

Pre-Course - Computer Programming
DSSC - 2021/2022

Unit 4

Ex. 1

Write a function that takes an integer number n and a natural number m and returns n^m .

Ex. 2

Write a function that takes two natural numbers n and m and returns $\lfloor \sqrt[m]{n} \rfloor$.

Ex. 3

The **Fibonacci sequence** is sequence of natural numbers F_0, F_1, \dots such that $F_0 = 1, F_1 = 1$ and $F_{m+2} = F_{m+1} + F_m$ for all $m \in \mathbb{N}$. Write a function that takes a natural number n and returns F_n .

Ex. 4

The **roots of a function** $f : \text{Dom} \mapsto \text{Cod}$ are all the elements $x \in \text{Dom}$ such that $f(x) = 0$.

The **bisection method** is an algorithm that takes as input

1. an interval $[a, b]$ over the reals
2. a function $f : \mathbb{R} \mapsto \mathbb{R}$, continuous in $[a, b]$, and such that $f(a) * f(b) < 0$
3. a number $\delta \in \mathbb{R}$ greater than 0

and returns a δ -approximated root of f (i.e., if the algorithm returns y , then $|f(y)| < \delta$).

The algorithm performs the following steps. First of all, it computes the mean point m in the interval $[a, b]$. If $|f(m)| < \delta$, then the algorithm returns m . Otherwise, it focus either on the interval $[a, m]$, if $f(a)$ and $f(m)$ have different signs, or on $[m, b]$, in the other way round. The algorithm keeps repeating all the process on the selected interval.

Write a function that takes a floating point number δ and computes a δ -approximated root of $f(x) = 2x^3 - 4x + 1$ in the interval $[0, 1]$.

Ex. 5

Write a function to compute the square root of any *double* by using the bisection method.