21 sett

Teor $\forall n \in \mathbb{N}$ e $\forall a \geq -1$ si ha

$$(1+a)^n \geq 1+na \qquad P_n$$

Dim Ieri abbiamo visto che P_1 è vera.

Supponiamo sia vero che per un certo n si abbia $(1+a)^n \geq 1+na$

Dobbiamo dimostrare che è vera con $n+1$

$P_n \quad (1+a)^n \geq 1 + na \qquad \cdot \underbrace{(1+a)}_{\geq 0}$

$a \geq -1 \iff 1+a \geq 0$

$$(1+a)(1+a)^n \geq (1+na)(1+a)$$

$$(1+a)^{n+1} \geq 1 + a + na + na^2 = 1 + (n+1)a + \overset{\geq 0}{na^2}$$

$$\geq 1 + (n+1)a$$

$P_{n+1} \quad (1+a)^{n+1} \geq 1 + (n+1)a$

P_n vera $\Rightarrow P_{n+1}$ vera

P_n è vera $\quad \forall\, n \in \mathbb{N}$

$$(1+a)^n \geq 1+na \qquad \forall\, n \in \mathbb{N} \text{ e } \forall\, a \geq -1.$$

Supponiamo di avere $a_1, \ldots, a_n$ (n numeri). Scriveremo

$$\sum_{j=1}^{n} a_j = a_1 + \cdots + a_n$$

$$\sum_{j=1}^{1} a_j = a_1 \qquad \prod_{j=1}^{n} a_j = a_1 \cdots a_n$$

$$\sum_{j=1}^{0} a_j = 0$$

$$\sum_{j=1}^{n} a_j = \sum_{k=1}^{n} a_k$$

$$\left.\begin{array}{l} a_1, \ldots, a_n \\ b_1, \ldots, b_n \end{array}\right\} \Rightarrow a_1+b_1,\ a_2+b_2, \ldots, a_n+b_n$$

$$\sum_{j=1}^{n} (a_j+b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} \lambda a_j = \lambda \sum_{j=1}^{n} a_j$$

<u>Teor</u> $\forall\, n \in \mathbb{N}$ si ha $\displaystyle\sum_{j=1}^{n} j = \frac{(n+1)\,n}{2}$. (P_n)

<u>Dim</u>

1) Per $n = 1$ $\displaystyle\sum_{j=1}^{1} j = j\big|_{j=1} = 1$

$$\frac{(1+1)\,1}{2} = \frac{2}{2} = 1$$

$$\sum_{j=1}^{n} j = 1 + 2 + 3 + \cdots\cdots + n-2 + n-1 + n$$

n pai

$$= [n+1] + [(n-1)+2] + [(n-2)+3] + \cdots$$

$$= \underbrace{[n+1] + [n+1] + [n+1] + \cdots + [n+1]}_{\frac{n}{2}}$$

$$= \frac{n}{2}(n+1)$$

<u>Teor</u> (somma geometrica di ragione x) $x \neq 1$

$\forall\, x \in \mathbb{R}$, $x \neq 1$ e $\forall\, n \in \mathbb{N} \cup \{0\}$ si ha

$$\sum_{j=0}^{n} x^j = \frac{1-x^{n+1}}{1-x}$$

dove $x^0 = 1 \quad \forall\, x \in \mathbb{R} \quad (0^0 = 1)$

Es. per $x=0$

$$\frac{1-x^{n+1}}{1-x}\bigg|_{x=0} = \frac{1-0}{1-0} = 1$$

$$\sum_{j=0}^{n} 0^j = 0^0 + \sum_{j=1}^{n} \underbrace{0^j}_{0} = 0^0 = 1$$

$$\sum_{j=0}^{n} x^j = \frac{1-x^{n+1}}{1-x}$$

1) $n=0$

$$\sum_{j=0}^{0} x^j = x^0 = 1$$

$$\frac{1-x^1}{1-x} = \frac{\cancel{1-x}}{\cancel{1-x}} = 1$$

$$x \neq 1 \Longleftrightarrow 1-x \neq 0$$

2) $n \Longrightarrow n+1$

$$\sum_{j=0}^{n} x^j = \frac{1-x^{n+1}}{1-x}$$

$$\sum_{j=0}^{n+1} x^j = \sum_{j=0}^{n} x^j + x^{n+1} = \frac{1-x^{n+1}}{1-x} + x^{n+1}\frac{(1-x)}{1-x}$$

$$= \frac{1-x^{n+1} + x^{n+1}(1-x)}{1-x} = \frac{1-\cancel{x^{n+1}}+\cancel{x^{n+1}}-x^{n+2}}{1-x}$$

$$= \frac{1-x^{n+2}}{1-x}$$

$$\sum_{j=0}^{n} x^j = \frac{1-x^{n+1}}{1-x} \qquad \cdot (1-x)$$

$$(1-x)\sum_{j=0}^{n} x^j = 1-x^{n+1}$$

$$= (1-x)\left(1+x+x^2+x^3+\dots+x^{n-2}+x^{n-1}+x^n\right)$$

$$= (1-x)+(1-x)x+(1-x)x^2+\dots+(1-x)x^{n-1}+(1-x)x^n$$

$$= 1-\cancel{x}+\cancel{x}-\cancel{x^2}+\cancel{x^2}-\cancel{x^3}+\dots+\cancel{x^{n-1}}-\cancel{x^n}+\cancel{x^n}-x^{n+1}$$

$$= 1-x^{n+1}$$

2) $n \Rightarrow n+1$

Dobbiamo dimostrare $\displaystyle\sum_{j=1}^{n+1} j = \frac{(n+2)(n+1)}{2}$

assumendo che ho $\displaystyle\sum_{j=1}^{n} j = \frac{(n+1)\,n}{2}$ è vera

$$\sum_{j=1}^{n+1} j = \sum_{j=1}^{n} j + n+1 = \frac{(n+1)\,n}{2} + n+1 = \frac{(n+1)\,n+2(n+1)}{2}$$

$$= \frac{(n+1)\,(n+2)}{2}$$