23 Settembre

Coefficienti binomiali e formula del binomio

Dato $n \in \mathbb{N} \cup \{0\}$ poniamo

$0! = 1$, se $n \geq 1$

$$n! = 1 \cdots n$$

n fattoriale

Siano $0 \le k \le n$ due interi definiamo

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{n-k}$$

$1! = 1$, $2! = 2$ $3! = 6$ $4! = 3!\,4 = 24$

$$n! = \underbrace{1 \cdot \ldots \cdot (n-1)}_{(n-1)!}\, n = (n-1)!\, n$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \binom{n}{n-k}$$

$$\binom{n}{0} = 1 = \binom{n}{n}$$

$$\binom{n}{0} = \frac{n!}{\underset{\overset{\shortparallel}{1}}{0!}\;\underbrace{(n-0)!}_{n!}} = \frac{\cancel{n!}}{\cancel{n!}} = 1$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \binom{n}{n-k}$$

$$\binom{n}{1} = n = \binom{n}{n-1}$$

$$\binom{n}{1} = \frac{n!}{\underbrace{1!}_{1}\;(n-1)!} = \frac{\cancel{(n-1)!}\;n}{\cancel{(n-1)!}} = n$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{\overbrace{1 \cdot 2 \cdot \ldots \cdot (n-k)}^{\cancel{(n-k)!}}\,(n-k+1)\ldots n}{k!\,\cancel{(n-k)!}}$$

$$= \frac{(n-k+1)(n-k+2)\ldots n}{k!}$$

Questo ci fornisce un'altra formula per il coeff. binomiale

$$\binom{n}{k} = \frac{(n-k+1)\ldots n}{k!} = \frac{(n-(k-1))\ldots n}{k!}$$

Lemma $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ ✓

Dim $\binom{n-1}{k-1} + \binom{n-1}{k} =$

$$= \frac{(n-1)!}{(k-1)!\,(n-k)!} + \frac{(n-1)!}{k!\,(n-1-k)!} =$$

$$= \frac{(n-1)!}{(k-1)!\,(n-1-k)!} \left[\frac{1}{n-k} + \frac{1}{k}\right] = \frac{(n-1)!}{(k-1)!\,(n-1-k)!} \cdot \frac{\cancel{k}+n-\cancel{k}}{k\cdot(n-k)} = \frac{n!}{k!\,(n-k)!}$$

Teor (Formula di Newton)

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$(a+b)^0 = 1$ ✓

$\binom{n}{0} = 1$

$$\sum_{k=0}^{0} \binom{0}{k} a^k b^{0-k} = \underbrace{\binom{0}{0}}_{1} a^0 b^0 = 1$$ ✓

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} \qquad 1 = \binom{n}{0} = \binom{n}{n}$$

$$(a+b)^1 = a+b$$

$$\sum_{k=0}^{1} \binom{1}{k} a^k b^{1-k} = \binom{1}{0} b + \binom{1}{1} a = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sum_{k=0}^{2} \binom{2}{k} a^k b^{2-k} = \binom{2}{0} b^2 + \binom{2}{1} ab + \binom{2}{2} a^2 = b^2 + 2ab + a^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\sum_{k=0}^{3} \binom{3}{k} a^k b^{3-k} = \binom{3}{0} b^3 + \binom{3}{1} a b^2 + \binom{3}{2} a^2 b + \binom{3}{3} a^3$$

$$= b^3 + 3ab^2 + 3a^2 b + a^3$$

$$\binom{3}{2} = \binom{3}{3-1} = \binom{3}{1}$$

$$\binom{n}{n-k} = \binom{n}{k}$$

$$(a+b)^{n-1} = \sum_{k=0}^{n-1} \binom{n-1}{k} a^k \, b^{n-1-k} \qquad j = k+1$$

$$(a+b)^n = (a+b)(a+b)^{n-1} = (a+b) \sum_{k=0}^{n-1} \binom{n-1}{k} a^k \, b^{n-1-k}$$

$$= \sum_{k=0}^{n-1} \binom{n-1}{k} a^{k+1} b^{n-1-k} + \sum_{k=0}^{n-1} \binom{n-1}{k} a^k \, b^{n-k}$$

$$= \sum_{k=1}^{n} \binom{n-1}{k-1} a^k \, b^{n-k} + \sum_{k=0}^{n-1} \binom{n-1}{k} a^k \, b^{n-k}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \binom{n-1}{k-1} a^k b^{n-k} + \sum_{k=1}^{n-1} \binom{n-1}{k} a^k \, b^{n-k}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \binom{n-1}{k-1} a^k b^{n-k} + \sum_{k=1}^{n-1} \binom{n-1}{k} a^k b^{n-k}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \left(\binom{n-1}{k-1} + \binom{n-1}{k} \right) a^k b^{n-k}$$

$$= a^n + b^n + \sum_{k=1}^{n-1} \binom{n}{k} a^k b^{n-k} =$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

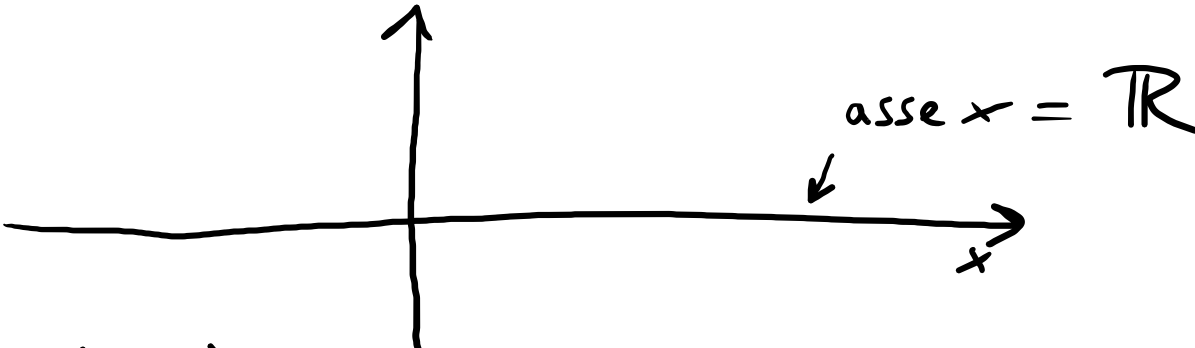
$$10 = 4 + 6$$

$$\boxed{\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}}$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Numeri complessi $\quad \mathbb{N}, \mathbb{Z}, \mathbb{R}$

$$\mathbb{C} = \mathbb{R}^2 = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$



$$(x,y) + (u,v) := (x+u, y+v)$$

$$(x,y) \cdot (u,v) := (xu - yv, xv + yu)$$

$$(x,y)\cdot(u,v)=(xu-yv,\ xv+yu)$$

$$(x,0)+(u,0)=(x+u,\ 0)$$

$$(x,0)\cdot(u,0)=(xu,\ 0)$$

$x\in\mathbb{R} \longrightarrow (x,0)$ sull'asse delle x

questa corrispondenza conserva somma e prodotto

$x \to (x, 0)$, $u \to (u, 0)$

$x + u \to (x+u, 0) = (x, 0) + (u, 0)$

$x \cdot u \to (xu, 0) = (x, 0) \cdot (u, 0)$