{\partial(p+\rho c^2)}{\partial x^\beta}\, n - (p+\rho c^2)\frac{\partial n}{\partial x^\beta}\right]$$

$$\partial \frac{1}{n} = -\frac{1}{n^2}\,\partial n$$

$$= \frac{1}{n}\left[\frac{\partial(p+\rho c^2)}{\partial x^\beta} - \frac{p+\rho c^2}{n}\frac{\partial n}{\partial x^\beta}\right]$$

SOSTITUENDOLO NELL' EQ PREC

$$u^\beta \left\{ n \frac{\partial}{\partial x^\beta} \left(\frac{p + \rho c^2}{n} \right) - \frac{\partial p}{\partial x^\beta} \right\} = 0 \quad \Rightarrow$$

$$u^\beta \left\{ n \left[\frac{\partial}{\partial x^\beta} \left(\frac{p}{n} \right) + \frac{\partial}{\partial x^\beta} \left(\frac{\rho c^2}{n} \right) \right] - \frac{\partial p}{\partial x^\beta} \right\} = 0$$

$$\stackrel{!}{=} u^\beta \left\{ n p \frac{\partial}{\partial x^\beta} \left(\frac{1}{n} \right) + \cancel{\frac{n}{n}} \cancel{\frac{\partial p}{\partial x^\beta}} + n \frac{\partial}{\partial x^\beta} \left(\frac{\rho c^2}{n} \right) - \cancel{\frac{\partial p}{\partial x^\beta}} \right\} = 0$$

$$= n u^\beta \left\{ p \frac{\partial}{\partial x^\beta} \left(\frac{1}{n} \right) + \frac{\partial}{\partial x^\beta} \left(\rho c^2 / n \right) \right\} = 0$$

$$dU = dQ + dL \qquad dQ = TdS \qquad dL = -pdV$$

I PRINCIPIO TERMODINAMICA

$$U = uV \qquad u = \rho c^2$$

$$TdS = dU + pdV$$

$\Rightarrow$ CI RIFERIAMO A UNA PARTICELLA

$$n = \frac{\#}{V} \qquad V = \frac{1}{n}$$

$$dU = d(\rho c^2 V)$$

$$\stackrel{!}{=} d\left(\frac{\rho c^2}{n}\right)$$

$$Td\sigma = d\left(\rho c^2/n\right) + pd\left(\frac{1}{n}\right) \Rightarrow$$

$$T \frac{\partial \sigma}{\partial x^\beta} dx^\beta = \frac{\partial}{\partial x^\beta}\left(\frac{\rho c^2}{n}\right) dx^\beta + p \frac{\partial}{\partial x^\beta}\left(\frac{1}{n}\right) dx^\beta \quad / ds$$

$$u^\beta = \frac{dx^\beta}{ds}$$

$$T \frac{\partial \sigma}{\partial x^\beta} u^\beta = u^\beta \left\{ \frac{\partial}{\partial x^\beta}\left(\frac{\rho c^2}{n}\right) + p \frac{\partial}{\partial x^\beta}\left(\frac{1}{n}\right) \right\} = 0$$

$$T \frac{\partial \sigma}{\partial x^\beta} u^\beta = 0 \quad \Rightarrow \quad u^\beta \frac{\partial \sigma}{\partial x^\beta} = 0$$

$$\gamma \frac{1}{c} \frac{\partial \sigma}{\partial t} + \gamma \frac{v_x}{c} \frac{\partial \sigma}{\partial x} + \gamma \frac{v_y}{c} \frac{\partial \sigma}{\partial y} + \gamma \frac{v_z}{c} \frac{\partial \sigma}{\partial z} = 0$$

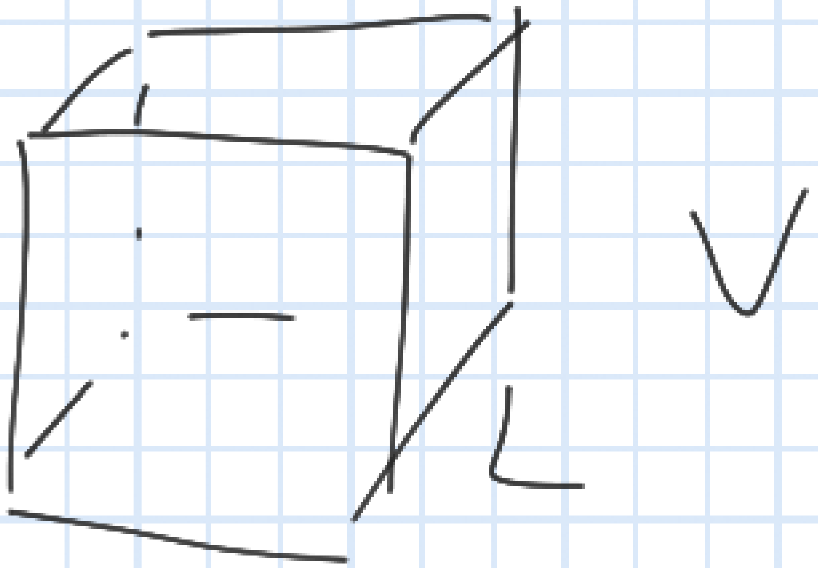
$$\frac{\partial \sigma}{\partial t} + (\bar{v} \cdot \bar{\nabla})\sigma = 0 \iff \frac{d\sigma}{dt} = 0$$

I

II

EULERIANO

LAGRAGIANO



$$dQ = 0 \Rightarrow dS = 0$$

$$dQ = dU + p\,dV \qquad U = \rho c^2 V$$

$$dQ = \rho c^2 dV + V d(\rho c^2) + p\,dV = 0$$

$$p = w \rho c^2 \qquad w = \text{cost} \qquad w = w(T)$$

$$(1+w)\rho c^2 dV = -V d(\rho c^2)$$

se $w = \text{cost}$

$$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow$$

$$d \ln \rho = -(1+w)\, d \ln V \Rightarrow \rho = K V^{-(1+w)}$$

$$\Rightarrow \rho V^{1+w} = \text{cost}$$

(1) PER GAS NON RELATIVISTICO $p \ll \rho_0 c^2$

$\Rightarrow w = 0$ $\rho = \rho_0$

$$\rho_0 V = \text{cost} \qquad V \propto L^3$$

$$\rho \propto 1/L^3$$

(2) GAS DI FOTONI

$v = c$

$$\rho_{rad} \propto T^4$$

$$p = \langle \rho V_x^2 \rangle = \frac{1}{3}\langle \rho \bar{v}^2 \rangle = \frac{1}{3}\langle \rho c^2 \rangle$$

$$p = \frac{1}{3}\rho c^2 \Rightarrow w = \frac{1}{3}$$

$$\rho V^{(1+w)} = \text{COST} \propto T^4 V^{4/3} \Rightarrow T V^{1/3} = \text{COST}$$

$$V \propto L^3 \Rightarrow T \propto \frac{1}{L}$$

$$\rho_{rad} V^{4/3} = \text{COST} \Rightarrow V^{4/3} = L^4 \Rightarrow \rho_{rad} \propto 1/L^4$$

(3) $p = -\rho c^2 \quad (w = -1) \qquad \rho V^0 = \text{COST}$

$\rho \neq f(V, L)$

$$dQ = \rho c^2 dV + V d(\rho c^2) + p dV = 0$$

$$dV(\rho c^2 + p) + V dp = 0 \Rightarrow \left(\rho + \frac{p}{c^2}\right) dV + V d\rho = 0$$

$$V = L^3 \qquad dV = 3L^2 dL$$

$$\left(\rho + \frac{p}{c^2}\right) 3L^2 dL + L^3 d\rho = 0 \quad / L^3 \Rightarrow$$

$$3\left(\rho + \frac{p}{c^2}\right)\frac{dL}{L} + d\rho = 0 \qquad \text{se } L(t)$$

$$\boxed{3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{L}}{L} + \dot{\rho} = 0}$$

$$\partial_\alpha T^{\beta\alpha} = 0 \qquad \Gamma^\alpha_{\beta\gamma} = 0$$

1. V POSTULATO EUCLIDE
2. TENSORE METRICO
3. TENSORI
4. CONCETTO DI CURVATURA $g_{i,s}$

$\Rightarrow \Gamma^{i}_{jk}$

5. DERIVATA NORMALE $\to$ DERIVATA COV

(6) TRASPORTO PARALLELO

$R^{i}{}_{jke}$

(7) R_{ij}, R

(8) G_{ij}

$\updownarrow$

(9) $T_{\alpha\beta}$