Testi del Syllabus

Resp. Did.	**CUCCAGNA SCIPIO**	**Matricola: 015277**

Docente	**CUCCAGNA SCIPIO, 6 CFU**

Anno offerta:	**2021/2022**
Insegnamento:	**525SM - TOPICS IN ADVANCED ANALYSIS 2**
Corso di studio:	**SM34 - MATEMATICA**
Anno regolamento:	**2020**
CFU:	**6**
Settore:	**MAT/05**
Tipo Attività:	**C - Affine/Integrativa**
Anno corso:	**2**
Periodo:	**Primo Semestre**
Sede:	**TRIESTE**

Testi in italiano

Lingua insegnamento	English
Contenuti (Dipl.Sup.)	The course is focused on the incompressible Navier Stokes equation in the Euclidean space in dimensions 2 and especially 3. For 2 weeks the course will be run as guest speaker by prof. Joachim Krieger from EPFL Lausanne who willspeak about Spectral Methods for Nonlinear Stability Problems. 1. Some basics of Harmonic and Functional Analysis: Berstein inequalities. , the homogenous Sobolev spaces, boundedness properties of the Hardy Littlewood Maximal Function (in particular the Marcinkiewicz interpolation theorem), the Hardy Littlewood Sobolev fractional integration theorem, Sobolev's embedding theorem for homogeneous Sobolev spaces and, time permitting, Calderon Zygmund kernels and their corresponding boundedness theory in L^p spaces. 2. Definition of weak solutions of the Navier Stokes (NS) equation in Euclidean spaces 2 and 3. Notion of Leray Hopf solutions. Proof of the existence of global existence of Leray Hopf solutions. Strong solutions and proof tha in dim 2 weak solutions are strong. 3. Various fixed point theorems about the existence of mild solutions in dimension 3 and proof of the regolarity of such solutions. 4. Serrin's regularity criterion. 5. Notion of suitable solution and two local regularity criteria by Caffarelli, Kohn and Nirenberg.
Testi di riferimento	Along with some instructor's notes, we will use the following bibliography 1) Bahouri, Chemin, Danchin: Fourier analysis and nonlinear partial differential equations. Springer 2) Cazenave, Haraux: An introduction to semilinear evolution equations. Oxford Univ.Press. 3) Chemin, Desjardins, Gallagher, Grenier: Mathematical Geophisics. Oxford Univ.Press. 5) Robinson, Rodrigo, Sadowski: The three dimensional Navier Stokes Equations,

	Cambridge Univ. Press. 6) Stein: Singular Integrals and Differentiability Properties of Functions. Princeton University Press. 7) Stein: analysis: real-variable methods, orthogonality, and oscillatory integrals. Princeton University Press.
Obiettivi formativi	The purpose of the course is to introduce the students to the rigorous analytic theory of the incompressible Navier Stokes Equation in Euclidean space and to show the deep differences between the notion of PDE and of ODE. PDE's are intrinsically more complex and in modern Mathematical Analysis are understood and treated through the framework of Functional Analysis. In particular, students of Mathematics will see in this course various application of ideas of Functional Analysis, like the notion of weak topology, that can be fully appreciated only when applied to concrete problems. The course assumes some background in the theory of Banach spaces, Fourier transform, distributions, tempered distributions, which are taught in the master course Advanced Analysis by professors Zagatti and Del Santo. Interested students who don't have such a mathematical background shouldn't be discouraged to attend the course, because to get a sense of what will be taught in the course it is not necessary to have rigorous understanding but rather an intuitive grasp of such background that the lecturer can provide to these students upon request. However, to fully benefit from this course, especially in view of an autonomous elaboration of the mathematical formalism of the NS or of other equations in their future research, it is necessary that the students fill the background gaps.
Prerequisiti	Functional analysis, specifically Sobolev spaces and, broadly speaking, the topics of the 1st year courses ADVANCED ANALYSIS parts A e B.
Metodi didattici	The course consists of lectures during which the Instructor discusses all the details of the topics covered, answers student's questions and tries to get them involved. The students will receive before the lectures the lecture notes of the Instructor.
Altre informazioni	The lecture notes and other information will be available through Moodle
Modalità di verifica dell'apprendimento	The exam consists of a student seminar of about 30 minutes on a topic arranged with the Instructor, during which the student will show whether or not is able to apply the main ideas presented during the lectures by the Instructor in specific and analogous contexts. The Instructor might ask some questions on the topics covered during the course in class.
Programma esteso	.

Testi in inglese

	English
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