The Tully-Fisher and Faber-Jackson Relations

There are three important properties for galaxies: radius R, luminosity L, and mass M, which are connected to "observables" with:

$$\theta = R/d \tag{1}$$

$$f = L/(4\pi d^2) \tag{2}$$

$$v^2 = GM/R \tag{3}$$

where θ is the angular diameter, d is the distance galaxy-observer, and the third relation is the virial theorem. The surface brightness I is independent of distance and is given by:

$$I = f/\theta^2 = Lv^4/(4\pi G^2 M^2) \tag{4}$$

and then

$$L = v^4/I \times 1/[4\pi G^2 (M/L)^2] \tag{5}$$

where M/L is the mass-to-light ratio. If **we can assume** that, for a give class of galaxies, the central surface brightness I and the M/L are constant, then

$$L \propto v^4 \tag{6}$$

which is the basis of **the most useful distance indicators in cosmology**.

For galaxy disks, where the rotation velocity is measured from HI 21cm line, the observed relation "the Tully-Fisher relation", is:

$$L \propto V_{rot}^{3-4}, \tag{7}$$

where the precise exponent depend on the band of observations, e.g L_B or L_R...

For elliptical parameters, the appropriate velocity is the central velocity dispersion which can be measured from the size of the absorption lines in the stellar spectrum. The "the Faber-Jackson relation" is observed:

$$L \propto \sigma_0^4. \tag{8}$$

However, this relation is very scattered and there is the need to introduce another parameter.

The Fundamental Plane

The observational quantities are: effective radius r_e, luminosity L, mean surface brightness $< I >_e$ (called also $< \mu_e >$), velocity dispersion σ_V.

From the observational point of view, $< I_e >$ correlates with L and σ_V correlates with r_e, but a much better correlation is obtained combining σ_V, $< I_e >$, and r_e, i.e.

The origin of the fundamental plane is the following. Assume that ellipticals form a "homologous" family, e.g. dispersion σ_V; note that L, $< I >_e$, $< \mu_e >$ are not all independent since they are connected by the typical I profile of elliptical galaxies (de Vaucouleurs law):

$$L = c_1 I_e r_e^2, \tag{9}$$

Moreover, from the virial theorem

$$M = c_2 \frac{\sigma_0^2 r_e}{G}. \tag{10}$$

Combining them, we obtain

$$r_e = c_2/c_1 \times (M/L)^{-1} \sigma_0^2 I_e^{-1}. \tag{11}$$

If one has $M/L \propto L^{\sim 0.2}$, one recovers the observed fundamental plane. Alternatively, one could have constant M/L with a structure which varies relative to one of more of the fundamental variables. This study of this topic is ongoing.